

Supply Chain Multi-Item Inventory Model for Quadratic Deterioration with Expiration

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Abstract: *Inventory decisions in supply chain are crucial for its success. Making these decisions in inventory systems with multiple items is a challenging task for managers. These decisions become more important for the items with expiration date. Quadratic rate of deterioration is taken in this article. Most approaches in the literature for optimizing decisions in such an environment consider only a single item inventory. This paper presents a multi item inventory model to optimize the unit time profit of inventory management for the products having an expiration date after which the product can not be sold. As on one side the shortage costs are significant, on the other side, to maintain appropriate inventory levels for such type of products and avoid shortages is a very problematic job. This article offers an approach for optimization and thus has business significance.*

I. Introduction

It is important to control and maintain the inventories of deteriorating items for the modern business environment. In general, deterioration is defined as the damage, spoilage, dryness, vaporization, etc., that result in decrease of usefulness of the original one. In recent years, inventory problems for deteriorating items have been widely studied after Ghare and Schrader(1963).The effect of deterioration plays an important role in the storage of some commonly used decaying items like, breakable items, (glass, china clay, ceramic goods etc.), radioactive substances, perishable goods etc. In these cases, a certain fraction of these goods are either damaged or decayed and are not in a perfect condition to satisfy the future demand of customers for good items. Deterioration in such items is continuous and time independent or time dependent and/or dependent on on-hand inventory. A number of research papers have already been published on above type of items. Goswami and Chaudhuri(1992) put forward a model with and without shortages by assuming a linear time dependent demand. Bhunia and Maiti(1 998)corrected this model for its shortcomings and considered deterioration in the stock. Singh and Singh(2009) developed a continuous production control inventory model for deteriorating items with linear demand rate in the environment of permissible delay of payments. Tayal et al.(2014) developed a two echelon supply chain model for deteriorating items with effective investment in preservation technology. Singh and Singh(2014) assumed optimal ordering policy for decaying items with stock-dependent demand under inflation in a supply chain. Singh and Jain(2014) explored a deterministic inventory model for a deteriorating item in an inflation-induced environment. Yadav etl.(2013) presented Integrated 3-echolen Supply Chain Inventory Models for perishable items for quadratic demand and production rate dependant on demand and two parametric deterioration. Hsu et al.(2007) presented an optimal ordering decision for deteriorating items with expiration date and uncertain lead time in which the demand for the product decreases as the product is nearer to the expiration date.

Supply chain for the products with high fluctuation in demand patterns and a fix time of useable period necessitates better and at times different planning approach for market, based on forecast data. The shortage costs in such cases are significant and any loss in sales badly impacts on the balance sheet. The problem gets compounded when the executives are required to deal with variety. The model presents a new scheme to arrive at the inventory replenishment levels and tries to improve the pull in the system. This is usually taken care of by maintaining high inventory levels at the cost. It may become even more complicated if the deterioration rates and lead time for the items are also taken into account. For such requirements, the conventional optimization models based on inventory costs are extremely difficult to practice. The modification in the periodic review with the cyclic replenishments based on forecasts and the rhythm followed may offer a better way of modeling the inventory patterns in system to take care of real-life issues like shortage. The behaviour of the demand and the forecasting method employed provides an approach to obtain the optimal ordering quantity. The ultimate aim of inventory management is to serve the customers with highest possible service levels and flexibility keeping the nature of the system in consideration. Many of the classical inventory models concern with single-item model. Ben-Daya and Raouf(2014) have developed approach for a more realistic and general SPIP (Single Period Inventory Problem), they consider a multi-item with budgetary and floor- or shelf- space constraints, they

assume that, the demand of the items follows uniform probability distribution. Also, they have discussed a multi-item inventory model with stochastic demand subject to the restrictions on available space and budget. Bhattacharya(2005) has studied a two item inventory for deteriorating items with a linear stock-dependent demand rates. Lenard and Roy(1995) are define another approach for the determination of inventory policies based on the notion of efficient policy surface and extend this notion to multi-item inventory control by defining the concepts of family and aggregate item. Mathematicians like Worell and Hall(1982) have applied different programming methods to solve multi-items inventory problems. Sulem(1986) has been determined the optimal ordering policy for impulse control of a deterministic two product inventory system subject to constant demand rates, linear storage and shortage costs and economies of joint ordering. An important assumption in inventory models found in the existing literature is that the lifetime of an item is infinite while it is in storage.

In realistic business environment let us discuss another feature which comes frequently and which is shortage. It plays an important role and creates backlogging. There are two type of backlogging; complete backlogging and partial backlogging. In the most of the referred papers, complete backlogging of unsatisfied demand is assumed. In practice, there are customers who are willing to wait and receive their orders at the end of shortage period, while other is not. Mandal and Pal(1998) considered inventory model for exponentially decaying items by allowed shortages. Wu et al.(1999) related the backlogging rate to the waiting time up to the next replenishment. In the last few years, considerable attention has been paid to inventory models with partial as well as complete backlogging. The backlogging rate can be modeled taking into account the customer's behavior. The first paper in which customer's importance functions are proposed seems to be that by Abad(1996). Change and Dye(1999) developed a finite horizon inventory model using Abad's reciprocal backlogging rate. Singh et al.(2009) presented an EOQ model for perishable items with power demand and partial backlogging. Arya et al.(2009) developed an order level inventory model for perishable items with stock dependent demand and partial backlogging. Here in this work we have used partial backlogging which depends on waiting time. The demand is taken as a function of price and expiration date. Singh and Vishnoi(2013) introduced a supply chain inventory model for deteriorating and ameliorating items with price-dependent consumption rate. Singh et al.(2011) a soft computing based inventory model with deterioration and price dependent demand.

In this paper consideration was given to quadratic rate of deterioration, the control of multi product inventory under deterministic demands. The multi-item inventory models are more realistic than the single item model. So this study concern with multi-item inventory models. The purpose remains the same for single-item as well as for multi-item inventory. The analysis for a single-item inventory is almost parallel to that of multi-item inventory. This article considers the real-life requirements like product variety with expiration date and provides a simple as well as logical method which may be used for the inventory optimization to arrive at the overall results without tedious calculations. Much attention has been devoted to variant inventory models. Since in real life there are so many products which have a fix life cycle, and after they cannot be used. Demand for these product decreases as the product is nearer to expiration date. To maintain appropriate inventory levels for such type of products and avoid shortages is a very problematic job. This model is treated with multiple items having a fix expiration date for each product. The proposed model takes care of real-life business requirements. The empirical relation in the proposed model is used to calculate the items variability with a set of assumptions that are practically acceptable. There is no simple model that offers the optimal result for the variable quantity with expiration date. In the next section we present the multi item inventory model and derive the optimal control of the system. The present paper is organized as follow. Section 2 includes the assumptions and notations of this model. Section 3 is devoted to obtain the mathematical formulation of the optimal control problem of this model. In section 4 optimality of the model is proved. Section 5 provides a numerical example to clarify the proposed model. Finally observations and conclusion of the results are presented.

II. Assumptions And Notations

We consider a multi item inventory model with the following notation and assumptions for the products having a fix expiration date after that which can not be used.

1. Deterioration rate for the item is assumed as $a_i + b_i t + c_i t^2$ which is a linear function of time.
2. Demand rate for the product follows a function of price p_i and expiration date T_i and is given by $D_i(t) = \frac{T_i - t}{p_i^m - 1}$, where $0 < m < 1$,
3. The model is considered for multi items.
4. Shortages are allowed and partially backlogged at the rate of $B(h)$ where $B(h) = K_{i0} e^{-K_{i1} h}$, where $K_{i0} < 1$ and $K_{i1} > 1$.
5. The warehouse has unlimited capacity.
6. There is no replacement or repair of deteriorated items during a given cycle.

7. The shortage period can not exceed the cycle time.

T_i expiration date for the i^{th} product

l_i lead time for the i^{th} product

t_i time at which inventory level becomes zero

p_i the selling price per unit for the retailer

C_{pi} the selling price per unit for the vendor

C_{mi} the manufacturing cost per unit for the vendor

C_{di} deterioration cost per unit for the retailer

C_{ri} the lost sale cost per unit for the retailer

C_{hi} holding cost per unit per unit time

λ a constant

K_{i0}, K_{i1} positive constants

H waiting time upto the next replenishment

C_{oi} ordering cost for the retailer for i^{th} item

a_i, b_i, c_i positive constants

X_i the vendor's managing cost for i^{th} item

F_{iR} unit time profit for i^{th} item for the retailer

F_{iS} unit time profit for i^{th} item for the vendor

III. Modelling And Analysis

3.1 When The Supplier's Lead Time $L_i \leq 0$:

Retailer's Model

In this case the supplier completes the order before l_i unit of times. Shortages are occurring in the time interval $[t_i, T_i]$. As shown in Figure 1, we consider the following time intervals separately, $[0, t_i]$ and $[t_i, T_i]$. During the interval $[0, t_i]$ the inventory level decreases due to combined effect of demand and deterioration. Hence the inventory level for i^{th} item ($0 \leq i \leq n$) is governed by the following differential equations:

$$\frac{dI_{i1}(t)}{dt} + (a_i + b_i t + c_i t^2) I_{i1}(t) = -\frac{T_i - t}{p_i^m - 1} \quad \dots(1)$$

$$\frac{dI_{i2}(t)}{dt} = -\frac{T_i - t}{p_i^m - 1} \quad \dots(2)$$

Using boundary conditions $I_{i1}(t_i) = 0$ and $I_{i2}(t_i) = 0$, one can get

$$I_{i1}(t) = \frac{1}{p_i^m - 1} \left[\{ (t_i - t) + \frac{a_i}{2} (t_i^2 - t^2) + \frac{b_i}{6} (t_i^3 - t^3) + \frac{c_i}{12} (t_i^4 - t^4) \} - \left\{ \frac{1}{2} (t_i^2 - t^2) + \frac{a_i}{3} (t_i^3 - t^3) + \frac{b_i}{8} (t_i^4 - t^4) + \frac{c_i}{15} (t_i^5 - t^5) \right\} e^{a_i t + \frac{b_i t^2}{2} + \frac{c_i t^3}{3}} \right], \quad 0 \leq t \leq t_i \quad \dots(3)$$

$$I_{i2}(t) = \frac{1}{p_i^m - 1} \{ (t_i - t) - \frac{1}{2} (t_i^2 - t^2) \}, \quad t_i \leq t \leq T_i \quad \dots(4)$$

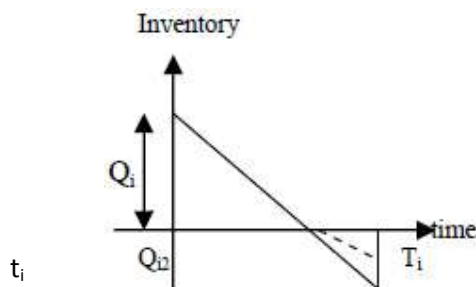


Figure 1. Retailer's inventory level.

From equation (3) using the condition $I_{i1}(0) = Q_i$ we get:

$$Q_i = \frac{1}{p_i^m - 1} \left\{ \left(t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12} \right) - \left(\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15} \right) \right\} \quad \dots(5)$$

Vendor's Inventory Model:

Now let $I_{is}(t)$ be the supplier's inventory level at t before the beginning of a cycle order before l_i units of time ie ($l_i \leq 0$) (Figure 2). The differential equation for the supplier is given by:

$$\frac{dI_{is}(t)}{dt} + (a_i + b_i t + c_i t^2) I_{is}(t) = 0, l_i \leq t \leq 0. \quad \dots(6)$$

Using boundary condition $I_{is}(0) = I_{i1}(0) = Q_i$, one can get:

$$I_{is}(t) = I_{i1}(0) e^{-\left(a_i t + \frac{b_i t^2}{2} + \frac{c_i t^3}{3}\right)}$$

$$= \frac{1}{p_i^m - 1} \left\{ \left(t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12} \right) - \left(\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15} \right) \right\} e^{-\left(a_i t + \frac{b_i t^2}{2} + \frac{c_i t^3}{3}\right)}, \quad \dots(7)$$

$$Q_{i1} = \int_0^{t_i} \frac{T_i - t}{p_i^m - 1} dt$$

$$Q_{i1} = \frac{t_i(2T_i - t_i)}{2(p_i^m - 1)} \quad \dots(8)$$

After the stock out, the arrival of inventory will be of fresh stock, so considering this assumption all the demand during stock out will be of fresh product.

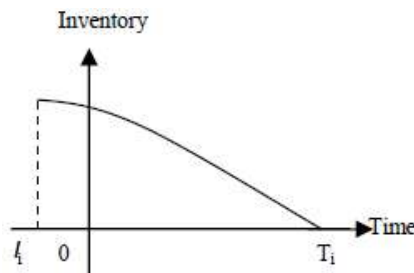


Figure 2. Vendor's inventory level when $l_i < 0$.

$$Q_{i2} = \int_{t_i}^{T_i} \frac{T_i}{p_i^m - 1} K_{i0} e^{-K_{i1}(T_i - t)} dt$$

$$Q_{i2} = \frac{K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - t_i)}) \quad \dots(9)$$

Sales Revenue = $p_i Q_{i1} + \lambda p_i Q_{i2}$

$$S.R_i = \frac{p_i t_i (2T_i - t_i)}{2(p_i^m - 1)} + \frac{\lambda p_i K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - t_i)}) \quad \dots(10)$$

The ordering quantity at each replenishment is:

$$Q_i = I_{i0}(0) + Q_{i2}$$

Purchasing cost

$$P.C_i = (I_{i0}(0) + Q_{i2}) C_{pi}$$

$$P.C_i = \frac{C_{pi}}{p_i^m - 1} \left\{ (t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12}) - (\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15}) \right\} + \frac{C_{pi} K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - t_i)}) \quad \dots (11)$$

The lost sale amount is

$$= \int_{t_i}^{T_i} \frac{T_i}{p_i^m - 1} K_{i0} e^{-K_{i1}(T_i - t)} dt$$

Cost of lost sale amount is

$$L.C_i = \frac{T_i}{p_i^m - 1} \left\{ T_i - t_i - \frac{K_{i0}}{K_{i1}} (1 - e^{-K_{i1}(T_i - t)}) \right\} C_{ri} \quad \dots (12)$$

Deterioration cost

$$D.C_i = \left\{ I_{i1}(0) - \int_0^{t_i} \frac{T_i - t}{p_i^m - 1} dt \right\} C_{di} \quad \dots (13)$$

$$= \frac{C_{di}}{p_i^m - 1} \left\{ (t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12}) - (\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15}) - \frac{(2T_i t_i - t_i^2)}{2} \right\}$$

Holding cost

$$H.C_i = \int_0^{t_i} I_{i1}(t) dt C_{hi}$$

$$= \int_0^{t_i} \frac{1}{p_i^m - 1} \left[\left\{ (t_i - t) + \frac{a_i}{2} (t_i^2 - t^2) + \frac{b_i}{6} (t_i^3 - t^3) + \frac{c_i}{12} (t_i^4 - t^4) \right\} - \left\{ \frac{1}{2} (t_i^2 - t^2) + \frac{a_i}{3} (t_i^3 - t^3) + \frac{b_i}{8} (t_i^4 - t^4) + \frac{c_i}{15} (t_i^5 - t^5) \right\} e^{\frac{a_i t + b_i t^2 + c_i t^3}{2}} dt \right] C_{hi}$$

$$= \frac{C_{hi}}{p_i^m - 1} \left\{ \left(\frac{t_i^2}{2} + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15} \right) - \left(\frac{1}{3} t_i^3 + \frac{a_i t_i^4}{4} + \frac{b_i t_i^5}{10} + \frac{c_i t_i^6}{18} \right) + \frac{a_i t_i^3}{6} + \frac{b_i t_i^4}{24} + \frac{c_i t_i^5}{36} - \frac{a_i t_i^4}{2} - \frac{b_i t_i^5}{30} - \frac{c_i t_i^6}{72} \right\}$$

$$= \frac{C_{hi}}{p_i^m - 1} \left\{ \left(\frac{t_i^2}{2} - \frac{1}{3} t_i^3 + a_i \left(\frac{t_i^3}{2} - \frac{3t_i^4}{4} \right) + b_i \left(\frac{t_i^4}{6} - \frac{2t_i^5}{15} \right) + c_i \left(\frac{17t_i^5}{180} - \frac{5t_i^6}{72} \right) \right) \right\}$$

as $a_i \geq 1, b_i \geq 1$ and $c_i \geq 1$... (14)

Ordering cost

$$O.C_i = C_{io} \quad \dots (15)$$

When the supplier's delivery is completed early, the retailer's unit time profit for i^{th} item without late delivery is:

$$F_{iR} = \frac{1}{T_i} [\text{sales revenue} - \text{purchasing cost} - \text{deterioration cost} - \text{inventory holding cost} - \text{lost sale cost} - \text{ordering cost}]$$

$$F_{iR} = \frac{1}{T_i} \left[\frac{p_i t_i (2T_i - t_i)}{2(p_i^m - 1)} + \frac{\lambda p_i K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - t_i)}) - \right]$$

$$\begin{aligned}
 & \frac{c_{pi}}{p_i^m - 1} \left\{ \left(t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12} \right) - \left(\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15} \right) \right\} + \frac{c_{pi} K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - t_i)}) - \\
 & \frac{C_{di}}{p_i^m - 1} \left\{ \left(t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12} \right) - \left(\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15} \right) - \frac{(2T_i t_i - t_i^2)}{2} \right\} - \\
 & \frac{C_{hi}}{p_i^m - 1} \left\{ \left(\frac{t_i^2}{2} - \frac{1}{3} t_i^3 + a_i \left(\frac{t_i^3}{2} - \frac{3t_i^4}{4} \right) + b_i \left(\frac{t_i^4}{6} - \frac{2t_i^5}{15} \right) + c_i \left(\frac{17t_i^5}{180} - \frac{5t_i^6}{72} \right) \right\} \right. \\
 & \left. - \frac{T_i}{p_i^m - 1} \left\{ T_i - t_i - \frac{K_{i0}}{K_{i1}} (1 - e^{-K_{i1}(T_i - t)}) \right\} C_{\bar{v}} - C_{oi} \right] \dots(16)
 \end{aligned}$$

Since the supplier suffers inventory holding cost until the target date due to early delivery by l_i unit of time, the supplier's unit time profit:

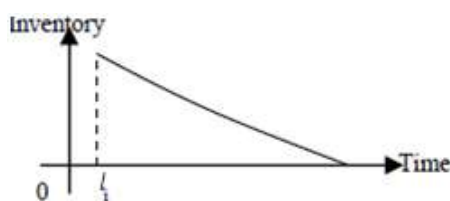
$$F_{is}(l_i \leq 0) = \frac{1}{T_i} [\text{sales revenue} - \text{purchasing cost} - \text{deterioration cost} - \text{inventory holding cost} - \text{managing cost}]$$

$$F_{is}(l_i \leq 0) = \frac{1}{T_i} \left[(I_{i1}(0) + Q_{i2}) (C_{pi} - C_{mi}) - (I_{is}(l_i) - I_{i1}(0)) C_{pi} - \int_{l_i}^0 I_{is}(t) dt - X_i \right]$$

$$\begin{aligned}
 F_{is}(l_i \leq 0) &= \frac{1}{T} \left[\left\{ \frac{1}{p_i^m - 1} \left\{ \left(t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12} \right) - \left(\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15} \right) \right\} (C_{pi} - C_{mi}) \right. \right. \\
 & \left. \left. + \frac{K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - t_i)}) \right\} (C_{pi} - C_{mi}) - \left\{ \frac{1}{p_i^m - 1} \left\{ \left(t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12} \right) - \right. \right. \right. \\
 & \left. \left. \left(\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15} \right) \right\} e^{-(a_i l_i + \frac{b_i l_i^2}{2} + \frac{c_i l_i^3}{3})} - \frac{1}{p_i^m - 1} \left\{ \left(t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12} \right) - \right. \right. \\
 & \left. \left. \left(\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15} \right) \right\} \right\} C_{mi} - \frac{1}{p_i^m - 1} \left\{ \left(t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12} \right) - \left(\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} \right. \right. \\
 & \left. \left. + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15} \right) \right\} \left(\frac{a_i l_i^2}{2} + \frac{b_i l_i^3}{3} + \frac{c_i l_i^4}{4} \right) - \xi_i \left. \right]
 \end{aligned}$$

$$\begin{aligned}
 F_{is}(l_i \leq 0) &= \frac{1}{T} \left[\frac{1}{p_i^m - 1} \left\{ \left(t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12} \right) - \left(\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15} \right) \right\} \right. \\
 & \left. \left\{ (C_{pi} - C_{mi}) + (1 - e^{-(a_i l_i + \frac{b_i l_i^2}{2} + \frac{c_i l_i^3}{3})}) C_{mi} - \left(\frac{a_i l_i^2}{2} + \frac{b_i l_i^3}{3} + \frac{c_i l_i^4}{4} \right) \right\} + \frac{K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - t_i)}) \right. \\
 & \left. (C_{pi} - C_{mi}) - X_i \right] \dots(17)
 \end{aligned}$$

**3.2 When Supplier's Lead Time $L_i > 0$
Retailer's Model**



When supplier's lead time $l_i > 0$ (Figure 3):

sales revenue = $p_i Q_{i1d} + \lambda p_i Q_{i2}$ (18)

where

$$Q_{i1d} = \int_0^{T_i} \frac{T_i - t}{p_i^m - 1} dt - \int_0^{l_i} \frac{T_i - t}{p_i^m - 1} dt$$

$$Q_{i1d} = \frac{(T_i - l_i)^2}{2(p_i^m - 1)}$$

$$Q_{i2} = \int_{l_i}^{T_i} \frac{T_i}{p_i^m - 1} K_{i0} e^{-K_{i1}(T_i - t)} dt$$

$$Q_{i2} = \frac{K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - l_i)})$$
(19)

Sales revenue = $p_i Q_{i1d} + \lambda p_i Q_{i2}$

$$S.R_i = \frac{p_i (T_i - l_i)^2}{2(p_i^m - 1)} + \frac{\lambda p_i K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - l_i)})$$
(20)

The order quantity at each replenishment is:

$$Q_{id} = [I_{i1}(t_i) + Q_{i2}]$$

Purchasing cost at each replenishment

$$C_{pi} = [I_{i1}(l_i) + Q_{i2}] \cdot C_{pi}$$

$$= \frac{C_{pi}}{p_i^m - 1} \left[\{ (t_i - l_i) + \frac{a_i}{2} (t_i^2 - l_i^2) + \frac{b_i}{6} (t_i^3 - l_i^3) + \frac{c_i}{12} (t_i^4 - l_i^4) \} - \right.$$

$$\left. \left\{ \frac{1}{2} (t_i^2 - l_i^2) + \frac{a_i}{3} (t_i^3 - l_i^3) + \frac{b_i}{8} (t_i^4 - l_i^4) + \frac{c_i}{15} (t_i^5 - l_i^5) \right\} \right] e^{a_i l_i + \frac{b_i l_i^2}{2} + \frac{c_i l_i^3}{3}} + \frac{C_{pi} K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - l_i)}),$$

.....(21)

The lost sale amount is

$$= \int_0^{l_i} \frac{T_i}{p_i^m - 1} dt + \int_{l_i}^{T_i} \frac{T_i}{p_i^m - 1} (1 - K_{i0} e^{-K_{i1}(T_i - t)}) dt$$

The lost sale cost

$$= \left[\frac{l_i (2T_i - l_i)}{p_i^m - 1} + \frac{T_i}{p_i^m - 1} \left\{ T_i - t_i - \frac{K_{i0}}{K_{i1}} (1 - e^{-K_{i1}(T_i - t_i)}) \right\} \right] C_{ri}$$

.....(22)

Deterioration cost

$$D.C_i = \{ I_{i1}(l_i) - \int_{l_i}^{T_i} \frac{(T_i - t)}{p_i^m - 1} dt \} C_{di}$$

$$= \left[\frac{1}{p_i^m - 1} \left[\{ (t_i - l_i) + \frac{a_i}{2} (t_i^2 - l_i^2) + \frac{b_i}{6} (t_i^3 - l_i^3) + \frac{c_i}{12} (t_i^4 - l_i^4) \} - \left\{ \frac{1}{2} (t_i^2 - l_i^2) + \frac{a_i}{3} (t_i^3 - l_i^3) + \frac{b_i}{8} (t_i^4 - l_i^4) + \frac{c_i}{15} (t_i^5 - l_i^5) \right\} \right] e^{a_i l_i + \frac{b_i l_i^2}{2} + \frac{c_i l_i^3}{3}} - \frac{2T_i (t_i - l_i) - t_i^2 + l_i^2}{p_i^m - 1} \right] C_{di}$$

.....(23)

Holding cost

$$\begin{aligned}
 H.C_i &= \int_{l_i}^{t_i} I_{il}(t) dt h_i \\
 &= \int_{l_i}^{t_i} \left[\frac{1}{p_i^m - 1} \left[\left\{ (t_i - t) + \frac{a_i}{2}(t_i^2 - t^2) + \frac{b_i}{6}(t_i^3 - t^3) + \frac{c_i}{12}(t_i^4 - t^4) \right\} - \left\{ \frac{1}{2}(t_i^2 - t^2) + \frac{a_i}{3}(t_i^3 - t^3) \right\} \right. \right. \\
 &\quad \left. \left. + \frac{b_i}{8}(t_i^4 - t^4) + \frac{c_i}{15}(t_i^5 - t^5) \right\} \right] e^{a_i t + \frac{b_i t^2}{2} + \frac{c_i t^3}{3}} dt \Big] C_{hi} \\
 &= \frac{C_{hi}}{p_i^m - 1} \left\{ \left(\frac{t_i^2}{2} + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15} \right) - \left(\frac{1}{3} t_i^3 + \frac{a_i t_i^4}{4} + \frac{b_i t_i^5}{10} + \frac{c_i t_i^6}{18} \right) + \right. \\
 &\quad \left. \frac{a_i t_i^3}{6} + \frac{b_i t_i^4}{24} + \frac{c_i t_i^5}{36} - \frac{a_i t_i^4}{2} - \frac{b_i t_i^5}{30} - \frac{c_i t_i^6}{72} \right\} - \frac{C_{hi}}{p_i^m - 1} \left\{ (t_i l_i - \frac{y_i^2}{2}) + \right. \\
 &\quad \left. \frac{a_i}{2} (t_i^2 l_i - \frac{y_i^3}{3}) + \frac{b_i}{6} (t_i^3 l_i - \frac{l_i^4}{4}) + \frac{c_i}{12} (t_i^4 l_i - \frac{l_i^5}{5}) \right\} - \left\{ \frac{1}{2} (t_i^2 l_i - \frac{l_i^3}{3}) + \frac{a_i}{3} (t_i^3 l_i - \frac{l_i^4}{4}) + \right. \\
 &\quad \left. \frac{b_i}{8} (t_i^4 l_i - \frac{y_i^5}{5}) + \frac{c_i}{15} (t_i^5 l_i - \frac{l_i^6}{6}) \right\} - \frac{C_{hi} t_i - \frac{C_{hi} t_i^2}{2}}{p_i^m - 1} \left(\frac{a_i l_i^2}{2} + \frac{b_i l_i^3}{6} + \frac{c_i l_i^4}{12} \right) + \frac{C_{hi}}{p_i^m - 1} \left(\frac{a_i l_i^3}{3} + \frac{b_i l_i^4}{8} + \frac{c_i l_i^5}{15} \right) \\
 &\quad - \frac{C_{hi}}{p_i^m - 1} \left(\frac{a_i l_i^4}{4} + \frac{b_i l_i^5}{10} + \frac{c_i l_i^6}{18} \right) \\
 &= \frac{C_{hi}}{p_i^m - 1} \left\{ \left(\frac{t_i^2}{2} - \frac{1}{3} t_i^3 + a_i \left(\frac{t_i^3}{2} - \frac{3t_i^4}{4} \right) + b_i \left(\frac{t_i^4}{6} - \frac{2t_i^5}{15} \right) + c_i \left(\frac{17t_i^5}{180} - \frac{5t_i^6}{72} \right) \right) \right\} \\
 &\quad - \frac{C_{hi}}{p_i^m - 1} \left\{ (t_i l_i - \frac{l_i^2}{2}) + \frac{a_i}{2} (t_i^2 l_i - \frac{l_i^3}{3}) + \frac{b_i}{6} (t_i^3 l_i - \frac{l_i^4}{4}) + \frac{c_i}{12} (t_i^4 l_i - \frac{l_i^5}{5}) \right\} - \left\{ \frac{1}{2} (t_i^2 l_i - \frac{l_i^3}{3}) + \right. \\
 &\quad \left. \frac{a_i}{3} (t_i^3 l_i - \frac{l_i^4}{4}) + \frac{b_i}{8} (t_i^4 l_i - \frac{l_i^5}{5}) + \frac{c_i}{15} (t_i^5 l_i - \frac{l_i^6}{6}) \right\} \\
 &\quad - \frac{C_{hi} t_i - \frac{C_{hi} t_i^2}{2}}{p_i^m - 1} \left(\frac{a_i l_i^2}{2} + \frac{b_i l_i^3}{6} + \frac{c_i l_i^4}{12} \right) + \frac{C_{hi}}{p_i^m - 1} \left(\frac{a_i l_i^3}{3} + \frac{b_i l_i^4}{8} + \frac{c_i l_i^5}{15} \right) - \frac{C_{hi}}{p_i^m - 1} \left(\frac{a_i l_i^4}{4} + \frac{b_i l_i^5}{10} + \frac{c_i l_i^6}{18} \right)
 \end{aligned}$$

As $a_i \square 1, b_i \square 1$ and $c_i \square 1$...(24)

Ordering cost

$O.C_i = C_{io}$...(25)

The retailer's unit time profit with late delivery by l_i unit time F_{iRd} is:

$$\begin{aligned}
 F_{iRd} &= \frac{1}{T_i} [\text{sales revenue} - \text{purchasing cost} - \text{lost sale cost} - \text{det cost} - \text{holding cost} - \\
 &\quad \text{ordering cost}] \\
 F_{iRd} &= \frac{1}{T_i} \left[\frac{l_i(2T_i - l_i)}{p_i^m - 1} + \frac{T_i}{p_i^m - 1} \left\{ T_i - t_i - \frac{K_{i0}}{K_{i1}} (1 - e^{-K_{i1}(T_i - t)}) \right\} C_{ri} - \right. \\
 &\quad \left. \frac{C_{pi}}{p_i^m - 1} \left[\left\{ (t_i - l_i) + \frac{a_i}{2}(t_i^2 - l_i^2) + \frac{b_i}{6}(t_i^3 - l_i^3) + \frac{c_i}{12}(t_i^4 - l_i^4) \right\} - \left\{ \frac{1}{2}(t_i^2 - l_i^2) + \right. \right. \right. \\
 &\quad \left. \left. \frac{a_i}{3}(t_i^3 - l_i^3) + \frac{b_i}{8}(t_i^4 - l_i^4) + \frac{c_i}{15}(t_i^5 - l_i^5) \right\} \right] e^{a_i l_i + \frac{b_i l_i^2}{2} + \frac{c_i l_i^3}{3}} + \frac{C_{pi} K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - t)}) \\
 &\quad - \frac{l_i(2T_i - l_i)}{p_i^m - 1} + \frac{T_i}{p_i^m - 1} \left\{ T_i - t_i - \frac{K_{i0}}{K_{i1}} (1 - e^{-K_{i1}(T_i - t)}) \right\} C_{ri}
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{C_{di}}{p_i^m - 1} \left[\{(t_i - l_i) + \frac{a_i}{2}(t_i^2 - l_i^2) + \frac{b_i}{6}(t_i^3 - l_i^3) + \frac{c_i}{12}(t_i^4 - l_i^4)\} - \right. \\
 & \left. \left\{ \frac{1}{2}(t_i^2 - l_i^2) + \frac{a_i}{3}(t_i^3 - l_i^3) + \frac{b_i}{8}(t_i^4 - l_i^4) + \frac{c_i}{15}(t_i^5 - l_i^5) \right\} \right] e^{a_i l_i + \frac{b_i l_i^2}{2} + \frac{c_i l_i^3}{3}} + \frac{2T_i(t_i - l_i) - t_i^2 + l_i^2}{p_i^m - 1} C_{di} \\
 & - \frac{C_{hi}}{p_i^m - 1} \left\{ \left(\frac{t_i^2}{2} - \frac{1}{3}t_i^3 + a_i \left(\frac{t_i^3}{2} - \frac{3t_i^4}{4} \right) + b_i \left(\frac{t_i^4}{6} - \frac{2t_i^5}{15} \right) + c_i \left(\frac{17t_i^5}{180} - \frac{5t_i^6}{72} \right) \right) \right. \\
 & \left. - \frac{C_{hi}}{p_i^m - 1} \left\{ (t_i l_i - \frac{l_i^2}{2}) + \frac{a_i}{2}(t_i^2 l_i - \frac{l_i^3}{3}) + \frac{b_i}{6}(t_i^3 l_i - \frac{l_i^4}{4}) + \frac{c_i}{12}(t_i^4 l_i - \frac{l_i^5}{5}) \right\} - \left\{ \frac{1}{2}(t_i^2 l_i - \frac{l_i^3}{3}) + \right. \right. \\
 & \left. \left. \frac{a_i}{3}(t_i^3 l_i - \frac{l_i^4}{4}) + \frac{b_i}{8}(t_i^4 l_i - \frac{l_i^5}{5}) + \frac{c_i}{15}(t_i^5 l_i - \frac{l_i^6}{6}) \right\} - \frac{C_{hi} t_i - \frac{C_{hi} t_i^2}{2}}{p_i^m - 1} \left(\frac{a_i l_i^2}{2} + \frac{b_i l_i^3}{6} + \frac{c_i l_i^4}{12} \right) \right. \\
 & \left. + \frac{C_{hi}}{p_i^m - 1} \left(\frac{a_i l_i^3}{3} + \frac{b_i l_i^4}{8} + \frac{c_i l_i^5}{15} \right) - \frac{C_{hi}}{p_i^m - 1} \left(\frac{a_i l_i^4}{4} + \frac{b_i l_i^5}{10} + \frac{c_i l_i^6}{18} \right) \right. \\
 & \left. - C_{oi} \right] \dots(26)
 \end{aligned}$$

Vendor’s Inventory Model:

The retailer maintains his profit regardless of the supplier’s delivery behaviour.

The supplier’s unit time profit for i^{th} item with late delivery by l_i unit time:

$$\begin{aligned}
 F_{isd} &= \left[I_{i1}(l_i) + Q_{i2}(C_{pi} - C_{im}) - (F_{iR} - F_{iRd}) - X_i \right] \\
 F_{isd} &= \frac{1}{T_i} \left[\{(t_i - l_i) + \frac{a_i}{2}(t_i^2 - l_i^2) + \frac{b_i}{6}(t_i^3 - l_i^3) + \frac{c_i}{12}(t_i^4 - l_i^4)\} - \left\{ \frac{1}{2}(t_i^2 - l_i^2) + \frac{a_i}{3}(t_i^3 - l_i^3) + \frac{b_i}{8}(t_i^4 - l_i^4) + \right. \right. \\
 & \left. \left. \frac{C_{pi}}{15}(t_i^5 - l_i^5) \right\} \right] e^{a_i l_i + \frac{b_i l_i^2}{2} + \frac{c_i l_i^3}{3}} + \frac{K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - t_i)}) (C_{pi} - C_{im}) - (F_{iR} - F_{iRd}) - X_i] \dots(27)
 \end{aligned}$$

The integrated unit time profit is the sum of unit time profit of vendor and retailer:

$$F_{i1} = F_{iR} + F_{is}$$

$$F_{id1} = F_{iRd} + F_{isd}$$

and

$$F_1 = \sum_{i=1}^n (F_{iR} + F_{is})$$

$$F_{d1} = \sum_{i=1}^n (F_{iRd} + F_{isd}) \dots (28)$$

Theorem

F_i is concave in t_i

$$F_{i1} = \frac{1}{T} \left[\frac{p_i t_i (2T_i - t_i)}{2(p_i^m - 1)} + \frac{\lambda p_i K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - t_i)}) - \right.$$

$$\begin{aligned} & \frac{C_{pi}}{p_i^m - 1} \left\{ (t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12}) - (\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15}) \right\} + \frac{C_{pi} K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - t_i)}) - \\ & \frac{C_{di}}{p_i^m - 1} \left\{ (t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12}) - (\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15}) - \frac{(2T_i t_i - t_i^2)}{2} \right\} - \\ & \frac{C_{hi}}{p_i^m - 1} \left\{ (\frac{t_i^2}{2} - \frac{1}{3} t_i^3 + a_i (\frac{t_i^3}{2} - \frac{3t_i^4}{4}) + b_i (\frac{t_i^4}{6} - \frac{2t_i^5}{15}) + c_i (\frac{17t_i^5}{180} - \frac{5t_i^6}{72})) \right\} \\ & - \frac{T_i}{p_i^m - 1} \left\{ T_i - t_i - \frac{K_{i0}}{K_{i1}} (1 - e^{-K_{i1}(T_i - t)}) \right\} C_{ri} - C_{oi}] \\ & - \frac{1}{T} \left[\frac{1}{p_i^m - 1} \left\{ (t_i + \frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{6} + \frac{c_i t_i^4}{12}) - (\frac{1}{2} t_i^2 + \frac{a_i t_i^3}{3} + \frac{b_i t_i^4}{8} + \frac{c_i t_i^5}{15}) \right\} \right. \\ & \left. \{ (C_{pi} - C_{mi}) + (1 - e^{-(a_i t_i + \frac{b_i t_i^2}{2} + \frac{c_i t_i^3}{3})}) C_{mi} - (\frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{3} + \frac{c_i t_i^4}{4}) \right\} + \frac{K_{i0}}{K_{i1}} \frac{T_i}{(p_i^m - 1)} (1 - e^{-K_{i1}(T_i - t_i)}) \right. \\ & \left. (C_{pi} - C_{mi}) - X_i \right] \\ & \frac{\partial F_{i1}}{\partial t_i} = \frac{1}{T} \left[\frac{p_i (2T_i - 2t_i)}{2(p_i^m - 1)} - \frac{\lambda p_i K_{i0}}{K_{i1}^2} \frac{T_i}{(p_i^m - 1)} e^{-K_{i1}(T_i - t_i)} - \right. \\ & \frac{C_{pi}}{p_i^m - 1} \left\{ (1 + a_i t_i + \frac{b_i t_i^2}{2} + \frac{c_i t_i^3}{3}) - (t_i + a_i t_i^2 + \frac{b_i t_i^3}{2} + \frac{c_i t_i^4}{3}) \right\} - \frac{C_{pi} K_{i0}}{K_{i1}^2} \frac{T_i}{(p_i^m - 1)} e^{-K_{i1}(T_i - t_i)} - \\ & \frac{C_{di}}{p_i^m - 1} \left\{ (1 + a_i t_i + \frac{b_i t_i^2}{2} + \frac{c_i t_i^3}{3}) - (t_i + a_i t_i^2 + \frac{b_i t_i^3}{2} + \frac{c_i t_i^4}{3}) - (T_i - t_i) \right\} - \\ & \frac{C_{hi}}{p_i^m - 1} \left\{ (t_i - t_i^2 + a_i (\frac{3t_i^2}{2} - 3t_i^3) + b_i (\frac{2t_i^3}{3} - \frac{2t_i^4}{3}) + c_i (\frac{17t_i^4}{36} - \frac{5t_i^5}{12})) \right\} - \\ & \frac{T_i}{p_i^m - 1} \left\{ -K_{i0} e^{-K_{i1}(T_i - t_i)} \right\} C_{ri} - C_{oi}] - \frac{1}{T} \left[\left\{ \frac{1}{p_i^m - 1} \left\{ (1 + a_i t_i + \frac{b_i t_i^2}{2} + \frac{c_i t_i^3}{3}) - (t_i + a_i t_i^2 + \frac{b_i t_i^3}{2} \right. \right. \right. \\ & \left. \left. + \frac{c_i t_i^4}{3}) \right\} \{ (C_{pi} - C_{mi}) + (1 - e^{-(a_i t_i + \frac{b_i t_i^2}{2} + \frac{c_i t_i^3}{3})}) C_{mi} - (\frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{3} + \frac{c_i t_i^4}{4}) \right\} - \frac{K_{i0} T_i}{(p_i^m - 1)} e^{-K_{i1}(T_i - t_i)} (C_{pi} - C_{mi}) \right] \left. \right] \\ & \frac{\partial^2 F_{i1}}{\partial t_i^2} = \frac{1}{T_i} \left[\frac{p_i}{(p_i^m - 1)} - \frac{\lambda p_i K_{i0} K_{i1} T_i}{(p_i^m - 1)} e^{-K_{i1}(T_i - t_i)} - \frac{C_{pi}}{p_i^m - 1} \left\{ (a_i + b_i t_i + c_i t_i^2) - (1 + 2a_i t_i + \frac{3b_i t_i^2}{2} + \frac{4c_i t_i^3}{3}) \right\} \right. \\ & - \frac{C_{pi} K_{i1} K_{i0} T_i}{(p_i^m - 1)} e^{-K_{i1}(T_i - t_i)} - \frac{C_{di}}{p_i^m - 1} \left\{ (a_i + b_i t_i + c_i t_i^2) - (1 + 2a_i t_i + \frac{3b_i t_i^2}{2} + \frac{4c_i t_i^3}{3}) + 1 \right\} - \frac{C_{hi}}{p_i^m - 1} \left\{ (1 \right. \\ & - 2t_i + a_i (3t_i - 6t_i^2) + b_i (6t_i^2 - \frac{8t_i^3}{3}) + c_i (\frac{17t_i^3}{9} - \frac{25t_i^4}{12}) - \frac{T_i}{p_i^m - 1} \left\{ -K_{i0} e^{-K_{i1}(T_i - t_i)} \right\} C_{ri} - C_{oi}] - \frac{1}{T} \left[\left\{ \right. \right. \\ & \left. \left. \frac{1}{p_i^m - 1} \left\{ (a_i + b_i t_i + c_i t_i^2) - (1 + 2a_i t_i + \frac{3b_i t_i^2}{2} + \frac{4c_i t_i^3}{3}) \right\} \{ (C_{pi} - C_{mi}) + (1 - e^{-(a_i t_i + \frac{b_i t_i^2}{2} + \frac{c_i t_i^3}{3})}) C_{mi} - \right. \right. \\ & \left. \left. (\frac{a_i t_i^2}{2} + \frac{b_i t_i^3}{3} + \frac{c_i t_i^4}{4}) \right\} - \frac{K_{i1} K_{i0} T_i}{(p_i^m - 1)} e^{-K_{i1}(T_i - t_i)} \right] \left. \right] \end{aligned}$$

$$\frac{\partial^2 F_{i1}}{\partial t_i^2} = \frac{1}{(p_i^m - 1)T_i} [p_i - 1 - \{C_{pi} + C_{di} + (C_{pi} - C_{mi})\} + (1 - e^{-(a_i t_i + \frac{b_i t_i^2}{2} + \frac{c_i t_i^3}{3})}) C_{mi} - (\frac{a_i l_i^2}{2} + \frac{b_i l_i^3}{3} + \frac{c_i l_i^4}{4}) \{ (a_i + b_i t_i + c_i t_i^2) - (1 + 2a_i t_i + \frac{3b_i t_i^2}{2} + \frac{4c_i t_i^3}{3}) \} - \frac{(\lambda p_i + C_{pi} + C_{ri}) K_{i0} K_{i1} T_i}{(p_i^m - 1)} e^{-K_{ri}(T_i - t_i)} - \frac{C_{hi}}{p_i^m - 1} \{ (1 - 2t_i + a_i(3t_i - 6t_i^2) + b_i(6t_i^2 - \frac{8t_i^3}{3}) + c_i(\frac{17t_i^3}{9} - \frac{25t_i^4}{12}) \}] \leq 0.$$

$$0 \leq [p_i - 1 - \{C_{pi} + C_{di} + (C_{pi} - C_{mi})\} + (1 - e^{-(a_i t_i + \frac{b_i t_i^2}{2} + \frac{c_i t_i^3}{3})}) C_{mi} - (\frac{a_i l_i^2}{2} + \frac{b_i l_i^3}{3} + \frac{c_i l_i^4}{4}) \{ (a_i + b_i t_i + c_i t_i^2) - (1 + 2a_i t_i + \frac{3b_i t_i^2}{2} + \frac{4c_i t_i^3}{3}) \} - (\lambda p_i + C_{pi} + C_{ri}) K_{i0} K_{i1} T_i e^{-K_{ri}(T_i - t_i)} - C_{hi} \{ (1 - 2t_i + a_i(3t_i - 6t_i^2) + b_i(6t_i^2 - \frac{8t_i^3}{3}) + c_i(\frac{17t_i^3}{9} - \frac{25t_i^4}{12}) \}]$$

This is the required condition.

IV. Numerical Example

Now we solve the model with the help of partially differentiating the equation with respect to continuous variable t_i and then obtain the values with the help of numerical using the software mathematica.

Item	Item1	Item2	Item3
T_i	20	25	30
K_{i0}	0.5	0.6	0.7
K_{i1}	1.2	1.4	1.6
a_i	.0001	.0002	.0003
b_i	.001	.0014	.0018
c_i	.0001	.0002	.0003
m	3	3	3
l_i	2	3	4
C_{pi}	15	16	17
C_{mi}	8	9	10
C_{hi}	0.5	0.55	0.6
C_{di}	16	17	18
C_{ri}	8	9	10
X_i	100	150	200
K	3	3	3
λ	0.8	0.8	0.8
t_i	17.6340	15.4369	14.2345
$I_{i1}(0)$	91.6466	116.4430	137.4030
Q_{i2}	9.6710	15.6463	91.0959
Q_i	101.367	164.6358	226.5679
Q_{i1}	85.3041	110.715	128.946
F_r	27.9078	27.9934	32.8493

It can be observed that

1. With the increments in deterioration factor a_i, b_i or c_i the unit time profit for all the items decreases.
2. As the value of m increases the unit time profit for all the items decreases.
3. With the increments in holding cost for all the items unit time profit shows the reverse effect.

V. Conclusion

The main contribution of this paper has been the development of a dynamic heuristic to determine replenishment cycle and economic order quantity of all the products. The heuristic provides an excellent performance, especially for larger problems which makes it very promising in applications of practical size. A multi-item inventory model of deteriorating items with expiration date is developed and analyzed. As lead time plays very important role in business decision therefore in the present study it is also taken into account. Numerical illustration with respect to different parameters is also presented in the model. In totality, the setup that has been chosen boasts of uniqueness in terms of the conditions under which the model has been developed.

The assumptions of the study impart exclusivity due to the combination of deterioration and expiration date for the products. The proposed model can be extended in numerous ways. For example, we may extend the demand to a more generalized demand pattern. We could generalize the model to allow for quantity discount. Also, we could consider the unit purchase cost, the inventory holding cost, and others as time dependent.

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