IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 1 Ver. I (Jan. - Feb. 2016), PP 50-52
www.iosrjournals.org

The Solution of Bessel Equation of Order Zero and Hermit Polynomial by Using the Differential Transform Method

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Abstract : *The differential Transform method is one of important methods to solve the differential equations. In this paper we show to that the differential transform method (DTM) is very Hermite equation.*
Keywords: *Differential transform method, Bessel equation, Hermite equation.*

I. Introduction

The basic definition of differential transform method is introduced after Taylor series as follows:-

II. Definition

The one-dimensional differential transform of the function f(x) is defined as

$$F(k) = \frac{1}{k!}\left[\frac{\partial^k}{\partial x^k} f(x)\right]_{x=0} \qquad (1)$$

and therefore the differential inverse transform of f(x) is given by

$$f(x) = \sum_{k=0} F(k)\, x^k \qquad (2)$$

III. Indentations And Equations

we can easily prove the following results satisfy by (DTM)

(a) if $f(x) = u(x) + v(x)$, then

$$F(k) = U(k) + V(k) \qquad (3)$$

(b) if $f(x) = \alpha\, u(x)$; α is constant then

$$F(k) = \alpha\, U(k) \qquad (4)$$

(c) if $\frac{\partial^r u(x)}{\partial x^r}$; then

$$\Gamma(k) = (k+1)(k+2) \ldots (k+r)\ U(k+r) \qquad (5)$$

IV. Formulation Of The Problem

(A) Solution of Bessel function of order zero let Bessel differential equation of order zero written as

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0 \qquad (6)$$

Now we are giving to apply the differential transform (DT) on equation (6), but first of all we compute the following

$$(DT) \quad of \quad x\frac{d^2y}{dx^2} = \frac{1}{k!}\frac{\partial^k}{\partial x^k}(x\,y'')$$

$$= \frac{1}{k!}\left[x\frac{\partial^k}{\partial x^k} y'' + k\,\frac{\partial^{k-1}}{\partial x^{k-1}} y''\right]_{x=0} = \frac{1}{(k-1)!}$$

$$\therefore \quad DT\left(x\frac{d^2y}{dx^2}\right) = k\,(k+1)\,Y(k) \qquad (7)$$

where $Y(k) = DT\,(y(x))$

also

$$(DT) \quad of \quad \frac{dy}{dx} = \frac{1}{k!}\left[\frac{\partial^k}{\partial x^k}\,y'\right]_{x=0} = \frac{1}{k!}\left[\frac{\partial^{k+1}}{\partial x^{k+1}}\,y\right]_{x=0}$$

$$= (k+1)\;Y(k+1)$$

$$\therefore \quad DT\left(\frac{dy}{dx}\right) = (k+1)\,Y(k+1) \qquad (8)$$

and

$$DT(xy) = \frac{1}{k!}\left[\frac{\partial^k}{\partial x^k}\,xy\right]_{x=0} = \frac{1}{k!}\left[x\,\frac{\partial^k}{\partial x^k}\,y + k\,\frac{\partial^{k-1}}{\partial x^{k-1}}y\right]_{x=0}$$

$$i.e \quad DT\,(xy) = Y(k+1) \qquad (9)$$

substitute (7),(8) and (9) in equation (6) we find

$$[\,k\,(k+1) + (k+1)\,]\;Y(k+1) = -Y(k-1)$$

$$i.e \quad Y(k+1) = -\frac{Y(k-1)}{(k+1)^2} \qquad (10)$$

assume that $Y(0) = a_1$, $Y(1) = a_2$ then $\forall k \geq 1$ we have

$$Y(2) = -\frac{a_1}{2^2} \quad , \quad Y(3) = -\frac{a_2}{3^2}$$

$$Y(4) = -\frac{a_1}{2^2\,.\,4^2} \quad , \quad Y(5) = -\frac{a_2}{3^2\,.\,5^2}$$

$$Y(6) = -\frac{a_1}{2^2\,.\,4^2\,.\,6^2} \quad , \quad Y(7) = -\frac{a_2}{3^2\,.\,5^2\,.\,7^2}$$

For the special case when $a_2 = 0$ we have

$$Y(2k) = \frac{(-1)^k}{2^{2k}\,(k!)^2} \qquad (11)$$

taking the inverse of (DT) we get

$$y(x) = \sum_{k=0}^{\infty} Y(2k)\;x^{2k}$$

$$\therefore \quad J_0(x) = \sum_{k=0}^{\infty} Y(2k)\;x^{2k} \qquad (12)$$

(B) Solution of Hermite Polynomials The Hermite Polynomials satisfy the differential equation

$$x\,\frac{d^2y}{dx^2} - 2\,x\,\frac{dy}{dx} + 2\,n\,y = 0 \qquad (13)$$

By using the (DTM) for equation (13) we get

$$DT\left(\frac{d^2y}{dx^2}\right) = \frac{1}{k!}\,\frac{\partial^k}{\partial x^k}(\,y''\,)\;at\;\;x = 0$$

$$= \frac{1}{k!}\left[\frac{\partial^{k+2}}{\partial x^{k+2}}\,y\right]_{x=0} = \frac{1}{(k-1)!}$$

$$DT\left(\frac{d^2y}{dx^2}\right) = (k+2)(k+1)Y(k+2) \qquad (14)$$

also

$$DT(x\,y') = \frac{1}{k!}\frac{\partial^k}{\partial x^k}(x\,y')\ at\ x=0$$

$$= \frac{1}{k!}\left[x\frac{\partial^{k+1}}{\partial x^{k+1}}y + k\frac{\partial^k}{\partial x^k}y\right]_{x=0} = \frac{1}{k!}\left[\frac{\partial^k}{\partial x^k}y\right]_{x=0}$$

$$\therefore \quad DT(x\,y') = k\,Y(k) \qquad (15)$$

Substitute (14), (15) in equation (13) leading to

$$(k+2)(k+1)\,Y(k+2) - 2\,k\,Y(k) + 2\,n\,Y(k) = 0$$

$$i.e \qquad Y(k+2) = -\frac{2(n-k)}{(k+2)(k+1)}Y(k) \qquad (16)$$

clearly at $n = k$ $\quad Y(n+2) = 0$ $\quad$ and then the solution is a polynomial of degree n

Now if $\quad Y(0) = a_1 \quad , \quad Y(1) = a_2$

$$\therefore\ Y(2) = -\frac{2\,n}{2\,.\,1}a_1 \quad , \quad Y(3) = -\frac{2(n-1)}{3\,.\,2}a_2 \qquad (17)$$

$$Y(4) = -\frac{2\,(n-2)}{4\,.\,3}\left(\frac{-2\,n}{2!}\right)a_1 = 2^2\frac{n\,(n-2)}{4!}a_1 \qquad (18)$$

$$Y(5) = -\frac{2\,(n-3)}{5\,.\,4}\left(\frac{-2\,(n-1)}{3!}\right)a_2 = 2^2\frac{(n-1)(n-3)}{5!}a_2 \qquad (19)$$

V. Conclusion

Generally

$$Y(2\,k) = \frac{(-1)^k\,2^k}{(2\,k)!}\,n\,(n-2)\ldots(n-2\,k+2)\,a_1 \qquad (20)$$

$$Y(2\,k+1) = \frac{(-1)^k\,2^k}{(2\,k+1)!}(n-1)(n-3)\ldots(n-2\,k+1)\,a_2 \qquad (21)$$

Hence

$$H_n(x) = \sum_{k=0}^{\left[\frac{1}{2}n\right]}\frac{(-1)^k}{2^{2\,k}\,k!\,(n-2\,k)!}\,x^{n-2\,k}\,a_1 \qquad (22)$$

where

$$\left[\frac{1}{2}n\right] = \begin{cases}\frac{1}{2}\,n & if\ \ n\ \ is\ an\ even\\ \frac{1}{2}\,(n-1) & if\ \ n\ \ is\ an\ odd\end{cases}$$

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