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Homomorphism and Anti-homomorphism of Multi-Fuzzy Ideal and Multi-Anti Fuzzy Ideal of a Ring

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Abstract: *In this paper, we discuss the properties of image of multi- fuzzy ideal of a ring under homomorphism and anti homomorphism and the properties of image of multi-anti fuzzy ideal of a ring under homomorphism and anti homomorphism*
Keywords: *multi-anti fuzzy ideals, homomorphism and anti homomorphism of multi-fuzzy ideal and multi-anti fuzzy ideal of a ring.*

I. Introduction

The innovative works of Zadeh [16] and Rosenfeld [12] led to the fuzzification of algebraic structures. The idea of anti fuzzy subgroup was introduced by Biswas [3] which was extended by many researchers. F.A.Azam, A.A. Mamum, and F.Nasrin [2] apply the idea of Biswas to the theory of ring. They introduced a notion of anti fuzzy ideal A of a ring X.

Sabu Sebastian and T.V. Ramakrishnan [13] introduced the theory of multi-fuzzy sets in terms of multi-dimensional membership functions and investigated some properties of multi level fuzziness. After introducing multi-fuzzy subsets of a crisp set, they have also introduced and studied some elementary properties of multi-fuzzy subgroups. R. Muthuraj and S. Balamurugan [9] introduced the concept of multi-anti fuzzy subgroup and discussed some of its properties.

In [11], we extended the concept of multi-anti fuzzy subgroup to multi-anti fuzzy ideal of a ring. and introduced a notion of multi-anti fuzzy ideal A of a ring X and some of its properties are discussed.

In this paper, we discuss the properties of image of multi- fuzzy ideal of a ring under homomorphism and anti homomorphism and the properties of image of multi-anti fuzzy ideal of a ring under homomorphism and anti homomorphism.

1.1 Basic Concepts

The theory of multi-fuzzy set is an extension of theories of fuzzy sets. The membership function of a multi-fuzzy set is an ordered sequence of membership functions of a fuzzy set. The notion of multi-fuzzy sets provides a new method to represent some problems which are difficult to explain in other extensions of fuzzy set theory.

Throughout this paper, we will use the following notations (i) R and S for ring . (ii) J for index set, (iii) X for the Universal set, (iv) I for the unit interval [0, 1] and (v) I^X for the set of all functions from X to I respectively

1.2 Definition [13]

Let X is a non-empty set. A multi-fuzzy set A in X is a set of ordered sequences

$A = \{(x, \mu_1(x), \mu_2(x), \dots \mu_k(x), \dots): x \in X\}$ where $\mu_j(x) \in I$, $\forall$ j = 1,2,…,k,…

Remarks [13]

i. If the sequences of the membership functions have only k-terms, then k is called the dimension of A.
ii. The set of all multi-fuzzy sets in X of dimension k is denoted by $M^K FS(X)$.
iii. The multi-membership function A(x) of dimension k is denoted by
$A(x) = (\mu_1(x), \mu_2(x), \dots, \mu_k(x))$, for all $x \in X$.

1.3 Definition [13]

Let k be a positive integer and A and B be a multi-fuzzy set of dimension k on X.
That is, $A = \{(x, \mu_1(x), \mu_2(x) \dots \mu_k(x)), x \in X\}$ and $B = \{(x, \gamma_1(x), \gamma_2(x) \dots \gamma_k(x)), x \in X\}$ where $\mu_j(x), \gamma_j(x) \in I$, $\forall$ j = 1,2,…,k

Then we have the following relations and operations for all $x \in X$

i. $A = B$ iff $\mu_i(x) = \gamma_j(x)\ \forall\ j = 1,2,\ldots,k$
ii. $A \leq B$ iff $\mu_i(x) \leq \gamma_j(x)\ \forall\ j = 1,2,\ldots,k$
iii. $A \cup B = \{(x, \max\{\mu_1(x), \gamma_1(x)\},\ldots,\max\{\mu_k(x),\gamma_k(x)\}) : x \in X\}$
iv. $A \cap B = \{(x, \min\{\mu_1(x), \gamma_1(x)\},\ldots, \min\{\mu_k(x), \gamma_k(x)\}) : x \in X\}$

1.4 Definition [6]

A mapping f from a ring R to a ring S (both R and S not necessarily commutative) is called an anti-homomorphism if for all $x, y \in R$

i. $f(x + y)= f(y) + f(x)$ and
ii. $f(xy)= f(y)f(x)$.

A surjective anti-homomorphism is called an anti-epimorphism.

1.5 Definition

Let f be a mapping from a set R to a set S and let A be a multi-fuzzy subset in R. Then A is called f-invariant if $f(x) = f(y)$ implies $A(x) = A(y)$ for all $x, y \in R$.
Clearly, if A is f-invariant, then $f^{-1}(f(A)) = A$.

II. Properties Multi-Fuzzy Ideal Of A Ring

In this section, we discuss some results on multi-fuzzy ideal of a ring under homomorphism and anti-homomorphism

2.1 Definition [10]

A multi-fuzzy set A on a ring R is said to be a multi-fuzzy ring on R if for every $x, y \in R$,

i. $A(x - y) \geq \min\{A(x), A(y)\}$ and
ii. $A(xy) \geq \min\{A(x), A(y)\}$.

2.2 Definition [10]

A multi-fuzzy ring A on R is said to be

i. a multi-fuzzy left ideal if $A(xy) \geq A(y)$, for all $x, y \in R$ and
ii. a multi-fuzzy right ideal if $A(xy) \geq A(x)$, for all $x, y \in R$.

2.3 Definition [10]

A multi-fuzzy ring A on a ring R is called a multi-fuzzy ideal if it is both a multi-fuzzy left ideal and a multi-fuzzy right ideal.

In other words, a multi-fuzzy set A on R is a multi-fuzzy ideal of a ring if

i. $A(x - y) \geq \min\{A(x), A(y)\}$ and
ii. $A(xy) \geq \max\{A(x), A(y)\}$, for all $x, y \in R$.

2.4 Definition

A multi-fuzzy set A in X has the sup property if, for any subset T of X, there exists

$t_0 \in T$ such that $A(t_0) = \sup_{t \in T} A(t)$.

2.5 Definition

Let f be a mapping from a set X to a set Y, and let A and B be multi-fuzzy subsets in X and Y respectively.

i. f(A), the image of A under f, is a multi-fuzzy subset in Y. For all $y \in Y$, we define,

iii.
$$f(A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} A(x) & \text{if } f^{-1}(y) \neq \varphi \\ 0, & \text{Otherwise} \end{cases}$$

ii. $f^{-1}(B)$, is the pre-image of B under f, is a multi-fuzzy set in X.
That is, $f^{-1}(B)(x) = B(f(x))$ for all $x \in R$.

2.6 Theorem

Let f be a homomorphism from a ring R into a ring S and let B be a multi-fuzzy left ideal of S. Then the pre-image, $f^{-1}(B)$ is a multi-fuzzy left ideal left of R.

Proof

Consider a ring homomorphism $f : R \rightarrow S$
Let B be a multi- fuzzy left ideal of S.
For all $x, y \in R$

i. $f^{-1}(B)(x-y) = Bf(x-y)$
$= B(f(x) - f(y))$
$\geq \min\{Bf(x), Bf(y)\}$
$= \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(x-y) \geq \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

ii. $f^{-1}(B)(xy) = B(f(xy))$
$= B(f(xy))$
$\geq \max\{Bf(x), Bf(y)\}$
$= \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(xy) \geq \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

iii. $f^{-1}(B)(xy) = B\, f(xy)$
$= B(f(x)f(y))$
$\geq B(f(y))$
$= f^{-1}(B)(y))$
$f^{-1}(B)(xy) \geq f^{-1}(B)(y))$

Therefore, $f^{-1}(B)$ is a multi-fuzzy left ideal of R.

2.7 Theorem

Let f be a homomorphism from a ring R into a ring S and let B be a multi-fuzzy right ideal of S. Then the pre-image, $f^{-1}(B)$ is a multi-fuzzy right ideal of R.

Proof

Consider a ring homomorphism $f : R \rightarrow S$
Let B be a multi-fuzzy right ideal of S.
For all $x, y \in R$

i. $f^{-1}(B)(x-y) = Bf(x-y)$
$= B(f(x) - f(y))$
$\geq \min\{Bf(x), Bf(y)\}$
$= \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(x-y) \geq \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

ii. $f^{-1}(B)(xy) = B(f(xy))$
$= B(f(xy))$
$\geq \max\{Bf(x), Bf(y)\}$
$= \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(xy) \geq \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$

iii. $f^{-1}(B)(xy) = Bf(xy)$
$= B(f(x)f(y))$
$\geq B(f(x))$
$= f^{-1}(B)(x))$
$f^{-1}(B)(xy) \geq f^{-1}(B)(x))$

Therefore, $f^{-1}(B)$ is a multi-fuzzy right ideal of R.

2.8 Theorem

Let f be a homomorphism from a ring R into a ring S, and let B be a multi-fuzzy ideal of S. Then the pre-image, $f^{-1}(B)$ is a multi-fuzzy ideal of R.

Proof

It is clear.

2.9 Theorem

Let f be a homomorphism from a ring R into a ring S, and let A be a multi-fuzzy left ideal of a ring R with sup property. Then the image, f(A) is a multi-fuzzy left ideal of a ring S.

Proof

Consider a ring homomorphism $f : R \to S$
Let A be a multi- fuzzy left ideal of R.
For all $x, y \in R$

i. $f(A)(f(x)-f(y) = f(A)\, f(x-y)$
$= A(x-y)$
$\geq \min\{A(x), A(y)\}$
$= \min\{ f(A)(x), f(A)(y)\}$
$f(A)(f(x)-f(y) \geq \min\{ f(A)(x), f(A)(y)\}$.

ii. $f(A)(f(x)f(y)) = f(A)f(x\,y)$
$= A(xy)$
$\geq \max\{A(x), A(y)\}$
$= \max\{ f(A)(x), f(A)(y)\}$
$f(A)(f(x)f(y)) \geq \max\{ f(A)(x), f(A)(y)\}$

iii. $f(A)(f(x)f(y)) = f(A)f(x\,y)$
$= A(xy)$
$\geq A(y)$
$= f(A)(f(y))$
$f(A)(f(x)f(y)) \geq f(A)(f(y))$

Therefore, f(A) is a multi-fuzzy left ideal of S,

2.10 Theorem

Let f be a homomorphism from a ring R into a ring S, and let A be a multi-fuzzy right ideal of a ring R with sup property. Then the image, f(A) is a multi-fuzzy right ideal of a ring S.

Proof

Consider a ring homomorphism $f : R \to S$
Let A be a multi- fuzzy right ideal of R.
For all $x, y \in R$

i. $f(A)(f(x)-f(y) = f(A)\, f(x-y)$
$= A(x-y)$
$\geq \min\{A(x), A(y)\}$
$= \min\{ f(A)(x), f(A)(y)\}$
$f(A)(f(x)-f(y) \geq \min\{ f(A)(x), f(A)(y)\}$.

ii. $f(A)(f(x)f(y)) = f(A)f(x\,y)$
$= A(xy)$
$\geq \max\{A(x), A(y)\}$
$= \max\{ f(A)(x), f(A)(y)\}$
$f(A)(f(x)f(y)) \geq \max\{ f(A)(x), f(A)(y)\}$.

iii. $f(A)(f(x)f(y)) = f(A)f(x\,y)$
$= A(xy)$
$\geq A(x)$
$= f(A)(f(x))$
$f(A)(f(x)f(y)) \geq f(A)(f(x))$

Therefore, f(A) is a multi-fuzzy right ideal of S

2.11 Theorem

Let f be a homomorphism from a ring R into a ring S, and let A be a multi-fuzzy ideal of a ring R with sup property. Then the image, f(A) is a multi-fuzzy ideal of a ring S.

Proof

It is clear.

2.12 Theorem

Let f be an anti- homomorphism from a ring R into a ring S, and let B be a multi-fuzzy left ideal of S. Then the pre-image, $f^{-1}(B)$ is a multi-fuzzy right ideal of R.

Proof

Consider a ring anti-homomorphism $f : R \rightarrow S$
Let B be a multi- fuzzy left ideal of S.
For all $x, y \in R$

i. $f^{-1}(B)(x - y) = Bf(x - y)$
$= B(f(y) - f(x))$
$\geq \min\{Bf(y), Bf(x)\}$
$= \min\{f^{-1}(B)(y), f^{-1}(B)(x)\}$
$= \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(x - y) \geq \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$.

ii. $f^{-1}(B)(xy) = B(f(xy))$
$= B(f(y)f(x))$
$\geq \max\{Bf(y), Bf(x)\}$
$= \max\{f^{-1}(B)(y), f^{-1}(B)(x)\}$
$= \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(xy) \geq \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$.

iii. $f^{-1}(B)(xy) = B\,f(xy)$
$= B(f(y)f(x))$
$\geq B(f(x))$
$= f^{-1}(B)(x))$
$f^{-1}(B)(xy) \geq f^{-1}(B)(x))$.

Therefore, $f^{-1}(B)$ is a multi-fuzzy right ideal of R.

2.13 Theorem

Let f be an anti- homomorphism from a ring R into a ring S, and let B be a multi-fuzzy right ideal of S. Then the pre-image, $f^{-1}(B)$ is a multi-fuzzy left ideal of R.

Proof

Consider a ring anti-homomorphism $f : R \rightarrow S$
Let B be a multi- fuzzy right ideal of S.
For all $x, y \in R$

i. $f^{-1}(B)(x - y) = Bf(x - y)$
$= B(f(y) - f(x))$
$\geq \min\{Bf(y), Bf(x)\}$
$= \min\{f^{-1}(B)(y), f^{-1}(B)(x)\}$
$= \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(x - y) \geq \min\{f^{-1}(B)(x), f^{-1}(B)(y)\}$.

ii. $f^{-1}(B)(xy) = B(f(xy))$
$= B(f(y)f(x))$
$\geq \max\{Bf(y), Bf(x)\}$
$= \max\{f^{-1}(B)(y), f^{-1}(B)(x)\}$
$= \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(xy) \geq \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$.

iii $f^{-1}(B)(xy) = B\,f(xy)$
$= B(f(y)f(x))$
$\geq B(f(y))$
$= f^{-1}(B)(y))$
$f^{-1}(B)(xy) \geq f^{-1}(B)(y))$

Therefore, $f^{-1}(B)$ is a multi-fuzzy left ideal of R.

2.14 Theorem

Let f be an anti- homomorphism from a ring R into a ring S, and let B be a multi-fuzzy left (right) ideal of S. Then the pre-image, $f^{-1}(B)$ is a multi-fuzzy right(left) ideal of R.

Proof

It is clear.

2.15 Theorem

Let f be an anti- homomorphism from a ring R into a ring S, and let A be a multi-fuzzy left ideal of a ring R with sup property. Then the image, f(A) is a multi-fuzzy right ideal of a ring S.

Proof

Consider a ring anti-homomorphism $f : R \rightarrow S$

Let A be a multi- fuzzy left ideal of R.

For all $x, y \in R$

i. $f(A)(f(x)-f(y)) = f(A)\, f(y-x)$
$= A(y-x)$
$\geq \min\{A(y), A(x)\}$
$= \min\{ f(A)(y), f(A)(x)\}$
$= \min\{ f(A)(x), f(A)(y)\}$
$f(A)(f(x)-f(y)) \geq \min\{ f(A)(x), f(A)(y)\}$.

ii. $f(A)(f(x)f(y)) = f(A)f(yx)$
$= A(yx)$
$\geq \max\{A(y), A(x)\}$
$= \max\{ f(A)(y), f(A)(x)\}$
$= \max\{ f(A)(x), f(A)(y)\}$
$f(A)(f(x)f(y)) \geq \max\{ f(A)(x), f(A)(y)\}$.

iii. $f(A)(f(x)f(y)) = f(A)f(yx)$
$= A(yx)$
$\geq A(x)$
$= f(A)(f(x))$
$f(A)(f(x)f(y)) \geq f(A)(f(x))$.

Therefore, f(A) is a multi-fuzzy right ideal of S

2.16 Theorem

Let f be an anti-homomorphism from a ring R into a ring S, and let A be a multi-fuzzy right ideal of a ring R with sup property. Then the image, f (A) is a multi-fuzzy left ideal of a
ring S.

Proof

Consider a ring anti-homomorphism $f : R \rightarrow S$

Let A be a multi- fuzzy right ideal of R.

For all $x, y \in R$

i. $f(A)(f(x)-f(y)) = f(A)\, f(y-x)$
$= A(y-x)$
$\geq \min\{A(y), A(x)\}$
$= \min\{ f(A)(y), f(A)(x)\}$
$= \min\{ f(A)(x), f(A)(y)\}$
$f(A)(f(x)-f(y)) \geq \min\{ f(A)(x), f(A)(y)\}$

ii. $f(A)(f(x)f(y)) = f(A)f(y\, x)$
$= A(yx)$
$\geq \max\{A(y), A(x)\}$
$= \max\{ f(A)(y), f(A)(x)\}$
$= \max\{ f(A)(x), f(A)(y)\}$.

f(A) (f(x)f(y)) $\geq$ max{ f(A) (x), f(A)(y)}

iii. f(A) (f(x)f(y)) = f(A)f(y x)
= A(yx)
$\geq$ A(y)
= f(A) (f(y))
f(A) (f(x)f(y)) $\geq$ f(A) (f(y))

Therefore, f(A) is a multi-fuzzy left ideal of S

2.17 Theorem

Let f be an anti- homomorphism from a ring R into a ring S, and let A be a multi-fuzzy left(right) ideal of a ring R with sup property. Then the image, f(A) is a multi-fuzzy right (left) ideal of a ring S.

Proof

It is clear.

III. Properties Multi-Anti Fuzzy Ideal Of A Ring

In this section, we discuss some results on multi-anti fuzzy ideal of a ring under homomorphism and anti-homomorphism

3.1 Definition [11]

A multi-fuzzy set A of X is called a multi- anti fuzzy left (respectively right) ideal of X if for all x, y $\in$ X,

i. $A(x-y) \leq \max\{A(x), A(y)\}$
ii. $A(xy) \leq \max\{A(x), A(y)\}$
iii. $A(xy) \leq A(y)$ (respectively right $A(xy) \leq A(x)$).

3.2 Definition [11]

A MFS A of X is called a multi-anti fuzzy ideal of X if it is a multi-anti fuzzy left ideal as well as a multi-anti fuzzy right ideal of X.

Remark

i. A MFS A of X is a multi-anti fuzzy left (respectively right) ideal of X if and only if A^C is multi fuzzy left (respectively right) ideal of X.
ii. Every multi-anti fuzzy left (right) ideal of X is an additive multi-anti fuzzy subgroup of X.

3.3 Definition [11]

If A is a multi-anti fuzzy ideal of X, Then for all x, y $\in$ X,

i. $A(x-y) \leq \max\{A(x), A(y)\}$
ii. $A(xy) \leq \max\{A(x), A(y)\}$.

3.4 Definition

A multi-fuzzy set A in X has the inf property if, for any subset T of X, there exists $t_0 \in T$ such that

$A(t_0) = \inf_{t \in T} A(t)$.

3.5 Definition [11]

Let f be a mapping from a set X to a set Y, and let A and B be multi-fuzzy subsets in X and Y respectively.

i. f(A), the anti image of A under f, is a multi-fuzzy subset in Y. For all y $\in$ Y,

I. we define, $f(A)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} A(x) & \text{if } f^{-1}(y) \neq \varphi \\ 0 & \text{Otherwise} \end{cases}$

ii. $f^{-1}(B)$, is the anti pre-image of B under f, is a multi-fuzzy set in X.
That is, $f^{-1}(B)(x) = B(f(x))$ for all x $\in$ R.

3.6 Theorem

Let f be a homomorphism from a ring R onto a ring S, and let B be a multi-anti fuzzy left ideal of S. Then the anti pre-image, $f^{-1}(B)$ is a multi-anti fuzzy left ideal of R.

Proof

Consider a ring homomorphism $f : R \rightarrow S$

Let B be a multi-anti fuzzy left ideal of S.

For all x, y∈ R

i. $f^{-1}(B)(x-y) = Bf(x-y)$
$= B(f(x) - f(y))$
$\leq \max\{Bf(x),Bf(y)\}$
$= \max\{f^{-1}(B)(x),f^{-1}(B)(y)\}$
$f^{-1}(B)(x-y) \leq \max\{f^{-1}(B)(x),f^{-1}(B)(y)\}$.

ii. $f^{-1}(B)(xy) = B(f(xy))$
$= B(f(xy))$
$\leq \max\{Bf(x),Bf(y)\}$
$= \max\{f^{-1}(B)(x),f^{-1}(B)(y)\}$
$f^{-1}(B)(xy) \leq \max\{f^{-1}(B)(x),f^{-1}(B)(y)\}$.

iii. $f^{-1}(B)(xy) = B\, f(xy)$
$= B(f(x)f(y))$
$\leq B(f(y))$
$= f^{-1}(B)(y))$
$f^{-1}(B)(xy) \leq f^{-1}(B)(y))$

Therefore, $f^{-1}(B)$ is a multi-anti fuzzy left ideal of R.

3.7 Theorem

Let f be a homomorphism from a ring R into a ring S, and let B be a multi-anti fuzzy right ideal of S. Then the anti pre-image, $f^{-1}(B)$ is a multi-anti fuzzy right ideal of R.

Proof

Consider a ring homomorphism $f : R \rightarrow S$

Let B be a multi-anti fuzzy right ideal of S.

For all x, y∈ R

i. $f^{-1}(B)(x-y) = Bf(x-y)$
$= B(f(x) - f(y))$
$\leq \max\{Bf(x),Bf(y)\}$
$= \max\{f^{-1}(B)(x),f^{-1}(B)(y)\}$
$f^{-1}(B)(x-y) \leq \max\{f^{-1}(B)(x),f^{-1}(B)(y)\}$.

ii. $f^{-1}(B)(xy) = B(f(xy))$
$= B(f(xy))$
$\leq \max\{Bf(x),Bf(y)\}$
$= \max\{f^{-1}(B)(x),f^{-1}(B)(y)\}$
$f^{-1}(B)(xy) \leq \max\{f^{-1}(B)(x),f^{-1}(B)(y)\}$.

iii. $f^{-1}(B)(xy) = B\, f(xy)$
$= B(f(x)f(y))$
$\leq B(f(x))$
$= f^{-1}(B)(x))$
$f^{-1}(B)(xy) \leq f^{-1}(B)(x))$

Therefore, $f^{-1}(B)$ is a multi-anti fuzzy right ideal of R.

3.8 Theorem

Let f be a homomorphism from a ring R into a ring S, and let B be a multi-anti fuzzy left (right) ideal of S. Then the anti pre-image, $f^{-1}(B)$ is a multi-anti fuzzy left (right) ideal of R.

Proof

It is clear.

3.9Theorem

Let f be a homomorphism from a ring R into a ring S, and let A be a multi-anti fuzzy left ideal of a ring R with inf property. Then the anti- image, f(A) is a multi-anti fuzzy right ideal of a ring S.

Proof

Consider a ring homomorphism $f : R \to S$
Let A be a multi-anti fuzzy left ideal of R.
For all $x, y \in R$

i. $f(A)(f(x) - f(y)) = f(A)\, f(x - y)$
$= A(x - y)$
$\leq \max\{A(x), A(y)\}$
$= \max\{ f(A)(x), f(A)(y)\}$
$f(A)(f(x) - f(y)) \leq \max\{ f(A)(x), f(A)(y)\}$.

ii. $f(A)(f(x)f(y)) = f(A)f(xy)$
$= A(xy)$
$\leq \max\{A(x), A(y)\}$
$= \max\{f(A)(x), f(A)(y)\}$
$f(A)(f(x)f(y)) \leq \max\{ f(A)(x), f(A)(y)\}$.

iii. $f(A)(f(x)f(y)) = f(A)f(xy)$
$= A(xy)$
$\leq A(y)$
$= f(A)(f(y))$
$f(A)(f(x)f(y)) \leq f(A)(f(y))$

Therefore, f(A) is a multi-fuzzy left ideal of S

3.10 Theorem

Let f be a homomorphism from a ring R onto a ring S, and let A be a multi-anti fuzzy right ideal of a ring R with inf property. The anti- image, f(A) is a multi-anti fuzzy right ideal of a ring S.

Proof

Consider a ring homomorphism $f : R \to S$
Let A be a multi-anti fuzzy right ideal of R.
For all $x, y \in R$

i. $f(A)(f(x) - f(y)) = f(A)\, f(x - y)$
$= A(x - y)$
$\leq \max\{A(x), A(y)\}$
$= \max\{ f(A)(x), f(A)(y)\}$
$f(A)(f(x) - f(y)) \leq \max\{ f(A)(x), f(A)(y)\}$.

ii. $f(A)(f(x)f(y)) = f(A)f(xy)$
$= A(xy)$
$\leq \max\{A(x), A(y)\}$
$= \max\{ f(A)(x), f(A)(y)\}$
$f(A)(f(x)f(y)) \leq \max\{ f(A)(x), f(A)(y)\}$.

iii. $f(A)(f(x)f(y)) = f(A)f(xy)$
$= A(xy)$
$\leq A(x)$
$= f(A)(f(x))$
$f(A)(f(x)f(y)) \leq f(A)(f(x))$

Therefore, f(A) is a multi-fuzzy right ideal of S

3.11 Theorem

Let f be a homomorphism from a ring R into a ring S, and let A be a multi-anti fuzzy ideal of a ring R with inf property. Then the anti- image, f(A) is a multi-anti fuzzy ideal of a ring S.

Proof

It is clear.

3.12Theorem

Let f be an anti- homomorphism from a ring R into a ring S, and let B be a multi-anti fuzzy left ideal of S. Then the anti pre-image, $f^{-1}(B)$ is a multi-fuzzy right ideal of R.

Proof

Consider a ring anti-homomorphism $f : R \rightarrow S$
Let B be a multi-anti fuzzy left ideal of S.
For all x, $y \in R$

i. $f^{-1}(B)(x-y) = Bf(x-y)$
$= B(f(y) - f(x))$
$\leq \max\{Bf(y), Bf(x)\}$
$= \max\{f^{-1}(B)(y), f^{-1}(B)(x)\}$
$= \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(x-y) \leq \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$.

ii. $f^{-1}(B)(xy) = B(f(xy))$
$= B(f(y)f(x))$
$\leq \max\{Bf(y), Bf(x)\}$
$= \max\{f^{-1}(B)(y), f^{-1}(B)(x)\}$
$f^{-1}(B)(xy) \leq \max\{f^{-1}(B)(y), f^{-1}(B)(x)\}$.

iii. $f^{-1}(B)(xy) = B\, f(xy)$
$= B(f(y)f(x))$
$\leq B(f(x))$
$= f^{-1}(B)(x))$
$f^{-1}(B)(xy) \leq f^{-1}(B)(x))$

Therefore, $f^{-1}(B)$ is a multi-fuzzy right ideal of R.

3.13 Theorem

Let f be an anti- homomorphism from a ring R into a ring S, and let B be a multi-anti fuzzy right ideal of S. Then the anti pre-image, $f^{-1}(B)$ is a multi-fuzzy left ideal of R.

Proof

Consider a ring anti-homomorphism $f : R \rightarrow S$
Let B be a multi-anti fuzzy right ideal of S.
For all x, $y \in R$

i. $f^{-1}(B)(x-y) = Bf(x-y)$
$= B(f(y) - f(x))$
$\leq \max\{Bf(y), Bf(x)\}$
$= \max\{f^{-1}(B)(y), f^{-1}(B)(x)\}$
$= \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(x-y) \leq \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$.

ii. $f^{-1}(B)(xy) = B(f(xy))$
$= B(f(y)f(x))$
$\leq \max\{Bf(y), Bf(x)\}$
$= \max\{f^{-1}(B)(y), f^{-1}(B)(x)\}$
$= \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$
$f^{-1}(B)(xy) \leq \max\{f^{-1}(B)(x), f^{-1}(B)(y)\}$.

iii. $f^{-1}(B)(xy) = B\, f(xy)$
$= B(f(y)f(x))$
$\leq B(f(y))$
$= f^{-1}(B)(y))$
$f^{-1}(B)(xy) \leq f^{-1}(B)(y))$

Therefore, $f^{-1}(B)$ is a multi-fuzzy left ideal of R.

3.14 Theorem

Let f be an anti- homomorphism from a ring R into a ring S, and let B be a multi-anti fuzzy left (right) ideal of S. Then the anti pre-image, $f^{-1}(B)$ is a multi-fuzzy right (left) ideal of a ring R.

Proof

It is clear.

3.15Theorem

Let f be an anti-homomorphism from a ring R onto a ring S, and let A be a multi-anti fuzzy left ideal of a ring R with inf property. Then the anti- image, f(A) is a multi- anti fuzzy right ideal of a ring S.

Proof

Consider a ring anti-homomorphism f : R→S

Let A be a multi- anti fuzzy left ideal of R.

For all x, y∈ R

i. $f(A)(f(x)-f(y)) = f(A)\, f(y-x)$
$= A(y-x)$
$\leq \max\{A(y), A(x)\}$
$= \max\{ f(A)(y), f(A)(x)\}$
$= \max\{ f(A)(x), f(A)(y)\}$
$f(A)(f(x)-f(y)) \leq \max\{ f(A)(x), f(A)(y)\}$.

ii. $f(A)(f(x)f(y)) = f(A)f(y\,x)$
$= A(yx)$
$\leq \max\{A(y), A(x)\}$
$= \max\{ f(A)(y), f(A)(x)\}$
$f(A)(f(x)f(y)) \leq \max\{ f(A)(y), f(A)(x)\}$.

iii. $f(A)(f(x)f(y)) = f(A)f(y\,x)$
$= A(yx)$
$\leq A(x)$
$= f(A)(f(x))$
$f(A)(f(x)f(y)) \leq f(A)(f(x))$

Therefore, f(A) is a multi-anti fuzzy right ideal of S.

3.16 Theorem

Let f be an anti-homomorphism from a ring R into a ring S, and let A be a multi-anti fuzzy right ideal of a ring R with inf property. Then the anti- image, f(A) is a multi- anti fuzzy left ideal of a ring S.

Proof

Consider a ring anti-homomorphism f: R→S

Let A be a multi- anti fuzzy right ideal of R.

For all x, y∈ R

i. $f(A)(f(x)-f(y)) = f(A)\, f(y-x)$
$= A(y-x)$
$\leq \max\{A(y), A(x)\}$
$= \max\{ f(A)(y), f(A)(x)\}$
$f(A)(f(x)-f(y)) \leq \max\{ f(A)(x), f(A)(y)\}$.

ii. $f(A)(f(x)f(y)) = f(A)f(yx)$
$= A(yx)$
$\leq \max\{A(y), A(x)\}$
$= \max\{ f(A)(y), f(A)(x)\}$
$f(A)(f(x)f(y)) \leq \max\{ f(A)(x), f(A)(y)\}$.

iii. $f(A)(f(x)f(y)) = f(A)f(yx)$
$= A(yx)$
$\leq A(y)$
$= f(A)(f(y))$
$f(A)(f(x)f(y)) \leq f(A)(f(y))$

Therefore, f(A) is a multi-fuzzy left ideal of S

3.17 Theorem

Let f be an anti-homomorphism from a ring R into a ring S, and let A be a multi-anti fuzzy left(right) ideal of a ring R with inf property. Then the anti- image, f(A) is a multi- anti fuzzy right (left) ideal of a ring S.

Proof

It is clear.

IV. Conclusion

In this paper, we discussed the properties of image of a multi- fuzzy ideal of a ring under homomorphism and anti homomorphism and the properties of image of multi-anti fuzzy ideal of a ring under homomorphism and anti homomorphism

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