IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 6 Ver. III (Nov. - Dec. 2015), PP 01-12
www.iosrjournals.org

Stochastic Analysis and Simulation Studies of Time to Hospitalization and Hospitalization Time with Prophylactic Treatment for Diabetic Patient with Two Failing Organs

Rajkumar.A[1], Gajivaradhan.P[2],Ramanarayanan.R[3]
[1]*Assistant Professorof Mathematics,SriRam Engineering College, Veppampattu, Thiruvallur, 602024, India.*
[2]*Principal & Head Department of Mathematics, Pachaiyappas College, Chennai, 600030, India.*
[3]*Professor of Mathematics Retired, VelTech University, Chennai, India.*

Abstract:*Diabetic mellitus is a chronic disease of pancreatic origin which is not fully curable once a person becomes diabetic.This paper assumes that one organ A of a diabetic person is exposed to organ failure due to a two phase risk process and another organ B has a random failure time. Two models are treated. In model I, his hospitalization for diabetes starts when any one of the organsA or B fails or when prophylactic treatment starts after an exponential time.In model II, his hospitalization for diabetes starts when the two organs A and B are in failed state or when prophylactic treatment starts after an exponential time. The expected time to hospitalization for treatment and expected treatment time are obtained for numericalStudies. Simulation studies are under taken using linear congruential generator and minimum value methods. EC distribution is considered for the general distribution of life time of organ B since it is the best approximation for a general distribution and Erlang distributions are considered for hospital stage treatment times.Random values of all the variables are generated to present simulated values of time to hospitalization and hospitalization times for various parameter values of time to prophylactic treatment.*

Keywords: *Diabetic mellitus, Prophylactic treatment, PH phase2 distribution, Erlang-Coxian2 distribution, Simulation study, Linearcongruential generator.*

I. Introduction

In Medical Science studies and real life situations, prevention of the disease is given very much importance as that would prevent loss of life. Several issues on risk factors ofdiabetes mellitus have been analyzed byBhattacharya S.K., Biswas R., Ghosh M.M., and Banerjee in [1].Foster D.W., Fauci A.S., Braunward E., Isselbacher K.J., Wilson J.S., Mortin J.B and Kasper D.L have studied diabetes mellitus in [2].Kannell W.B and McGee D.L[3] have analyzed Diabetes and Cardiovascular Risk Factors.King H, Aubert R.E and Herman W.H in [4] have concentrated on the global burden of diabetes during the period 1995-2025.King H and Rewers M [5] have presented global estimates for prevalence of diabetes mellitus and impaired glucose tolerance in adults. Mathematical studies have been presented by Usha and Eswaripremin [6] where they have focused their discussions onthe models with metabolic disorder. Eswariprem, Ramanarayanan and Usha [7] have analyzed such models with prophylactic treatment for thedisease. Mathematical models play a great and distinctive role in this area.Any study with prophylactic treatment will be very beneficial to the society. Moreover cure from thedisease after treatment is time consuming and above all seldom achieved in many cases. This paperconcentrates on situations of prophylactic treatment for the disease when one organ A of a person is exposed tofailure due to a two phase failure process and another organ B fails after a random time. Recent advancements in Probability, Operations Research and Simulation methods are utilized for the presentation of the results.Analyzing real life stochastic models researchers do collect data directly from the source/ hospitals (primary data) or use secondary data from research organizations or use simulated data for studies. Simulation studies are more suitable in this area since in most of the cases in general, hospitable real life data may not be sufficiently available and at times they may heavily depend on the biased nature of the data collectors. They may vary hospital to hospital and they may not be genuine enough for the study since many other factors such as the quality of nursing and medical treatments provided to the patients by hospitals are involved. The alertness of the patients in expressing the symptoms and timely assistance of insurance and finance agencies involved may play always the leading role in providing proper treatment. These are necessary to generate perfect and genuine data. Since the reputation of many connected organizations are involved,there may not be anybody to take the responsibility of the perfectness of the data provided. In this area not much of significant simulation studies are availableor taken up so far at any depth. For simulation analysismodels with general random variables present real life-like situations and results. It is well known that any general distribution may be well approximated by Erlang-Coxian 2 (EC) Phase type distributionand the details of the same are presented byT.Osogami and M.H.Balter [8]. For the simulationanalysis here, Martin Haugh [9] results

on minimum value methods andLaw and Kelton methodsusing Hull and Dobell results [10] are utilized to generate uniform and all other random values required.

In this paper two models are treated. In model 1,the person is provided treatment when any one of the organs A or B fails or when he is admitted for prophylactic treatment.In model II, he is provided treatment when the two organs A and B are in failed state or when he is admitted for prophylactic treatment. In real life situations, it is often seen that the failure of organ A producing insulin (pancreatic failure) may not be noticed by the patients until organ B fails (kidney failure) or organ B failure (kidney failure) may not be noticed until organ A fails (pancreatic failure) and the treatment begins when two organs A and B are in failed state. This situation arises when the patient is unable to identify and reveal the symptoms on time for treatment. This case is treated in the model II. The joint Laplace-Stieltjes transform of time to hospitalization for treatment and hospital treatment time, the expected time to hospitalization for treatment and expected treatment time for the modelsare derived.Numerical examples are studied assuming exponential life time for organ B.Simulation studies are provided considering Erlang-Coxian 2 life time for organ B and Erlang treatment times for the two models. Varying the parameter values of time to prophylactic treatment several simulated values are generated for time to hospitalization and hospitalization times.

II. Model I: One Organ Failure And Prophylactic Treatment

The general assumptions of the model I studied are given below.

2.1. Assumptions

(i) Affected organ A segregating insulin of a patient functions in two phases of damaged levels namely damaged level 1 (phase 1) and damaged level 2 (phase 2) where level 1 is considered to be a better level of the two with lesser transition rates to level 2 and lesser failure rate. Due to pre-hospital medication the organ A may move to level 1 from level 2 and due to negligence it may move to level 2 from level 1. The failed level of the organ A is level 3. The transition rates of the organ A to the failed level 3 from level 1 and from level 2 are respectively $\lambda_1 and\ \lambda_2\ with\ \lambda_2 > \lambda_1$.The transition rates from level 1 to level 2 is $\mu_1$and from level 2 to level 1 is μ_2. At time 0 the organ level is 1and let the organ A life time also be denoted by A.

(ii) Another organ B of the person has general life time with cumulative distribution function (Cdf) F_B(x).

(iii) Independent of the status of the organs the person may be admitted for prophylactic treatment in a random time with exponential distribution with parameter α.

(iv) The hospitalization begins when any one of the two organs fails or the prophylactic treatment starts.

(v) Thehospital treatment time of organ A and treatment time of the organ B are random variables $H_1, and\ \ H_2$with Cdfs$H_1(x)\ and\ \ H_2(x)$ respectively and the prophylactic treatmenttime in the hospital is a random variable H_3 with Cdf $H_3(x)$.

2.2.Analysis

To study the above model the probability distribution of time to failure of the organ A from the level 1 at time 0 is to be obtained.Levels 1 and 2 of the organ A may be considered as phases 1 and 2 respectively of PH phase 2 distribution. Considering the hospitalization as absorbing state 3, the infinitesimal generator describing the transitions is given by

$$Q=\begin{bmatrix} -(\lambda_1+\mu_1) & \mu_1 & \lambda_1 \\ \mu_2 & -(\lambda_2+\mu_2) & \lambda_2 \\ 0 & 0 & 0 \end{bmatrix}. \quad (1)$$

Various transition probabilities may be derived as follows. Before absorption, the organ A may be in state 1 or2. Let $P_{i,j}(t)$ = P (At time t the organ A is in level j and it has not failed during (0, t) / at time 0 it is in level i) (2) for i, j = 1, 2. The probability $P_{1,1}(t)$ may be written considering the two possibilities that (i) the organ A may remain in level 1 during (0, t) without a failure or (ii) it moves to level 2 at time u for u ϵ (0, t) and it is in level 1 at time t without a failure . It may be seen that

$P_{1,1}(t) = e^{-(\lambda_1+\mu_1)t} + \int_0^t \mu_1\, e^{-(\lambda_1+\mu_1)u} P_{2,1}(t-u)\text{du}$.(3)

Using similar arguments it may be seen that

$P_{2,1}(t) = \int_0^t \mu_2\, e^{-(\lambda_2+\mu_2)u}\, P_{1,1}(t-u)\ \text{du}$,(4)

$P_{1,2}(t) = \int_0^t \mu_1\, e^{-(\lambda_1+\mu_1)u}\, P_{2,2}(t-u)\ \text{du}$ (5)

and$P_{2,2}(t) = e^{-(\lambda_2+\mu_2)t} + \int_0^t \mu_2\, e^{-(\lambda_2+\mu_2)u} P_{1,2}(t-u)\ \text{du}.$ (6)

The above four equations (3) to (6) may be solved using Laplace transform to obtain results as follows.

$P_{1,1}(t)= (\frac{1}{2})e^{-at}(e^{bt} + e^{-bt}) +(\frac{1}{4b})\ (\lambda_2 - \lambda_1 + \mu_2 - \mu_1)e^{-at}(e^{bt} - e^{-bt}).$ (7)

$P_{1,2}(t) = (\frac{\mu_1}{2b})e^{-at}(e^{bt} - e^{-bt})$. (8)

$P_{2,2}(t) = (\frac{1}{2})e^{-at}(e^{bt} + e^{-bt}) + (\frac{1}{4b})(\lambda_1 - \lambda_2 + \mu_1 - \mu_2)e^{-at}(e^{bt} - e^{-bt})$. (9)

$P_{2,1}(t) = (\frac{\mu_2}{2b})e^{-at}(e^{bt} - e^{-bt})$.(10)

Here a =$(\frac{1}{2})(\lambda_1 + \lambda_2 + \mu_1 + \mu_2)$; b = $(\frac{1}{2})\sqrt{(\lambda_1 - \lambda_2 + \mu_1 - \mu_2)^2 + 4\mu_1\mu_2}$ and

$a^2 - b^2 = \lambda_1\lambda_2 + \lambda_1\mu_2 + \lambda_2\mu_1$.(11)

The absorption to state 3 can occur from any phase 1 or 2 since the organ A may fail from level 1 or level 2. Theprobability density function (pdf) $p_{1,3}(t)$of time to failure of organ A, starting from level 1 at time 0 is written using the absorption rates from levels 1 and 2 as follows.For convenience let $p_{1,3}(t) = f_A(t)$. After simplification

$p_{1,3}(t) = \lambda_1 P_{1,1}(t) + \lambda_2 P_{1,2}(t) = (\frac{a+b-\lambda_1}{2b})$ (a - b)$e^{-(a-b)t}$ - $(\frac{a-b-\lambda_1}{2b})$ (a + b)$e^{-(a+b)t} = f_A(t)$.(12)

Its Cdf$F_A$is$F_A(t) = \int_0^t f_A(u)$du = 1 – $(\frac{a+b-\lambda_1}{2b})e^{-(a-b)t} + (\frac{a-b-\lambda_1}{2b})e^{-(a+b)t}$. (13)

To study the model I, the joint pdf of two variables (T, H) where variable T is the time to hospitalization which is theminimum of {The absorption time of PH phase 2 (Organ A failure time), The failure time of organ B ,The time to prophylactic treatment} and variable H is the hospitalization time where H=$H_1 or\ H_2 or H_3$according to treatment begins when Organ A or organ B fails or the patient is admitted for prophylactic treatment respectively. The joint pdfof (T, H) is f(x, y) = $f_A(x)e^{-\alpha x}\overline{G(x)}\ h_1(y) + \overline{F_A(x)}e^{-\alpha x}$ g(x) h_2(y) +$\overline{F_A(x)}\overline{G(x)}\alpha e^{-\alpha x} h_3$(y) . (14)

Here $\bar{V}$ (x) = 1 – V (x) for any function V (x).The first term of the RHS of (14) is the pdf-part that the organ A fails before the failure of the organ B and before the completion of the time to prophylactic treatment and the hospitalization is provided for the failure of the organ A. The second term is the pdf-part that the organ B fails before the failure of organ A and before the completion of the time to prophylactic treatment and the hospitalization is provided for the failure of the organ B. The third term is the pdf-part that the patient is admitted for prophylactic treatment before the failures of organ A and organ B and the prophylactic treatment is provided. The double Laplace transform of the pdf of (T, H) is given by f*(ξ, η) = $\int_0^\infty \int_0^\infty e^{-\xi x - \eta y}$ f(x, y) dx dy. (15)

Here * indicates Laplace transform. The equation (15) using the structure of equation (14) becomes a single integral.

f*(ξ, η) =$\int_0^\infty e^{-\xi x}[f_A(x)e^{-\alpha x}\overline{F_B}(x)h^*_1(\eta) + \overline{F_A}(x)e^{-\alpha x} f_B(x)h^*_2(\eta) +$

$FA(x)\ FB(x)\ \alpha\ e{-}\alpha x h{*}3\eta]dx$.(16)Using the equations (12) and (13) the above integral (16) may be simplified as follows.f*(ξ, η) =$(\frac{a+b-\lambda_1}{2b})\overline{F_B}$*(ξ + α +a -b)[(a-b) $h^*_1(\eta)$+ α$h^*_3(\eta)$-(ξ + α + a - b) $h^*_2(\eta)$]

-$(\frac{a-b-\lambda_1}{2b})\ \overline{F_B}$*(ξ + α +a +b) [(a+b) $h^*_1(\eta)$ +α$h^*_3(\eta)$ -(ξ + α + a + b) $h^*_2(\eta)$] + $h^*_2(\eta)$. (17)

The Laplace transform of the pdf of the time to hospitalization T may be obtained after simplification by taking η = 0 in equation (17).f*(ξ, 0) = 1 - $\frac{\xi}{2b}$ (a + b - λ_1) $\overline{F_B}$*(ξ + α +a -b) + $\frac{\xi}{2b}$ (a - b - λ_1) $\overline{F_B}$*(ξ + α +a +b).(18)

E (T) =$-\frac{d}{d\xi}$ f*(ξ, 0) $|_{\xi=0}$gives, E (T) =$\frac{1}{2b}$ (a + b - λ_1) $\overline{F_B}$*(α +a -b) - $\frac{1}{2b}$ (a - b - λ_1) $\overline{F_B}$*(α +a +b).(19)

Using equation (17) the Laplace transform of the pdf of the hospitalization time H may be obtained by taking ξ = 0.

f*(0, η) =$(\frac{a+b-\lambda_1}{2b})\ \overline{F_B}$*(α +a -b)[(a-b) $h^*_1(\eta)$ + α$h^*_3(\eta)$-(α + a - b) $h^*_2(\eta)$]

-$(\frac{a-b-\lambda_1}{2b})\ \overline{F_B}$*(α +a +b) [(a+b) $h^*_1(\eta)$ +α$h^*_3(\eta)$ -(α + a + b) $h^*_2(\eta)$] + $h^*_2(\eta)$.(20)

E (H) = $-\frac{d}{d\eta}$ f*(0, η) $|_{\eta=0}$gives

E (H) =$(\frac{a+b-\lambda_1}{2b})\ \overline{F_B}$*(α +a -b)[(a-b) E($H_1$) + αE($H_3$) -(α + a - b)E($H_2$)]

-$(\frac{a-b-\lambda_1}{2b})\ \overline{F_B}$*(α +a +b) [(a+b)E($H_1$) +α E($H_3$) -(α + a + b)E($H_2$))] + E($H_2$) . (21)

Inversion of Laplace transform of (18) and (20) are straight forward. The Cdf of time to hospitalization T is

P(T ≤ t) = F_T (t) = 1 - $\frac{1}{2b}$ (a + b - λ_1) $\overline{F_B}$(t)$e^{-(\alpha+a-b)t}$+ $\frac{1}{2b}$ (a - b - λ_1) $\overline{F_B}$(t) $e^{-(\alpha+a+b)t}$. (22)

The Cdf of hospitalization time H is given below.

P (H ≤ t) =$(\frac{a+b-\lambda_1}{2b})\ \overline{F_B}$*(α +a -b)[(a-b) H_1(t) + αH_3(t) -(α + a - b)H_2(t)]

-$(\frac{a-b-\lambda_1}{2b})\ \overline{F_B}$*(α +a +b) [(a+b) H_1(t) +αH_3(t) -(α + a + b)H_2(t)] + H_2(t) (23)

2.3. Special cases:

(i)When the life time ofthe organ B has exponential distribution with parameter θ then

E (T) = $\frac{1}{2b}$ (a + b - λ_1) ($\frac{1}{\theta+\alpha+a-b}$) - $\frac{1}{2b}$ (a - b - λ_1) ($\frac{1}{\theta+\alpha+a+b}$). (24)

To write down E (H), $\overline{F_B}$*(s) may be replaced by $\frac{1}{\theta+s}$ for (i) s = α +a –b and (ii) s = α +a +b as the case may be.

(ii)Let the life time of the organ B have EC distributionwith parameter set (k, θ,θ_1, θ_2, p) which is the Cdf of thesum of Erlang (k, θ) and Coxian-2 (θ_1, θ_2, p) random variables where the Erlang has k phases with parameter θ and the Coxian 2 hasthe infinitesimal generator describing the transition as

Q'=$\begin{bmatrix} -\theta_1 & p\theta_1 & q\theta_1 \\ 0 & -\theta_2 & \theta_2 \\ 0 & 0 & 0 \end{bmatrix}$(25)

for p + q = 1with starting phase 1. By comparing Q' with Q in (1) the pdf of Coxian 2 may be written as

$g_1(x) = (\frac{q\theta_1-\theta_2}{\theta_1-\theta_2})\theta_1 e^{-\theta_1 x}+(\frac{p\theta_1}{\theta_1-\theta_2})\theta_2 e^{-\theta_2 x}$ and $G_1(x)$ = 1- $(\frac{q\theta_1-\theta_2}{\theta_1-\theta_2})\, e^{-\theta_1 x}$- $(\frac{p\theta_1}{\theta_1-\theta_2})e^{-\theta_2 x}$. The Laplace transform of the Coxian 2 is $g_1^*(s) = (\frac{q\theta_1-\theta_2}{\theta_1-\theta_2})\,(\frac{\theta_1}{\theta_1+s})+(\frac{p\theta_1}{\theta_1-\theta_2})(\frac{\theta_2}{\theta_2+s})$. (26)

The CdfF_B(x) of organ Bis the Cdf of the sum of Erlang and Coxian. So the Laplace transform of its pdf is

f_B*(s) = $(\frac{\theta}{\theta+s})^k g_1^*(s)$. (27)

Using the fact$\overline{F_B}$(x) =1-F_B(x), the Laplace transform of $\overline{F_B}$(x) is obtained from (26) and (27) as follows.

$\overline{F_B}$*(s) = $\frac{1-(\frac{\theta}{\theta+s})^k[(\frac{q\theta_1-\theta_2}{\theta_1-\theta_2})(\frac{\theta_1}{\theta_1+s})+(\frac{p\theta_1}{\theta_1-\theta_2})(\frac{\theta_2}{\theta_2+s})]}{s}$.

(28)The expected time to hospitalization E (T) may be written considering equation (19) and (28) for s = α +a – b and s = α +a +b as per the case.To write down the expected hospitalization (treatment) time E (H),$\overline{F_B}$*(s) of equation (28) may be again used in (21) for s = α +a –b and s = α +a +b as per the case.

III. Model II: Both Organs Failure And Prophylactic Treatment

The general assumptions of the model II studied here are given below which is a variation of Model I. In real life situations in many cases it may be seen that due to ignorance and negligence the patient may not be sent for hospitalization unless both organs become failed. It is very common in many cases of patients unless another organ is affected the insulin/ pancreas affected patient may not be aware of the lower insulin level / higher sugar level till a second organ gets affected.

3.1.Assumptions

(i) Affected organ A segregating insulin of a patient functions in two phases of damaged levels namelydamaged level 1 (phase 1) and damaged level 2 (phase 2) where level 1 is considered to be a better level of the two with lesser transition rates to level 2 and lesser failure rate. Due to pre-hospital medication the organ A may moveto level 1from level 2 and due to negligence it may move to level 2 from level 1. The failed level of the organ A is level 3. The transition rates of the organ A to the failed level 3 from level 1 and from level 2 are respectively$\lambda_1 and\ \lambda_2\ with\ \lambda_2 > \lambda_1$.The transition rates from level 1 to level 2 is $\mu_1$and from level 2 to level 1 is μ_2. At time 0 the organ level is 1and let the organ A life time also be denoted by A.

(ii) Another organ B of the patient has general life time with CdfF_B(x). (iii)

Independent of the status of the organs the patient may be admitted for prophylactic treatment in the hospital in a random time with exponential distribution with parameter α.

(iv) The hospitalization begins when the two organs are in failed state or the prophylactic treatment starts.

(v) The combined hospital treatment time of the two organs is a random variable H_4 with Cdf$H_4(x)$and the prophylactic treatment time in the hospital is a random variable H_5 with Cdf $H_5(x)$.

3.2. Analysis

To study the model II, the joint pdf of two variables (T, H) where T is the time to hospitalization and H is the hospitalization time is required. Here variable T is the minimum of {Maximum of the life times of the two organs (A, B), The time to prophylactic treatment}and variable H is the hospitalization time =$H_4 or\ H_5$according as the hospitalization begins when the two organs are in failed state or the patient is admitted for prophylactic treatment respectively. The joint pdfof (T, H) is

f (x, y) = $e^{-\alpha x}[F_A(x)f_B(x) + f_A(x)F_B(x)]\, h_4$(y) + (1 - $F_A(x)F_B(x)$)α$e^{-\alpha x}\, h_5$(y) .(29)

The first term (withthe square bracket) of the RHS of (29) is the pdf-part that the two organs A and B fail before the completion of the time to prophylactic treatment and the hospitalization is provided for the failure of the organs. The second term is the pdf-part that the patient is admitted for prophylactic treatment before the failures of the two organs A and B and the hospitalization for the prophylactic treatment is provided. The double Laplace transform of (T, H) is given by f*(ξ, η) = $\int_0^\infty \int_0^\infty e^{-\xi x-\eta y}$ f(x, y) dx dy.(30)

The Cdf and the pdfof the organ A life time $F_A(x)$and $f_A(x)= p_{1,3}(t)$are derived in (13) and (12). Using them in (29), equation (30) becomes

$f^*(\xi, \eta) = h_4^*(\eta)\big[f_B^*(\xi + \alpha) - (\frac{a+b-\lambda_1}{2b})\, f_B^*(\xi +\alpha + a - b) + (\frac{a-b-\lambda_1}{2b})\, f_B^*(\xi +\alpha + a + b)$
$+ (\frac{a+b-\lambda_1}{2b})\,(a\text{-}b)\, F_B^*(\xi +\alpha + a - b) - (\frac{a-b-\lambda_1}{2b})\,(a + b)\, F_B^*(\xi +\alpha + a + b)\big]$
$+ \alpha h_5^*(\eta)\big[\frac{1}{\alpha+\xi} - [F_B^*(\xi + \alpha) - (\frac{a+b-\lambda_1}{2b})\, F_B^*(\xi +\alpha + a - b) + (\frac{a-b-\lambda_1}{2b})\, F_B^*(\xi +\alpha + a + b)]\big]$. (31)

The Laplace transform of the pdf of the time to hospitalization T may be obtained after simplification by taking $\eta = 0$ in (31).

$f^*(\xi, 0) = \frac{\alpha}{\alpha+\xi} + \xi F_B^*(\xi + \alpha) - \xi\,(\frac{a+b-\lambda_1}{2b})\, F_B^*(\xi +\alpha + a - b) + \xi\,(\frac{a-b-\lambda_1}{2b})\, F_B^*(\xi +\alpha + a + b)$.(32)

$E(T) = -\frac{d}{d\xi} f^*(\xi, 0)\,|_{\xi=0}$ gives $E(T) = \frac{1}{\alpha} - F_B^*(\alpha) + \frac{1}{2b}(a + b - \lambda_1)\, F_B^*(\alpha +a -b) - \frac{1}{2b}(a - b - \lambda_1)\, F_B^*(\alpha +a +b)$ (33)

Using equation (31) the Laplace transform of the pdf of the hospitalization time H may be obtained by taking $\xi = 0$.

$f^*(0, \eta) = h_4^*(\eta)\big[f_B^*(\alpha) - (\frac{a+b-\lambda_1}{2b}) f_B^*(\alpha + a - b) + (\frac{a-b-\lambda_1}{2b})\, f_B^*(\alpha + a + b)$
$+ (\frac{a+b-\lambda_1}{2b})\,(a\text{-}b)\, F_B^*(\alpha + a - b) - (\frac{a-b-\lambda_1}{2b})\,(a + b)\, F_B^*(\alpha + a + b)\big]$
$+ \alpha h_5^*(\eta)\big[\frac{1}{\alpha} - [F_B^*(\alpha) - (\frac{a+b-\lambda_1}{2b})\, F_B^*(\alpha + a - b) + (\frac{a-b-\lambda_1}{2b})\, F_B^*(\alpha + a + b)]\big]$. (34)

Since $E(H) = -\frac{d}{d\eta} f^*(0, \eta)\,|_{\eta=0}$,

$E(H) = E(H_4)\big[f_B^*(\alpha) - (\frac{a+b-\lambda_1}{2b})\, f_B^*(\alpha + a - b) + (\frac{a-b-\lambda_1}{2b})\, f_B^*(\alpha + a + b)$
$+ (\frac{a+b-\lambda_1}{2b})\,(a\text{-}b)\, F_B^*(\alpha + a - b) - (\frac{a-b-\lambda_1}{2b})\,(a + b)\, F_B^*(\alpha + a + b)\big]$
$+ \alpha E(H_5)\big[\frac{1}{\alpha} - [F_B^*(\alpha) - (\frac{a+b-\lambda_1}{2b})\, F_B^*(\alpha + a - b) + (\frac{a-b-\lambda_1}{2b})\, F_B^*(\alpha + a + b)]\big]$.(35)

Inversion of Laplace transform of (32) and (34) are straight forward. The Cdf of the time to hospitalization T is

$P(T \le t) = F_T(t) = 1 - e^{-\alpha t} + F_B(t)\, e^{-\alpha t} - \frac{1}{2b}(a + b - \lambda_1)\, F_B(t) e^{-(\alpha+a-b)t} + \frac{1}{2b}(a - b - \lambda_1)\, F_B(t)\, e^{-(\alpha+a+b)t}$. (36)

The Cdf of the hospitalization time H is given below.

$P(H \le t) = F_H(t) = H_4(t)\big[f_B^*(\alpha) - (\frac{a+b-\lambda_1}{2b})\, f_B^*(\alpha + a - b) + (\frac{a-b-\lambda_1}{2b})\, f_B^*(\alpha + a + b)$
$+ (\frac{a+b-\lambda_1}{2b})\,(a\text{-}b)\, F_B^*(\alpha + a - b) - (\frac{a-b-\lambda_1}{2b})\,(a + b)\, F_B^*(\alpha + a + b)\big]$
$+ \alpha H_5(t)\big[\frac{1}{\alpha} - [F_B^*(\alpha) - (\frac{a+b-\lambda_1}{2b})\, F_B^*(\alpha + a - b) + (\frac{a-b-\lambda_1}{2b})\, F_B^*(\alpha + a + b)]\big]$.(37)

3.3. Special cases:

(i)When thelife time of organ B has exponential distribution with parameter θ then

$E(T) = \frac{1}{(\theta+\alpha)} + \frac{1}{2b}(a + b - \lambda_1)\,(\frac{1}{\alpha+a-b})\,(\frac{\theta}{\theta+\alpha+a-b}) - \frac{1}{2b}(a - b - \lambda_1)\,(\frac{1}{\alpha+a+b})\,(\frac{\theta}{\theta+\alpha+a+b})$. (38)

For writing E (H), $f_B^*(s)$ may be replaced by $\frac{\theta}{(\theta+s)}$ and $F_B^*(s)$ may be replaced by $\frac{\theta}{s(\theta+s)}$ for (i) s = α; (ii) s = α +a –b and (iii) s = α +a +b as the case may be.

(ii) When the life time of the organ B has the ECCdf with parameter set (k, θ,θ_1, θ_2, p) as in model I,then noting

$F_B^*(s) = (f_B^*(s)/s) = \frac{(\frac{\theta}{\theta+s})^k[(\frac{q\theta_1-\theta_2}{\theta_1-\theta_2})(\frac{\theta_1}{\theta_1+s})+(\frac{p\theta_1}{\theta_1-\theta_2})(\frac{\theta_2}{\theta_2+s})]}{s}$(39)

E(T) can be written using (33) and (39) for (i) s = α; (ii) s = α +a –b and (iii) s = α +a +b as the case may be.

For writing E (H), equation (35) may be used where $f_B^*(s)$ may be replaced by $(\frac{\theta}{\theta+s})^k[(\frac{q\theta_1-\theta_2}{\theta_1-\theta_2})(\frac{\theta_1}{\theta_1+s}) + (\frac{p\theta_1}{\theta_1-\theta_2})(\frac{\theta_2}{\theta_2+s})]$ and $F_B^*(s)$ may be replaced by (39) for (i) s = α; (ii) s = α +a –b and (iii) s = α +a +b as the case may be.

IV. Numerical And Simulation Studies:

4.1. Numerical Studies:

As an application of the results obtained numerical studyis taken up to present the expected times E (T) and E (H) for various values of the parameters introduced for the two models.

Let $\lambda_1 = 1, \lambda_2 = 2, \mu_1 = 3, \mu_2 = 4$.Then from (11), a=5, b=$\sqrt{13}$, $\frac{a+b-\lambda_1}{2b} = \frac{4+\sqrt{13}}{2\sqrt{13}}$, $\frac{a-b-\lambda_1}{2b} = \frac{4-\sqrt{13}}{2\sqrt{13}}$, $a^2 - b^2$=12.

The organ A has the Cdfof life time, $F_A(t) = 1 - 1.054700196\, e^{-1.4\,t} + 0.054700196 e^{-8.6t}$ withpdf $f_A(t) = 1.4$ x $1.054700196\, e^{-1.4\,t} - 8.6$ x.$054700196 e^{-8.6t}$ and $f_A^*(s) = \frac{1.476580274}{s+1.4} - \frac{0.4704216856}{s+8.6}$ with Mean=0.7470524036.

Let us assumethat the organ B hasCdf of life time

F_B (t) = 1-$e^{-\theta t}$ with θ =10.
Let the parameter of the (exponential) time to prophylactic treatment be α = 0.5,1, 1.5, 2, and2.5.
Let the expected values of various treatment times be E(H_1)=6, E(H_2)=5,E(H_3)=4,E(H_4)=7 and E(H_5)=6. E (T) and E (H) values are calculated using (19), (21), (33) and (35) and presentedfor various values of α for the two models I and II in tables 1 and 2 respectively.The effect of variation of α on E(T) and E(H) may be seen clearly.

Table 1. Model I Numerical Case for θ = 10

α	0.5	1	1.5	2	2.5
E(T)	0.085808581	0.082304527	0.079074253	0.076086957	0.073316283
E(H)	5.056105611	5.012345679	4.972034716	4.934782609	4.900255754

Table 2. Model II Numerical Case for θ = 10

α	0.5	1	1.5	2	2.5
E(T)	0.560154152	0.443387173	0.366856628	0.312801932	0.272579671
E(H)	6.719922924	6.556612827	6.449715058	6.374396135	6.318550823

4.2. Simulation Studies:

In the numerical study the general life time of the organ B is assumed to follow an exponential distribution. When the general life time of B has unknown distribution, the researcher has to fit the distribution using the data available (primary / secondary). The method is to find the first three moments from the data. When the first three moments are available it has been established that the EC distribution which is the distribution of the sum of Erlang (k, θ) and Coxian (θ_1, p, θ_2) random variables is an excellent and more suitable approximation for any general distribution function using the method of comparison of first three moments by T. Osogamy and M.H.Balter [8] where the details of finding the exact EC distribution with its parameter values are also available. So the assumption and consideration of an EC distribution for the life time of organ B is ideal for simulation studies. Further such a study with EC distribution is almost a study on any general distribution. The simulation of time to hospitalization is taken up here assuming the organ A has Phase 2 type life distribution, the organ B has EC life distribution with parameter set {k, θ, θ_1, p, θ_2}and time to prophylactic treatment has exponential distribution with parameter α. As in the numerical case-study the same values for organ A transition rates are assumed which gives the life time of organ A has CdfF_A (t) =1 - 1.054700196 $e^{-1.4\,t}$+ 0.054700196$e^{-8.6t}$ withpdf
f_A (t)=1.4 x 1.054700196 $e^{-1.4\,t}$- 8.6 x.054700196$e^{-8.6t}$.

The method and study taken up here is valid for any set of values of the EC parameter set for the organ B. For the purpose of illustration the EC parameter set is considered as {5, 10, 20, 0.5, 30}. Let the parameter of the (exponential) time to prophylactic treatment be α = 0.5, 1, 1.5, 2 and 2.5.The simulated values of the life times of organ A, organ B and the time to prophylactic treatment are required for the study. They are generated using the methods presented by Martin Haugh [9] by generating uniform random values u using Linear Congruential Generator (LCG). The random values for the life time of the organ A are generated using x_A= min {x :$F_A(x) \geq u$}.

The life time of organ B is the sum of Erlang and Coxian times. Its Erlang random time part is generated by considering y = - $\frac{1}{10}$ ln $\prod_{i=1}^{5} u_i$ where ln is natural logarithm and $u_i$are generated by different LCGs for i=1 to 5.
The first and second exponential random time values of Coxian 2 random time part are generated by z= - $\frac{1}{20}$ ln u and w = - $\frac{1}{30}$ ln u. Considering if u ≤ p = 0.5 the Coxian time is z and if u > p = 0.5 the Coxian time is z + w, the simulated random time values for the life time of organ B is

x' = $\begin{cases} y + z if u \leq q = 0.5 \\ y + z + w if u > q = 0.5 \end{cases}$

Simulated exponential random values for the time to hospitalization for prophylactic treatment are generated by considering x" = - $\frac{1}{\alpha}$ ln u for α = 0.5, 1, 1.5, 2 and 2.5. This gives the simulated time to hospitalization for treatment is T = min {x_A, x', x"} for model I and T = min {max {x_A, x'}, x"} for model II. It may be noted that 10 random variables are to be considered for the time to hospitalization. For the organ A one random variable is considered with CdfF_A (t). For the organ B, 8 random variables are considered of which five are exponential random variables for Erlang part and three random variables for Coxian part of which two are exponentials and one for probability q =0.5 (thenon occurrence of second exponential time in Coxian).One

random variable for the exponential time for the prophylactic treatment is also required. In the parametric model one may assume the expected values of various treatment (hospitalization) times. The exact structures of the distribution functions may not be required and the values of expectations alone are sufficient. Simulation studies are different. One has to study the types and stages of treatments as explained byMark Fackrell [11] to get simulated hospital treatment times.

Model I considers three types of treatments H_1 , $H_2 and H_3$. Let five, three and two stages of treatments one by onerespectively for them be assumed as follows.

(i) Treatment H_1 : Emergency Department (ED) → Operation Theatre (OPT) → Intensive Care Unit (ICU) → High Dependency ward (HDW) → Ward (W) → Discharge
(ii) Treatment H_2 :ED → ICU → Ward → Discharge and
(iii) TreatmentH_3 : ED → Ward →Discharge.

The random hospitalization times H_1, $H_2$and $H_3$are accordingly assumed to have Erlang distributions with phases and parameter valuesas (i) 5 and 6, (ii) 3 and 6 and (iii)2 and 6 respectively.
Model II considers two types of treatments (i) and (ii) as stated above. The hospitalization times H_4 and H_5 have Erlang distributions with phases and parameter values as (i) 5 and 2 and (ii) 3 and 2 respectively.
Accordingly the simulated hospitalization times for the two models are H_1= -($\frac{1}{6}$)ln $\prod_{i=1}^{5} u_i$; H_2=-($\frac{1}{6}$)ln$\prod_{i=1}^{3} u_i$; H_3 = - ($\frac{1}{6}$)ln $\prod_{i=1}^{2} u_i$; H_4 =-($\frac{1}{2}$)ln $\prod_{i=1}^{5} u_i$ and H_5 = -($\frac{1}{2}$)ln $\prod_{i=1}^{3} u_i$.

The random values u_i appearing on the right side of H_j for j= 1,2,3,4,5 are generated by different LCGs for various values of i. It may be noted that for the two models 18 random variables are considered for hospital treatment times, namely, 5 random variables with exponential distribution for H_1 ; 3 random variables with exponential distributions for H_2; 2 random variables with exponential distributions for H_3; 5 random variables with exponential distribution for H_4 and 3 random variables with exponential distributions for H_5. Noting the total number of random variables are 28 for time to hospitalization and hospital treatment times 28 sets of uniform random values are to be generated. These 28 uniform random u_i for i=1 to 28 values are generated by linear congruential generators Z_{n+1}= (aZ_n + c)mod 16 with seed value Z_0whose short form representation is given by LCG(a, c, 16, Z_0) for different values of a, c and Z_0. This generates all the sixteen reminder values of division by 16 in random manner so that sixteen uniform random values u = $\frac{z}{16}$given by {0.0625, 0.375,0.9375, 0.75, 0.8125, 0.125, 0.6875, 0.5, 0.5625, 0.875, 0.4375, 0.25, 0.3125, 0.625, 0.1875, 0}are obtained in some order. It may be noted that for uniform distribution it is known that E(U)= 0.5 and Variance =1/12= 0.08333.They are comparable with average simulated value 0.46875 and its variance 0.083 of the data. The value u =0 in the above gives extreme value of the simulated random value since natural logarithm of u, namely, ln (u) is required for simulation. When u =0 is deleted the average becomes 0.5equal to E(U).The LCG (5, 1, 16, 1) is used for simulation of the life time of organ A. For the Erlangphase 5 life part of organ B LCG(9, 1,16, 2), LCG(13, 1,16, 3), LCG(1, 3,16, 4), LCG(5, 3,16, 5) and LCG(9, 3,16, 6) are used and for Coxianlife part of organ B LCG(13, 3,16, 7), LCG(5, 5,16, 9) are used for the exponential times and LCG(1, 5,16, 8) is used for the probability q of the non occurrence of the second exponential treatment time. The LCG (9, 5, 16, 10) is used for simulation of time to prophylactic treatment. For the Erlang phase 5 hospitalization time of H_1, LCG(13, 5,16, 11), LCG(1, 7,16, 12), LCG(5, 7,16, 13), LCG(9, 7,16, 14) and LCG(13, 7,16, 15) are used. For the Erlangphase 3 hospitalization time of H_2, LCG(1, 9,16, 1), LCG(5, 9,16, 2), and LCG(9, 9,16, 3) are used. For the Erlang phase 2 hospitalization time of H_3, LCG(13, 9,16, 4) and LCG(1, 11,16, 5) are used. For the Erlang phase 5 hospitalization time of H_4, LCG(5, 11,16, 6), LCG(9, 11,16, 7), LCG(13, 11,16, 8), LCG(1, 13,16, 9) and LCG(5, 13,16, 10) are used. For the Erlang phase 3 hospitalization time of H_5, LCG(9, 13,16, 11), LCG(13, 13,16, 12) and LCG(1, 15,16, 13) are used.All the LCGs' used above namely, the LCG (a, c, m, Z_0) for a = 1,5, 9,13; c= 1, 3, 5, 7, 9, 11, 13, 15; m =16 ;Z_0= 1 to 15 have full period of length 16. The values of a, c given above with Z_0= 1 to 15 and m = 16 satisfy the Hull and Dobell theorem [10] which guarantees the LCG to have full period.The following table 3 gives the failure times of organ A and B. Organ A failure time is simulated using minimum method and presented in red color. Erlang and exponential random times are simulated using inverse method described earlier. The failure time of Organ Bis presented in green color. Coxian's second exponential time is considered for addition when u >q by blue color.

Table3: Failure Times of Organs A and B

Organ A failure time t= $F^{-1}(U)$	Organ B Erang(5, 10) Part	Organ B Cox Phase 1 exp(20)	Transition when u>0.5	Organ B Cox Phase2 exp(30)	Organ B failure EC time
0.059610	0.72836924	0.041333929	0.5	0.019178805	0.769703169

0.371200	0.40568484	0.00667657	0.8125	0.069314718	0.481676128
2.018500	0.278388273	0.028768207	0.125	0.002151284	0.30715648
1.029000	0.235631795	0.034657359	0.4375	0.038771694	0.270289154
1.233750	0.537414989	0.018734672	0.75	0.004451046	0.560600708
0.117350	0.603092943	0.103972077	0.0625	0.012489782	0.719554802
0.868800	0.826452165	0.010381968	0.375	0.009589402	0.836834133
0.535000	0.3178298	0.014384104	0.6875	0.092419624	0.424633528
0.628130	0.49728085	0.003226926	0.3125	0.015666788	0.500507776
1.523500	0.735268527	0.049041463	0.625	0.027555952	0.811865942
0.447540	0.350520079	0.138629436	0.9375	0.023104906	0.512254421
0.236900	0.48959495	0.083698822	0.25	0.006921312	0.573293771
0.301440	0.162770059	0.023500181	0.5625	0.032694308	0.218964549
0.738500	0.210215857	0.05815754	0.875	0.055799214	0.324172612
0.175835	0.466265358	0.069314718	0.1875	0.046209812	0.535580076

Times for Prophylactic treatments are simulated using inverse method to get exponential randomtimes which are presented in table 4.

Table 4: Time for Prophylactic Treatment for Various Values of α= 0.5, 1, 1.5, 2 and 2.5

Prophylactic Exp(0.5)	Prophylactic Exp(1)	Prophylactic Exp(1.5)	Prophylactic Exp(2)	Prophylactic Exp(2.5)
0.940007258	0.4700036	0.313335753	0.235001815	0.188001452
0.129077042	0.0645385	0.043025681	0.032269261	0.025815408
0.575364145	0.2876821	0.191788048	0.143841036	0.115072829
5.545177444	2.7725887	1.848392481	1.386294361	1.109035489
0.267062785	0.1335314	0.089020928	0.066765696	0.053412557
3.347952867	1.6739764	1.115984289	0.836988217	0.669590573
2.32630162	1.1631508	0.775433873	0.581575405	0.465260324
4.158883083	2.0794415	1.386294361	1.039720771	0.831776617
1.653357146	0.8266786	0.551119049	0.413339287	0.330671429
2.772588722	1.3862944	0.924196241	0.693147181	0.554517744
1.15072829	0.5753641	0.383576097	0.287682072	0.230145658
1.961658506	0.9808293	0.653886169	0.490414627	0.392331701
0.749386899	0.3746934	0.249795633	0.187346725	0.14987738
1.386294361	0.6931472	0.46209812	0.34657359	0.277258872
0.41527873	0.2076394	0.138426243	0.103819682	0.083055746

In Model I the patient is sent for hospitalization when organ A or Organ B fails or when Prophylactic treatment is to be started (after exponential random time). This gives T, the time to hospitalization, which is the minimum of three random times as follows given in table 5 for various values of α.

Table 5: Model I-Time to Hospitalization T =Min{X, X', X"} Varying Values of α

	T for α=0.5	T for α=1	T for α=1.5	T for α=2	T for α=2.5
Simulated T1	**0.059610000**	**0.059610000**	**0.059610000**	**0.059610000**	**0.059610000**
Simulated T2	**0.129077042**	**0.064538521**	**0.043025681**	**0.032269261**	**0.025815408**

Simulated T3	**0.307156480**	**0.287682072**	**0.191788048**	**0.143841036**	**0.115072829**
Simulated T4	**0.270289154**	**0.270289154**	**0.270289154**	**0.270289154**	**0.270289154**
Simulated T5	**0.267062785**	**0.133531393**	**0.089020928**	**0.066765696**	**0.053412557**
Simulated T6	**0.117350000**	**0.117350000**	**0.117350000**	**0.117350000**	**0.117350000**
Simulated T7	**0.836834133**	**0.836834133**	**0.775433873**	**0.581575405**	**0.465260324**
Simulated T8	**0.424633528**	**0.424633528**	**0.424633528**	**0.424633528**	**0.424633528**
Simulated T9	**0.500507776**	**0.500507776**	**0.500507776**	**0.413339287**	**0.330671429**
Simulated T10	**0.811865942**	**0.811865942**	**0.811865942**	**0.693147181**	**0.554517744**
Simulated T11	**0.447540000**	**0.447540000**	**0.383576097**	**0.287682072**	**0.230145658**
Simulated T12	**0.236900000**	**0.236900000**	**0.236900000**	**0.236900000**	**0.236900000**
Simulated T13	**0.218964549**	**0.218964549**	**0.218964549**	**0.187346725**	**0.149877380**
Simulated T14	**0.324172612**	**0.324172612**	**0.324172612**	**0.324172612**	**0.277258872**
Simulated T15	**0.175835000**	**0.175835000**	**0.138426243**	**0.103819682**	**0.083055746**
Average T Model I	**0.341853267**	**0.327350312**	**0.305704295**	**0.262849443**	**0.226258042**

Red color- Hospitalization due to Organ A failure; Green color- Hospitalization due to Organ B failure; Black color- Hospitalization for Prophylactic Treatment

In Model II the patient is sent for hospitalization when both organs A and B are in failed state or when Prophylactic treatment is to be started. This gives T as follows in the table 6.

Table 6: Model II time T = Min { Max {X, X'},X"} Varying Values of α

	T for α=0.5	**T for α=1**	**T for α=1.5**	**T for α=2**	**T for α=2.5**
Simulated T1	**0.769703169**	**0.470003629**	**0.313335753**	**0.235001815**	**0.188001452**
Simulated T2	**0.129077042**	**0.064538521**	**0.043025681**	**0.032269261**	**0.025815408**
Simulated T3	**0.575364145**	**0.287682072**	**0.191788048**	**0.143841036**	**0.115072829**
Simulated T4	**1.029000000**	**1.029000000**	**1.029000000**	**1.029000000**	**1.029000000**
Simulated T5	**0.267062785**	**0.133531393**	**0.089020928**	**0.066765696**	**0.053412557**
Simulated T6	**0.719554802**	**0.719554802**	**0.719554802**	**0.719554802**	**0.669590573**
Simulated T7	**0.868800000**	**0.868800000**	**0.775433873**	**0.581575405**	**0.465260324**
Simulated T8	**0.535000000**	**0.535000000**	**0.535000000**	**0.535000000**	**0.535000000**
Simulated T9	**0.628130000**	**0.628130000**	**0.551119049**	**0.413339287**	**0.330671429**
Simulated T10	**1.523500000**	**1.386294361**	**0.924196241**	**0.693147181**	**0.554517744**
Simulated T11	**0.512254421**	**0.512254421**	**0.383576097**	**0.287682072**	**0.230145658**
Simulated T12	**0.573293771**	**0.573293771**	**0.573293771**	**0.490414627**	**0.392331701**
Simulated T13	**0.301440000**	**0.301440000**	**0.249795633**	**0.187346725**	**0.149877380**
Simulated T14	**0.738500000**	**0.693147181**	**0.462098120**	**0.346573590**	**0.277258872**
Simulated T15	**0.415278730**	**0.207639365**	**0.138426243**	**0.103819682**	**0.083055746**
Average T Model II	**0.639063924**	**0.560687301**	**0.465244283**	**0.391022079**	**0.339934112**

Purple color- Hospitalization due to both Organs A and B failure; Black color- Hospitalization for Prophylactic Treatment

The simulated values of T decrease as α increases in the two tables 5 and 6.The black color numbers above indicates the time at which the patient is admitted for Prophylactic treatment in the Models I and II.Various Erlang random times may be similarly simulated for different types of treatments H_i for i=1, 2, 3, 4 and 5 using LCGs stated earlier.The exponential distributions assumed for the time to prophylactic treatment

have the parameter values α= 0.5, 1, 1.5, 2, and 2.5. Hospitalization times may be different for different simulations and at times different also for α values. These are nicely exhibited in tables 7 and 8.

Table 7: Model I - Hospitalization Times for Various Values of α

	H for α=0.5	H for α=1	H for α=1.5	H for α=2	H for α=2.5
Simulation 1	0.178014133	0.178014133	0.178014133	0.178014133	0.178014133
Simulation 2	0.097055469	0.097055469	0.097055469	0.097055469	0.097055469
Simulation 3	0.429127163	0.510045132	0.510045132	0.510045132	0.510045132
Simulation 4	0.532300365	0.532300365	0.532300365	0.532300365	0.532300365
Simulation 5	0.143841036	0.143841036	0.143841036	0.143841036	0.143841036
Simulation 6	0.736915253	0.736915253	0.736915253	0.736915253	0.736915253
Simulation 7	0.830926943	0.830926943	0.35733001	0.35733001	0.35733001
Simulation 8	0.477074175	0.477074175	0.477074175	0.477074175	0.477074175
Simulation 9	0.487356437	0.487356437	0.487356437	0.309382998	0.309382998
Simulation 10	0.172844829	0.172844829	0.172844829	0.35733001	0.35733001
Simulation 11	1.281526251	1.281526251	0.08470414	0.08470414	0.08470414
Simulation 12	0.611566967	0.611566967	0.611566967	0.611566967	0.611566967
Simulation 13	0.312007725	0.312007725	0.312007725	0.693147181	0.693147181
Simulation 14	0.154597456	0.154597456	0.154597456	0.154597456	0.174227962
Simulation 15	0.364649883	0.364649883	0.2161137	0.2161137	0.2161137
Average H Model I	0.453986939	0.45938147	0.338117789	0.363961202	0.365269902

Red color- Organ A treatment time; Green color- Organ B treatment time ; Black color- Prophylactic Treatment time

Table 8: Model II -Hospitalization Times for Various Values of α

	H for α=0.5	H for α=1	H for α=1.5	H for α=2	H for α=2.5
Simulation 1	1.77301139	0.435007443	0.435007443	0.435007443	0.435007443
Simulation 2	0.535342791	0.535342791	0.535342791	0.535342791	0.535342791
Simulation 3	2.266788266	2.266788266	2.266788266	2.266788266	2.266788266
Simulation 4	2.450532299	2.450532299	2.450532299	2.450532299	2.450532299
Simulation 5	1.327402843	1.327402843	1.327402843	1.327402843	1.327402843
Simulation 6	1.608259607	1.608259607	1.608259607	1.608259607	2.314443356
Simulation 7	3.406844385	3.406844385	0.970519609	0.970519609	0.970519609
Simulation 8	4.166757262	4.166757262	4.166757262	4.166757262	4.166757262
Simulation 9	1.392868149	1.392868149	1.621296176	1.621296176	1.621296176
Simulation 10	3.261702958	2.661016947	2.661016947	2.661016947	2.661016947
Simulation 11	0.845379362	0.845379362	1.306991846	1.306991846	1.306991846
Simulation 12	2.568555777	2.568555777	2.568555777	1.485329318	1.485329318
Simulation 13	2.557319349	2.557319349	1.63395508	1.63395508	1.63395508
Simulation 14	1.223705246	0.81036596	0.81036596	0.81036596	0.81036596
Simulation 15	0.320878117	0.320878117	0.320878117	0.320878117	0.320878117
Average H Model II	1.98035652	1.823554571	1.645578002	1.573362904	1.620441821

Purplet color- Both Organs A& B treatment time; Black color- Prophylactic Treatment time

It is better for the patient if he is admitted for prophylactic treatment instead of any organ treatment. It may be seen in the two models that when α increases the patient is admitted in many simulations for prophylactic treatment.

Table 9: The Effect of α on Time to Hospitalization and Hospitalization Time

	α=0.5	α=1	α=1.5	α=2	α=2.5
Average of T Model I	0.341853267	0.327350312	0.305704295	0.262849443	0.226258042

Average of T Model II	**0.639063924**	**0.560687301**	**0.465244283**	**0.391022079**	**0.339934112**
Average H Model I	**0.453986939**	**0.45938147**	**0.338117789**	**0.363961202**	**0.365269902**
Average H Model II	**1.98035652**	**1.823554571**	**1.645578002**	**1.573362904**	**1.620441821**

From the valuesobtained and presented in table 9 and the figures 1, 2 and 3 for them, it may seen that the average of T and average of H for model I are less than those average values of T and H for model II.

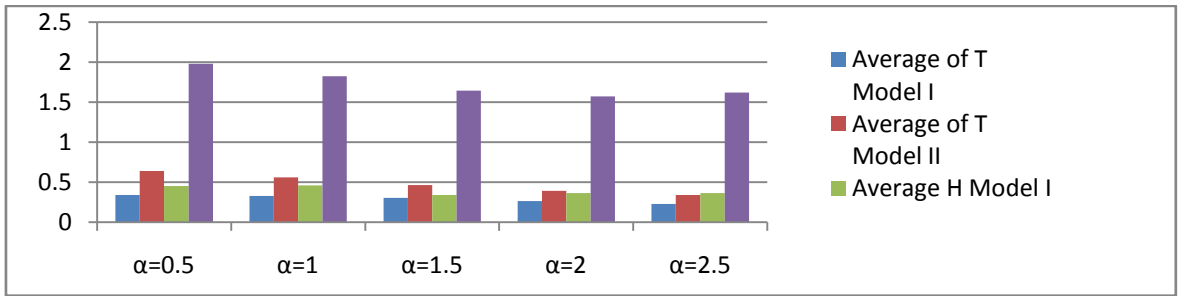
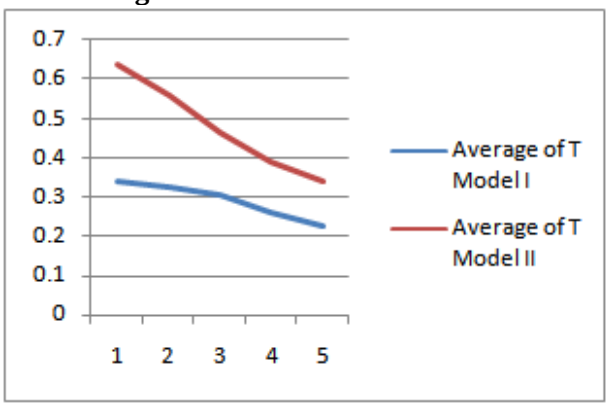


Figure1 : The Effect of α on Time to Hospitalization and Hospitalization Time



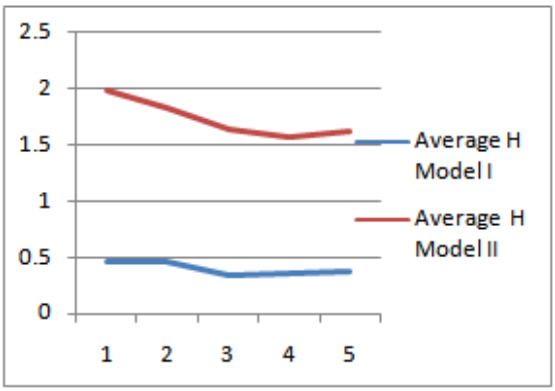


Figure2: The Effect of α on Time to Hospitalization; Figure 3 : The effect of α on Hospitalization Time

V. Conclusion

Two diabetic models are studied with prophylactic treatment where two organs are exposed to failures. In model I the patient is sent for hospitalization when an organ fails or when prophylactic treatment is provided. In model II he is sent for hospitalization when the two organs are in failed state or when he is provided prophylactic treatment. His organ A has two phase PH life time distribution and his organ B has general life time. The time to prophylactic treatment has exponential distribution.In model I the hospitalization times for organ A, organ B and prophylactic treatments have distinct distributions H_i for i=1, 2 and 3 respectively and in model II the hospitalization times for both organ A and organ B (combined failures treatment) and prophylactic treatment have distinct distributions H_i for i=4 and 5 respectively. The joint Laplace Stieltjes transform of the joint distribution of time to hospitalization and hospitalization time are obtained. The expected time to hospitalization and the expected hospitalization times are derived. Numerical studies are presented for the two models by fixing and varying parameter values. Simulation study has been presented in this area considering a set of parameter values of two phase life distribution of organ A, considering EC distribution for the life time of organ B, considering different parameter values of exponential time to prophylactic treatment and considering various Erlang distributions for hospitalization times for the two models. The averages for the two models for various parameter values are tabulated with graphical presentation. Since not much of simulation analysis are available in literature for diabetic models, this study opens up a real life like study in this area. Various other distributions if used for simulation studies also may produce more interesting results.

References

[1]. Bhattacharya S.K., Biswas R., Ghosh M.M., Banerjee., (1993), A Study of Risk Factors of Diabetes Mellitus, Indian Community Med., 18 (1993), p.7-13.
[2]. Foster D.W., Fauci A.S., Braunward E., Isselbacher K.J., Wilson J.S., Mortin J.B., Kasper D.L., (2002), Diabetes Mellitus, Principles of International Medicines,2, 15th edition p.2111-2126.
[3]. Kannell W.B., McGee D.L.,(1979), Diabetes and Cardiovascular Risk Factors- the Framingam Study, Circulation,59,p 8-13.

[4]. King H, Aubert R.E., Herman W.H., (1998), Global Burdon of Diabetes 1995-2025: Prevalence, Numerical Estimates and Projections, Diabetes Care 21, p 1414-1431.
[5]. King H and Rewers M.,(1993), Global Estimates for Prevalence of Diabetes Mellitus and Impaired Glucose Tolerance in Adults: WHO Ad Hoc Diabetes Reporting Group, Diabetes Care, 16, p157-177.
[6]. Usha K and Eswariprem., (2009), Stochastic Analysis of Time to Carbohydrate Metabolic Disorder, International Journal of Applied Mathematics,22,2, p317-330.
[7]. EswariPrem, R. Ramanarayanan, K. Usha, (2015),Stochastic Analysis of Prophylactic Treatment of a Diabetic Person with Erlang-2,IOSR Journal of Mathematics Vol. 11, Issue 5, Ver. 1 p01-07.
[8]. T.Osogami and M.H.Balter,(2003), A closed-Form Solution for Mapping General Distributions to Minimal PH Distributions,Carnegie Mellon Univ, Research Show case@CMU 9-2003, http://repository.cmu.edu/compsci.
[9]. Martin Haugh (2004) Generating Random Variables and Stochastic Processes, Monte Carlo Simulation: IEOR EA703
[10]. T.E. Hull and A.R. Dobell, (1962), Random Number Generators, Siam Review 4, No.3p230-249.
[11]. Mark Fackrell,(2009)Modeling Healthcare Systems with Phase-Type Distributions, Health Care Management Sci,12, p11-26, DOI 10,1007/s10729-008-9070-y.