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On Decomposition of Nano Continuity

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Abstract: *The aim of this paper is to obtain the decomposition of nano continuity in nano topological spaces.*
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I. Introduction and Preliminaries.

Continuity and its decomposition have been intensively studied in the eld of topology and other several branches of mathematics. Levine.N in [10] introduced the notion and decomposition of continuity in topological spaces. Jingcheng Tong in [6] in-troduced the notion of A-sets and A-continuity and established a decomposition of con-tinuity. Further, Jingcheng Tong in [5] introduced the notion of B-sets and B-continuity and established a decomposition of continuity. Ganster.M and Reilly.I.L in [2] improved Tong's decomposition result. Jingcheng Tong in [4] generalized Levine's[10] decomposi-tion theorem by introducing the notions of expansion of open sets in topological spaces. In recent years various classes of near to continuous functions were de ned by various authors. Recently, Maria Luisa Colasante in [13] studied almost continuity and the notions of expansion of open sets in topological spaces. Lellis Thivagar.M and Carmel Richard in [8] introduced the notion of Nano topology which was de ned in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. Lellis Thivagar.M and Carmel Richard in [9] studied a new class of functions called nano continuous functions and their characterizations in nano topo-logical spaces. In this paper, we study the notions of expansion of nano-open sets and obtain decomposition of nano continuity in nano topological spaces.In this connection, we refer [1],[3],[7],[11],[12],[14],[15] and [16].

De nition 1.1[8] Let U be a non-empty nite set of objects called the universe and R

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Be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let X U.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classi ed as X with respect to R and it is denoted byL_R(X).
That is, L_R(X)= [fR(x):R(x) Xg, where R(x) denotes the equivalence class deter-mined by x2U.
(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classi ed as X with respect to R and it is denoted byU_R(X). That is, U_R(X)= [fR(x):R(x)\X6= g.
(3) The boundary region of X with respect to R is the set of all objects, which can be classi ed neither as X nor as not-X with respect to R and it is denoted byB_R(X). That is, B_R(X)=U_R(X)-L_R(X).

Property 1.2[8]If (U,R) is an approximation space and X,Y U, then

(1) L_R(X) X U_R(X).

(2) L_R(')=U_R(')=' and L_R(U)=U_R(U)=U.

(3) U_R(X[Y)=U_R(X)[U_R(Y).

(4) U_R(X\Y) U_R(X)\U_R(Y).

(5) L_R(X[Y) L_R(X)[L_R(Y).

(6) L_R(X\Y)=U_R(X)\U_R(Y).

(7) $L_R(X)$ $L_R(Y)$ and $U_R(X)$ $U_R(Y)$ whenever X Y.

(8) $U_R(X^c)=[L_R(X)]^c$ and $L_R(X^c)=[U_R(X)]^c$.

(9) $U_RU_R(X)=L_RU_R(X)=U_R(X)$.

(10) $L_RL_R(X)=U_RL_R(X)=L_R(X)$.

De nition 1.3[8] Let U be the universe, R be an equivalence relation on U and $_R(X)$=fU, ,$L_R(X)$,$U_R(X)$,$B_R(X)$g where X U. Then by property 1.2, $_R(X)$ satis es the following axioms:

(1) U and 2 $_R(X)$.
(2) The union of the elements of any subcollection of $_R(X)$ is in $_R(X)$.

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(3) The intersection of the elements of any nite subcollection of $_R(X)$ is in $_R(X)$.

That is, $_R(X)$ is a topology on U called the nano topology on U with respect to X. We call (U, $_R(X)$) as the nano topological space. The elements of $_R(X)$ are called as nano-open sets. If (U, $_R(X)$) is a nano topological space[8] where X U and if A U, then the nano interior of A is de ned as the union of all nano-open subsets of A and it is denoted by NInt(A). NInt(A) is the largest nano-open subset of A. The nano closure of A is de ned as the intersection of all nano closed sets containing A and it is denoted by NCl(A). That is, NCl(A) is the smallest nano closed set containing A.

De nition 1.4[9]Let (U, $_R(X)$) and (V, $_R{}^0(Y)$)be two nano topological spaces. Then a mapping f:(U, $_R(X)$)!(V, $_R{}^0(Y)$) is nano continuous on U if the inverse image of every nano-open set in V is nano-open in U.

2. Expansion of nano-open sets.

De nition 2.1. Let (U, $_R(X)$)be a nano topological space, 2^U be the set of all subsets of U. A mapping A: $_R(X)$! 2^U is said to be an expansion on (U, $_R(X)$) if D AD for each D2 $_R(X)$.

Remark 2.2. Let us study the expansion of nano-open sets in nano topological spaces. Let (U, $_R(X)$) be an nano topological space,

(1) De ne CL: $_R(X)$! 2^U by CL(D)=NCl(D) for each D2 $_R(X)$.Then CL is an expan-sion on (U, $_R(X)$),because D CL(D) for each D2 $_R(X)$.

(2) Since for each D2 $_R(X)$, D is nano-open and hence NInt(D)=D,F(D) can be de ned as F: $_R(X)$! 2^U by $F(D)=(NCl(D)-D)^c$ .Then F is an expansion on (U, $_R(X)$). Here,

$F(D)=(NCl(D)-D)^c=(NCl(D)\backslash D^c)^c=(NCl(D))^c[D$ D for each D2 $_R(X)$.

(3) De ne NIntCL: $_R(X)$! 2^U by NIntCL(D)=NIntNCl(D) for each D2 $_R(X)$.Then NIntCL is an expansion on (U, $_R(X)$),because D NIntCL(D) for each D2 $_R(X)$.

(4) De ne F_s: $_R(X)$! 2^U by $F_s(D)=D[(NIntNCl(D))^c$ for each D2 $_R(X)$.Then F_s is an expansion on (U, $_R(X)$).

De nition 2.3. Let (U, $_R(X)$) be an nano topological space. A pair of expansion A,B on (U, $_R(X)$) is said to be mutually dual if AD\BD=D for each D2 $_R(X)$.

Example 2.4. Let U=fa,b,c,dg,X=fa; bg and ^U_R =ffag; fcg; fb; dgg, with nano topology $_R(X)$=fU; ;; fag; fa; b; dg; fb; dgg then CLfag=fa; cg , CLfb; dg=fb; c; dg , CLfUg=fUg , Ffag=fa; b; dg , Ffb; dg=fa; b; dg , FfUg=fUg Here CL and F are both mutually dual to

Proposition 2.5. Let (U, $_R(X)$)be a nano topological space. Then the expansions CL and F are mutually dual.

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Proof: Let D2 $_{R}$(X). Now,

CL(D)\F(D)=NCl(D)\(NCl(D)-D)^{c} =NCl(D)\(NCl(D)\D^{c})^{c}=NCl(D)\((NCl(D))^{c}[D) =(NCl(D)\(NCl(D))^{c})[((NCl(D))\D)= [D=D

That is, CL(D)\F(D)=D,for each D2 $_{R}$(X). Therefore the expansions CL and F are mutu-ally dual.

Remark 2.6. The identity expansion AD=D is mutually dual to any expansion B. The pair of expansions CL, F and NIntCL,F$_{s}$ are easily seen to be mutually dual.

De nition 2.7. A function f:(U, $_{R}$(X))!(V, $_{R}{}^{0}$(Y)) is said to be nano almost continu-ous if for each nano-open set E in V containing f(x), there exists an nano-open set D in U containing x such that f(D) NInt(NCl(E)).

Theorem 2.8. A function f:(U, $_{R}$(X))!(V, $_{R}{}^{0}$(Y)) is nano almost continuous if and only if f ^{1}(E) NInt(f ^{1}(NInt(NCl(E)))) for any nano-open set E in V.

Proof. Necessity: Let E be an arbitrary nano-open set in V and let x2f ^{1}(E) then f(x)2E. Since E is nano-open, it is a neighborhood of f(x) in V. Since f ia nano almost continuous at x, there exists a nano-open neighbourhood D of x in V such that f(D) NInt(NCl(E)). This implies that D f ^{1}(NInt(NCl(E))), thus x2D f ^{1}(NInt(NCl(E))).
Thus, f ^{1}(E) NInt(f ^{1}(NInt(NCl(E)))).

Su ciency : Let E be an arbitrary nano-open set in V such that f(x)2E. Then, x2f ^{1}(E) NInt(f ^{1}(NInt(NCl(E)))). Take D=NInt(f ^{1}(NInt(NCl(E)))),
then f(D) f(f ^{1}(NInt(NCl(E)))) NInt(NCl(E)) such that f(D) NInt(NCl(E)). By De ni-tion 2.7, f is nano almost continuous.

Proposition 2.9. If a function f:(U, $_{R}$(X))!(V, $_{R}{}^{0}$(Y)) is nano continuous, then f is nano almost continuous.

Proof: Let E be a nano-open set in V, then E NInt(NCl(E)). Since f is nano continuous, f ^{1}(E) is nano-open in U such that f(E) f(f ^{1}(NInt(NCl(E)))). Since f ^{1}(E)=NInt(f ^{1}(E)) in U, f ^{1}(E)=NInt(f ^{1}(E)) NInt(f ^{1}(NInt(NCl(E)))). By Theorem 2.8, f is nano almost continuous.

3.Decomposition of Nano continuity.

De nition 3.1. Let (U, $_{R}$(X)) and (V, $_{R}{}^{0}$(Y))be two nano topological spaces. A map-ping f:(U, $_{R}$(X))!(V, $_{R}{}^{0}$(Y)) is said to be A-expansion nano continuous if

f ^{1}(E) NIntf ^{1}(AE),for each E2 $_{R}{}^{0}$(Y).

Theorem 3.2. Let (U, $_{R}$(X)) and (V, $_{R}{}^{0}$(Y))be two nano topological spaces. Let A and B be two mutually dual expansion on V. Then a mapping f:(U, $_{R}$(X))!(V, $_{R}{}^{0}$(Y)) is nano continuous if and only if f is A-expansion nano continuous and B-expansion nano continuous. Proof. Necessity: Since A and B are mutually dual on V, AE\BE=E for each E2 $_{R}{}^{0}$(Y).
Let E2 $_{R}{}^{0}$(Y) then f ^{1}(E)=f ^{1}(AE)\f ^{1}(BE), Since f is nano continuous,

f ^{1}(E)=NIntf ^{1}(E). So, f ^{1}(E)=NInt(f ^{1}(AE)\f ^{1}(BE))=NIntf ^{1}(AE)\NIntf ^{1}(BE). Thus f ^{1}(E) NIntf ^{1}(AE) and f ^{1}(E) NIntf ^{1}(BE),for each E2 $_{R}{}^{0}$(Y).
Hence f is A-expansion nano continuous and B-expansion nano continuous.

Su ciency : Let A and B be two mutually dual expansion on (V, $_{R}{}^{0}$(Y)). Since f is A-expansion nano continuous,f ^{1}(E) NIntf ^{1}(AE),for each E2 $_{R}{}^{0}$(Y). Since f is B-expansion nano continuous, f ^{1}(E) NIntf ^{1}(BE),for each E2 $_{R}{}^{0}$(Y). Also AE\BE=E for each E2 $_{R}{}^{0}$(Y). Therefore, f ^{1}(AE)\f ^{1}(BE)=f ^{1}(E).

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Hence, NIntf ^{1}(E)=(NIntf ^{1}(AE)\NIntf ^{1}(BE)) f ^{1}(E)\f ^{1}(E)=f ^{1}(E).

So, NIntf ^{1}(E) f ^{1}(E). But,NIntf ^{1}(E) f ^{1}(E) always.Therefore,f ^{1}(E)=NIntf ^{1}(E). This implies that f ^{1}(E) is an open set in (U, $_{R}$(X)) for each E2 $_{R}{}^{0}$(Y). Therefore f is nano con-tinuous.

Corollary 3.3. A mapping f:(U, $_{R}$(X))!(V, $_{R}{}^{0}$(Y)) is nano continuous if and only if f is nano almost continuous and F_s-expansion nano continuous.

Proof : We have that the condition f is nano almost continuous is equivalent to f is NIntCL-expansion nano continuous, and the condition f is F_s-expansion nano continuous is equivalent to f ^{1}(E) NIntf ^{1}(E[(NIntNCl(E)^{c})), for each nano-open set E in V. Since NIntCL and F_s are mutually dual, the result follows from theorem 3.2.

Theorem 3.4. Let A be any expansion on (V, $_{R}{}^{0}$(Y)). Then the expansion BE = E[(AE)^{c} is the maximal expansion on (V, $_{R}{}^{0}$(Y)) which is mutually dual to A.

Proof: Let B_A be the set of all expansions on (V, $_{R}{}^{0}$(Y)) which are mutually dual to A. Since E AE, for any E2 $_{R}{}^{0}$(Y), AE can be written as AE = E[(AEnE). Let BE=E[(AE)^{c}=(AEnE)^{c}.
It is obvious that B is an expansion on (V, $_{R}{}^{0}$(Y)) and AE\BE=E for any E2 $_{R}{}^{0}$(Y). Thus
B2B_A. Given any expansion B^0 on (V, $_{R}{}^{0}$(Y)), write B^0 E=E[(B^0 EnE). If B^0 2B_A, then (AEnE)\(B^0 EnE)= , thus (B^0 EnE) (AEnE)^{c}. Therefore B^0 E BE and we have that B^0 <B,

That is, B is the maximal element of B_A.

De nition 3.5. Let (U, $_{R}$(X)) and (V, $_{R}{}^{0}$(Y))be two nano topological spaces. Let B be an expansion on (V, $_{R}{}^{0}$(Y)). A mapping f:(U, $_{R}$(X))!(V, $_{R}{}^{0}$(Y)) is said to be closed B-nano continuous if f ^{1}((BE)^{c}) is a nano closed set in (U, $_{R}$(X)) for each E2 $_{R}{}^{0}$(Y).

Proposition 3.6. A closed B-nano continuous mapping f:(U, $_{R}$(X))!(V, $_{R}{}^{0}$(Y)) is B-
expansion nano continuous.
Proof: We rst prove (f ^{1}((BE)^{c}))^{c}=f ^{1}(BE).Let x2(f ^{1}((BE)^{c}))^{c}.

Then x2=(f ^{1}((BE)^{c})). Hence f(x)2=(BE)^{c},this implies f(x)2(BE) and x2f ^{1}(BE). So, (f ^{1}((BE)^{c}))^{c} f ^{1}(BE).

Conversely,let x2f ^{1}(BE).Then f(x)2(BE). Hence,f(x)2=(BE)^{c},x2=(f ^{1}((BE)^{c})),this implies x2(f ^{1}((BE)^{c}))^{c}.
So, f ^{1}(BE) (f ^{1}((BE)^{c}))^{c}. Therefore, (f ^{1}((BE)^{c}))^{c}=f ^{1}(BE).

Since f:(U, $_{R}$(X))!(V, $_{R}{}^{0}$(Y)) is a B-nano continuous mapping,f ^{1}((BE)^{c}) is a nano closed set in (U, $_{R}$(X)). Hence f ^{1}(BE) is nano-open in (U, $_{R}$(X)) and

so f ^{1}(BE)=NIntf ^{1}(BE). Also, and this implies f ^{1}(E) f ^{1}(BE)=NIntf ^{1}(BE). There-fore f ^{1}(E) NInt(f ^{1}(BE)) for each E2 $_{R}{}^{0}$(Y). Hence f is B-expansion nano continuous.

Example 3.7. Consider the identity mapping I:(R, $_{R}$(X))!(R, $_{R}$(X)). Let U=R and X U, where R is the set of real numbers. De ne B: $_{R}$(X)!2^{R} such that BD=CL(D) for all D2 $_{R}$(X). Let X=f[0,2],(2,3),[3,5]g then $_{R}$(X)=fU, ,[0,4],[0,5],(4,5]g. When D=(4,5], then I ^{1}((BD)^{c}) is not nano closed. Therefore I is not closed B-nano continuous even though it is nano continuous.

De nition 3.8. An expansion A on (U, $_{R}$(X)) is said to be nano-open if AV2 $_{R}$(X) for all V2 $_{R}$(X).

De nition 3.9. An nano-open expansion A on (U, $_{R}$(X)) is said to be idempotent.

Example 3.10. The expansion FD=(NCl(D)-NInt(D))^{c} for each D2 $_{R}$(X) is idempotent.

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Open,

In fact the expansion F is nano- c = c c

NCl(D) NInt(D)) ((NCl(D) D))

F(FD)=F(c Fc c \ c [D)

=F((NCl(D)) [D)=(NCl((NCl(D)) [D)) [((NCl(D))

$=(NCl(NCl(D))^c[NCl(D))^c[((NCl(D))^c[D)=(NCl(NCl(D))^c)^c\backslash(NCl(D))^c[((NCl(D))^c[D) =((NCl(D))^c[D) =FD$

Remark 3.11. From the Example 3.7, we conclude that nano continuity does not im-ply closed B-nano continuity. Since f is nano continuous it is B-expansion nano continuous, but it is not closed B-nano continuous. The proposition gives a condition under which a B-expansion nano continuous function is closed B-nano continuous and vice versa.

Proposition 3.12. Let $f:(U, _R(X))!(V, _R{}^0(Y))$ and B be idempotent, then f is B-expansion nano continuous if and only if f is closed B-nano continuous.

Proof: The su ciency follows from Proposition 3.6.

Necessity: Let f be B-expansion nano continuous, where B is idempotent and E an nano-open subset of $(V, _R{}^0(Y))$. Since BE is nano-open on $(V, _R{}^0(Y))$ and B(BE)=BE, then f^{1}(BE) NIntf ^{1}(B(BE))=NIntf ^{1}(BE). Thus f^{1}(BE) is nano open in $(U, _R(X))$ and therefore f is closed B-nano continuous.

Corollary 3.13: Let A and B be two mutually dual expansion on $(V, _R{}^0(Y))$. If B is idempo-tent, then $f:(U, _R(X))!(V, _R{}^0(Y))$ is nano continuous if and only if f is A-expansion nano continuous and closed B-nano continuous.

VII. Conclusion.

In this paper, the notions of expansion of nano-open sets and decomposi-tion of nano continuity in nano topological spaces are studied. The theory of expansions and decomposition in nano topological spaces has a wide variety of applications in real life. The decomposition of nano topological space can be applied in the study of independence of real time problems and in de ning its attributes .

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