IOSR Journal of Mathematics (IOSR-JM)
e-ISSN:2278-5728, p-ISSN: 2319-765X.Volume12,Issue 1 Ver. III (Jan.-Feb.2016)PP 66-74
www.iosrjournals.org

Effect Of Viscous Dissipation On Power - Fluid Past A Permeable Stretching Sheet With Convective Boundary Condition

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Abstract: *The steady, two dimensional, laminar flow of a power - law fluid over a permeable a stretching sheet in the presence of viscous dissipation and convective boundary condition. Using the similarity transformations, the governing equations have been transformed into a system of ordinary differential equations. These differential equations are highly nonlinear which cannot be solved analytically. Therefore, bvp4c MATLAB solver has been used for solving it. Numerical results are obtained for the skin-friction coefficient and the local Nusselt number as well as the velocity and temperature profiles for different values of the governing parameters, namely suction parameter, power-law index parameter, Eckert number, convective parameter, Prandtl number.*
Keywords: *power-law fluid, viscous dissipation, stretching sheet, suction, convective boundary condition.*

I. Introduction

Since 1960, a considerable attention has been devoted to predict the drag force behavior and energy transport characteristics of the non-Newtonian fluid flows. The main reason for this is probably that fluids (such as molten plastics, pulps, slurries, emulsions), which do not obey the assumption of Newtonian fluids that the tress tensor is directly proportional to the deformation tensor, are found in various engineering applications. A variety of constitutive equations have been proposed to describe the flow and heat transfer non-Newtonian characteristics, among them the empirical Ostwaald-de Waele model, which is known as the so-called power-law model, gained much acceptance. Schowalter (1960) and Acrivos et al. (1960) successfully applied the power-law model to the boundary layer problems. Kumari et al. (1995) investigated the non-similar mixed convection flow of a non-Newtonian fluid past a vertical wedge. Nadeem and Akbar (2010) investigated the peristaltic flow of Walter's B fluid in a uniform inclined tube. Mahmoud (2011) investigated the effects of surface slip and heat generation (absorption) on the flow and heat transfer of a non-Newtonian power-law fluid on a continuously moving surface. Olanrewaju et al. (2013) investigated the thermal and thermo diffusion on convection heat and mass transfer in a power law flow over a heated porous plate in the presence of magnetic field. Yacob and Ishak (2014) investigated the laminar flow of a power-law fluid over a permeable shrinking sheet of constant surface temperature. Aziz et al. (2015) conducted the study of forced convective boundary layer flow of power-law fluid along with heat transfer over a porous plate in a porous medium. Hayat et al. (2015) investigated the effects of convective heat and mass transfer in the flow of Eyring-Powell fluid past an inclined exponential stretching surface. Rashidi et al. (2015) analyzed the convective flow of a third grade non-Newtonian fluid due to a linearly stretching sheet subject to a magnetic field.

Hady and Hassanien (1986) studied the magnetohydrodynamic and constant suction/injection effects of axisymetric stagnation point flow and mass transfer for power-law fluids. Jadhav and Waghmode (1990) investigated the effect of suction is to decrease in temperature and the rate of heat transfer, while reverse nature occurs for injection. Sahu and Mathu (1996) concluded that the suction influence decreases the skin-friction. Olanrewaju and Makinde (2011) studied the free convective heat and mass transfer fluid past a moving vertical plate in the presence of suction and injection with thermal diffusion (Soret) and diffusion-thermo (Dufour) effects. Ishak (2012) studied the micropolar fluid flow towards a stretching/shrinking sheet in a porous medium with suction. Raju and Varma (2014) investigated the unsteady MHD free convective of non-Newtonian fluid through porous medium bounded by an infinite porous plate in the presence of constant suction. Chinyoka and Makinde (2015) analyzed the unsteady and porous media flow of reactive non-Newtonian fluids subjected to buoyancy and suction/injection.

Viscous dissipation which, appears as a source term in the fluid flow generates appreciable temperature, gives the rate at which mechanical energy is converted into heat in a viscous fluid per unit volume. This effect is of particular significant in natural convection in various devices that are subjected to large variation of gravitational force or that operate at high rotational speeds, pointed by Gebhart (1962) in his study of viscous dissipation on natural convection in fluids. Lawal and Mujumdar (1992) investigated the forced convection heat transfer to power-law fluids in arbitrary cross-sectional ducts with finite viscous dissipation. Tso et al. (2010) studied the hydro-dynamically and thermally fully developed laminar heat transfer of non-

Newtonian fluids between fixed parallel plates by taking into account the effect of viscous dissipation of the flowing fluid. Kairi et al. (2011) concluded that the heat transfer coefficient increases with increasing in the power law index n and viscosity parameter, while it decreases with the dissipation parameter. Boubaker et al. (2012) investigated the effects of viscous dissipation on the thermal boundary layer of pseudoplastic power-law non-Newtonian fluids.

The present study investigates the steady, two dimensional, laminar flow of a power - law fluid over a permeable a stretching sheet in the presence of viscous dissipation and convective boundary condition. Using the similarity transformations, the governing equations have been transformed into a set of ordinary differential equations, which are nonlinear and cannot be solved analytically, therefore, bvp4c MATLAB solver has been used for solving it. The results for velocity and temperature functions are carried out for the wide range of important parameters namely; namely, suction/injection parameter, power-law index parameter, convective parameter, Prandtl number and Eckert number. The skin friction, the couple wall stress and the rate of heat transfer have also been computed.

II. Mathematical Formulation

Consider a steady, two-dimensional laminar viscous flow of a power-law fluid over a permeable shrinking sheet of constant surface temperature T_w as shown in Fig. A. The shrinking velocity is assumed to be of the form $U_w(x) = ax^{1/3}$, where a is a positive constant. The x-axis extends parallel, while the y-axis extends upwards, normal to the surface of the sheet. The boundary layer equations are (Howell et al. (1997); Wang (1994); Xu & Liao (2009), and Yacob and Ishak (2014)):

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

Momentum equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial \tau}{\partial y} \tag{2.2}$$

Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.3}$$

The boundary conditions are

$$u = -U_w(x), v = V_w(x), -k\frac{\partial T}{\partial y} = h\left(T_w - T\right) \quad \text{at} \quad y = 0$$

$$u \to 0, T \to T_\infty \quad \text{as} \quad y \to \infty \tag{2.4}$$

where u and v are the velocity components along the x and y directions, respectively, τ_{xy} is the shear stress, h is the convective heat transfer coefficient, k is the thermal conductivity of the fluid, c_p is the specific heat, υ is the kinematic viscosity of the fluid, ρ is the fluid density and $V_w(x)$ (which will be defined later) is the mass transfer velocity at the surface of the sheet.

The stress tensor is defined as (Andersson and Irgens (1990); Wilkinson (1960)),

$$\tau_{ij} = 2K\left(2D_{ij}D_{kl}\right)^{(n-1)/2} D_{ij} \tag{2.5}$$

Where

$$D_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2.6}$$

denotes the stretching tensor, K is called the consistency coefficient and n is the power-law index. The index n is non-dimensional and the dimension of K depends on the value of n. The two-parameter rheological (2.5) is known as the Ostwald-de-Waele model or, more commonly, the power-law model. The parameter n is an important index to subdivide fluids into pseudoplastic fluids (n <1) and dilatant fluids (n>1). For n = 1, the fluid is simply the Newtonian fluid. Therefore, the deviation of n from unity indicates the degree of deviation from Newtonian behavior (Wang 1994). With $n \neq 1$, the constitutive (2.5) represents shear-thinning (n <1) and shear-thickening (n >1) fluids. Using (2.5) and (2.6), the shear stress appearing in (2.2) can be written as:

$$\tau_{xy} = K\left(\frac{\partial u}{\partial y}\right)^n \tag{2.7}$$

Now the momentum (2.2) becomes:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{K}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n \tag{2.8}$$

The continuity (2.1) is satisfied by introducing a stream function ψ such that

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \tag{2.9}$$

The momentum (2.8) and the energy (2.3) can be transformed into the corresponding ordinary differential equations by the following transformation (Xu & Liao 2009):

$$\eta = \frac{y}{x}(\mathrm{Re}_x)^{1/(n+1)}, \psi = U_w x (\mathrm{Re}_x)^{-1/(n+1)} f(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{2.10}$$

Where η is the similarity variable, f(η) is the dimensionless stream function and $Re_x = \rho x^n U_w^{2-n}/K$ is the local Reynolds number.

Thus the mass transfer velocity $V_w(x)$ may be defined as:

$$V_w(x) = -\frac{2}{3}a\left(\frac{\rho a^{2-n}}{K}\right)^{1/(n+1)} x^{-1/3} f_w \tag{2.11}$$

where f_w is the suction/injection parameter with $f_w > 0$ is for suction, $f_w < 0$ is for injection and $f_w = 0$ corresponds to an impermeable plate.

The transformed nonlinear ordinary differential equations are:

$$n(f''(\eta))^{n-1} f'''(\eta) + \frac{2}{3} f(\eta) f''(\eta) - \frac{1}{3} f'(\eta)^2 = 0 \tag{2.12}$$

$$\frac{1}{\mathrm{Pr}}\theta''(\eta) + \frac{2}{3} f\theta' + Ecf''(\eta)^2 = 0 \tag{2.13}$$

The boundary conditions become,

$$f(0) = f_w, f'(0) = -1, \theta'(0) = -Bi(1 - \theta(0))$$

$$f'(\eta) \to 0, \theta(\eta) \to 0 \qquad as \ \eta \to \infty \tag{2.14}$$

Here primes denote differentiation with respect to η.

Pr is the Prandtl number, Ec is the Eckert number and Bi is the convective parameter defined respectively as

$$\mathrm{Pr} = \frac{a}{\alpha}\left(\frac{\rho a^{2-n}}{K}\right)^{2/(n+1)}, Ec = \frac{\upsilon U_w^2}{c_p(T_w - T_\infty)}, Bi = \frac{hx}{k}(\mathrm{Re}_x)^{-1/(n+1)} \tag{2.15}$$

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\tau_w}{\rho U_w^2/2}, Nu_x = \frac{xq_w}{k(T_w - T_\infty)} \tag{2.16}$$

where the surface shear stress τ_w and the surface heat flux q_w are given by

$$\tau_w = \left[K\left(\frac{\partial u}{\partial y}\right)^n\right]_{y=0}, q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{2.17}$$

Using the similarity variables (2.10), we obtain

$$\frac{1}{2}C_f \mathrm{Re}_x^{1/(n+1)} = [f''(0)]^n, \frac{Nu_x}{\mathrm{Re}_x^{1/(n+1)}} = -\theta'(0) \tag{2.18}$$

III. Solution Of The Problem

The set of equations (2.12) to (2.14) were reduced to a system of first-order differential equations and solved using a MATLAB boundary value problem solver called **bvp4c**. This program solves boundary value problems for ordinary differential equations of the form $y' = f\left(x, y, p\right), a \le x \le b$, by implementing a collocation method subject to general nonlinear, two-point boundary conditions $g\left(y(a), y(b), p\right)$. Here $\boldsymbol{p}$ is a vector of unknown parameters. Boundary value problems (BVPs) arise in most diverse forms. Just about any BVP can be formulated for solution with **bvp4c**. The first step is to write the *ODEs* as a system of first order ordinary differential equations. The details of the solution method are presented in Shampine and Kierzenka (2000).

IV. Results And Discussion

The governing equations (2.12) - (2.13) subject to the boundary conditions (2.14) are integrated as described in section 3. In order to get a clear insight of the physical problem, the velocity and temperature have been discussed by assigning numerical values to the parameters encountered in the problem.

Figs. 1 & 2 illustrate the effect of power – index parameter (n) on the velocity and temperature respectively. We observed that the velocity and temperature decreases with increasing n and consequently decreases the momentum and thermal boundary layer thickness. Figs. 3 & 4 show the effect of the suction parameter (f_w) on velocity and temperature profiles. We observe that the velocity and temperature decreases with the increases the values of f_w and consequently momentum and thermal boundary layer thickness decreases.

Figs. 5 illustrate the effect of Eckert number (Ec) on the temperature respectively. It is observed that the temperature increases with increasing Ec. Fig 6 illustrate the effects of the convective parameter (*Bi*) on temperature respectively. It is observed that the temperature of the fluid increases with a rising the values of Bi. Figs. 7 illustrate the effect of Prandtl number (Pr) on the temperature respectively. It is observed that the temperature decreases with increasing Pr.

Fig. 8 shows the effects of n and f_w on skin friction. From Fig. 8 it is seen that the skin friction increases with an increasing the values of n or *fw*. The effect of n and f_w on local Nusselt number is shown in fig.9. It is found that the local Nusselt number enhances with an increasing the values of n or f_w. The effect of *Ec* and *Bi* on local Nusselt number is shown in fig.10. It is found that the local Nusselt number enhances with an increasing the values of Bi whereas it decreases with increasing the values if Ec. Tables.1 shows that the present results perfect agreement to the previously published data.

V. Conclusions

In the present paper, the steady, two dimensional, laminar flow of a power - law fluid over a permeable a stretching sheet in the presence of viscous dissipation and convective boundary condition. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. It has been found that

1. The momentum and thermal boundary layer thickness decreases with an increase in the power-law index parameter and suction parameter.
2. The fluid temperature enhances in the presence of viscous dissipation and convective boundary condition.
3. The skin friction increases with an increase the suction parameter or the power-law index parameter.
4. The local Nusselt number enhances with an increase in the suction parameter or the power-law index parameter and convective parameter whereas local Nusselt number reduces with increase the Eckert number.

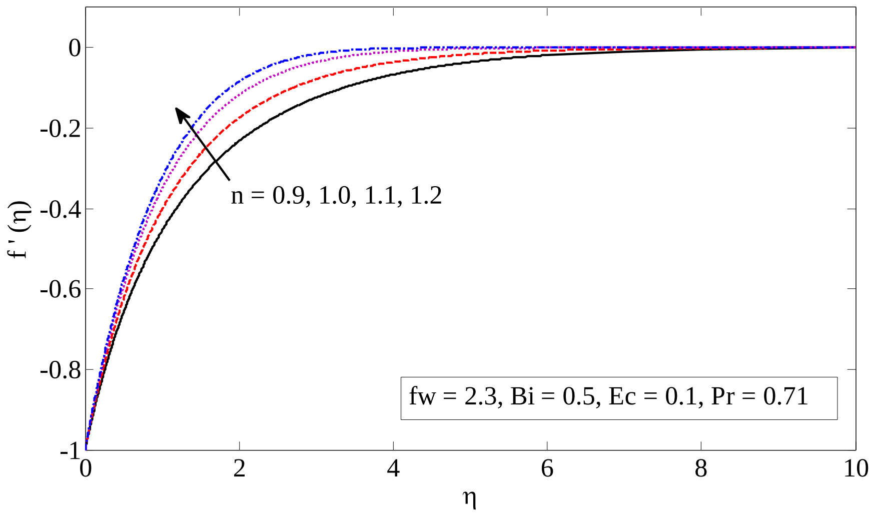

Fig.1 Velocity for various values of *n*

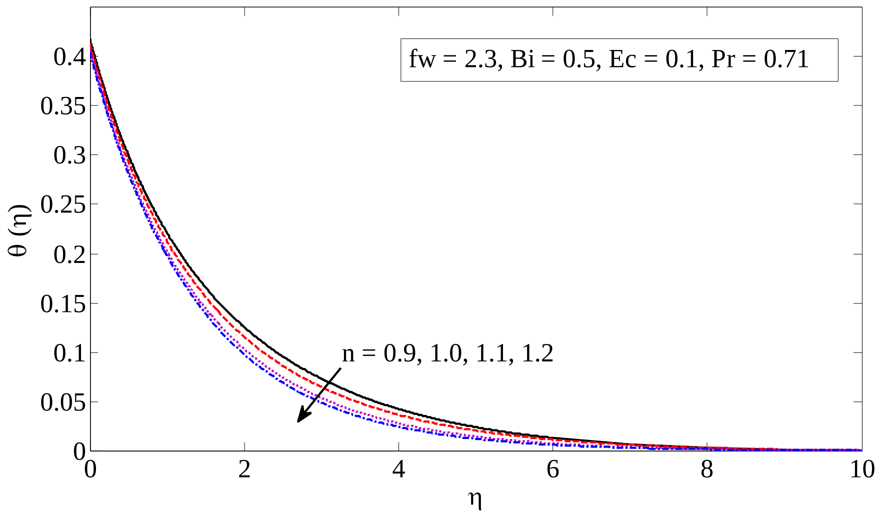

Fig.2 Temperature for various values of *n*

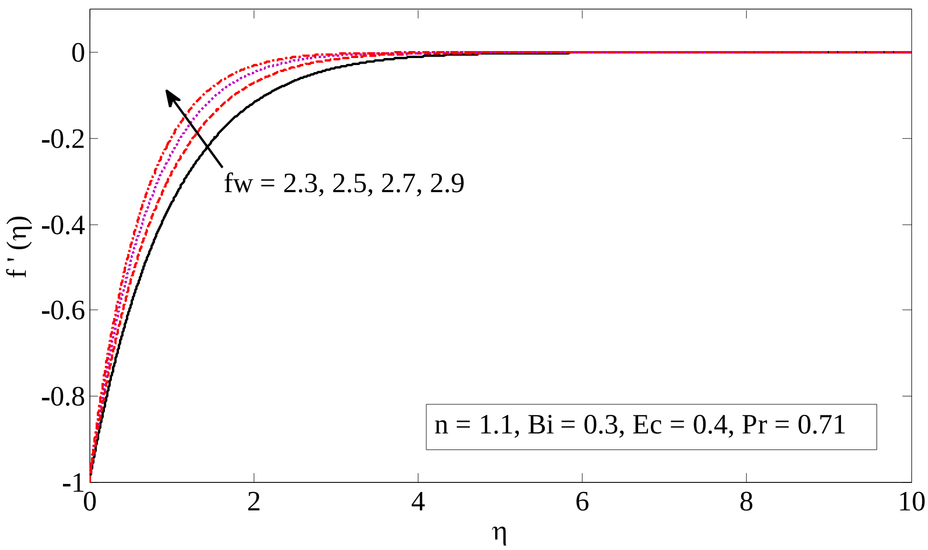

Fig.3 Velocity for various values of f_w

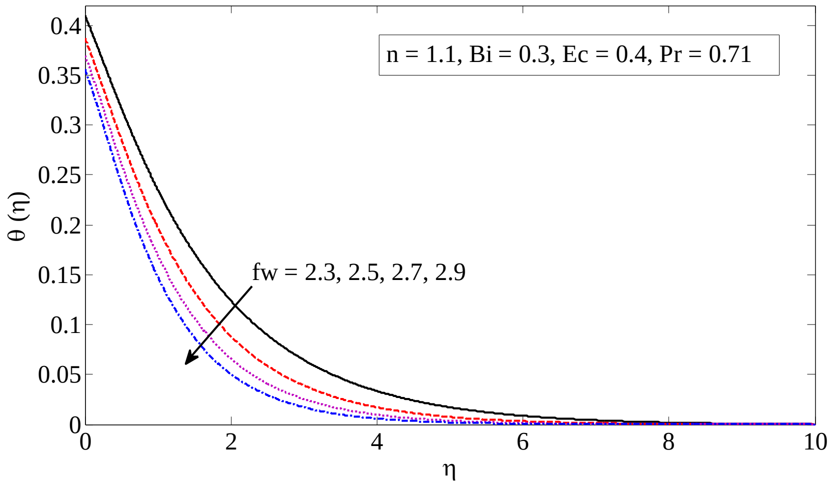

Fig.4 Temperature for various values of f_w

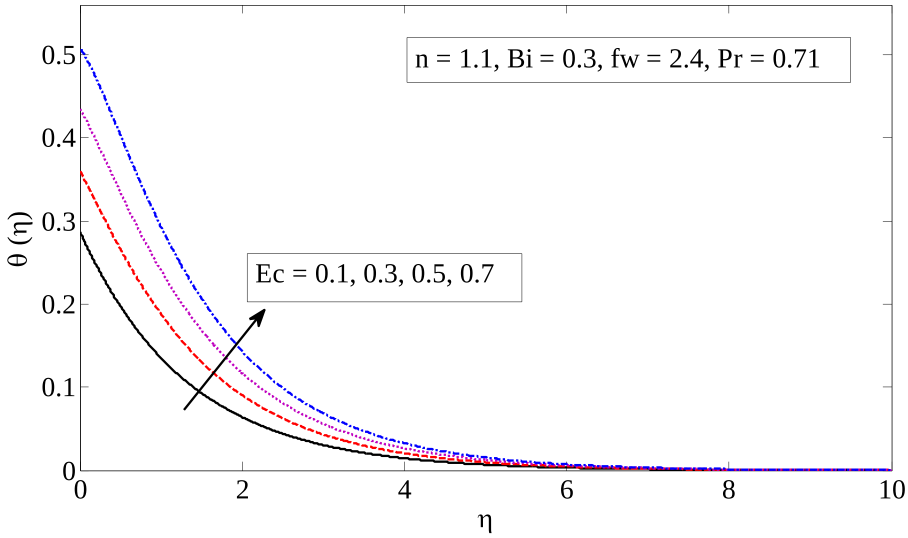

Fig.5 Temperature for various values of Ec

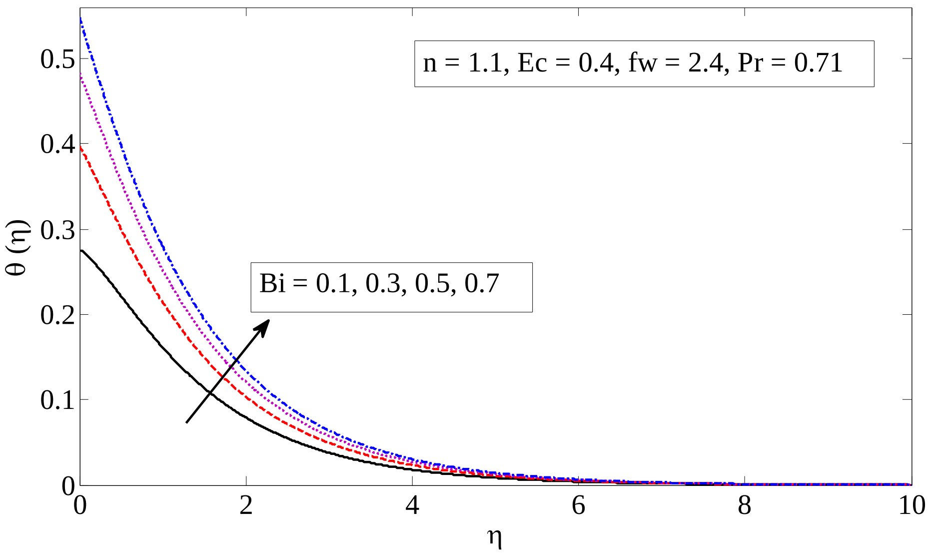

Fig.6 Temperature for various values of *Bi*

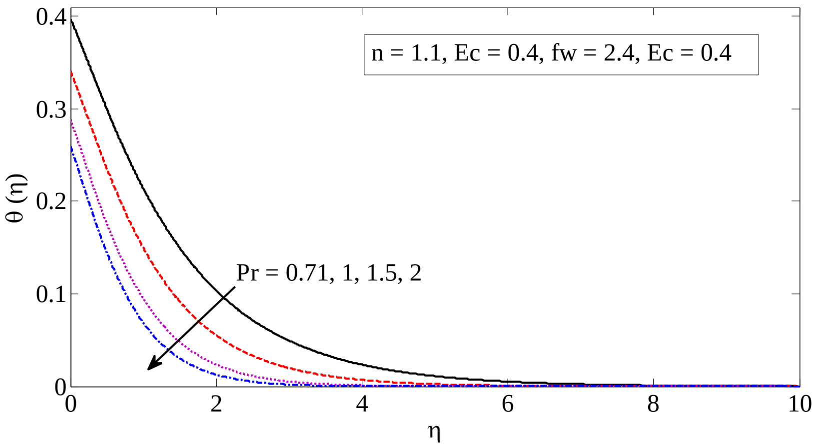

Fig.7 Temperature for various values of *Pr*

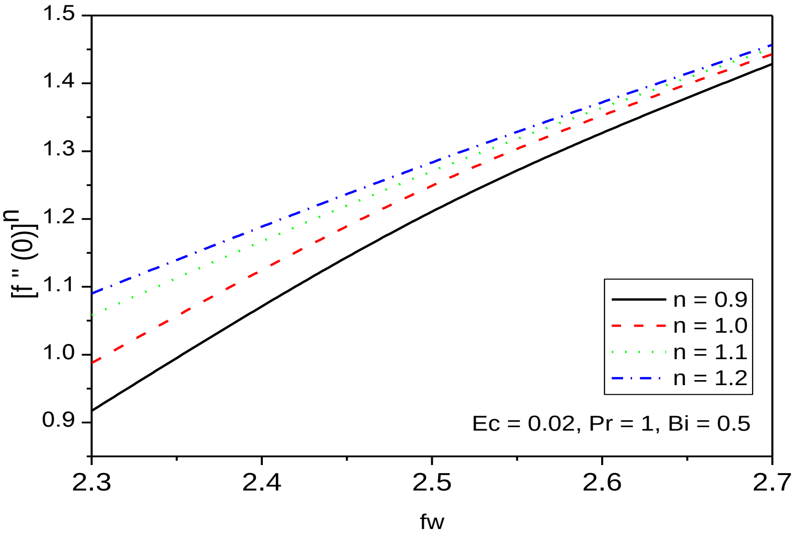

Fig.8 skin-friction for different values of f_w and *n*.

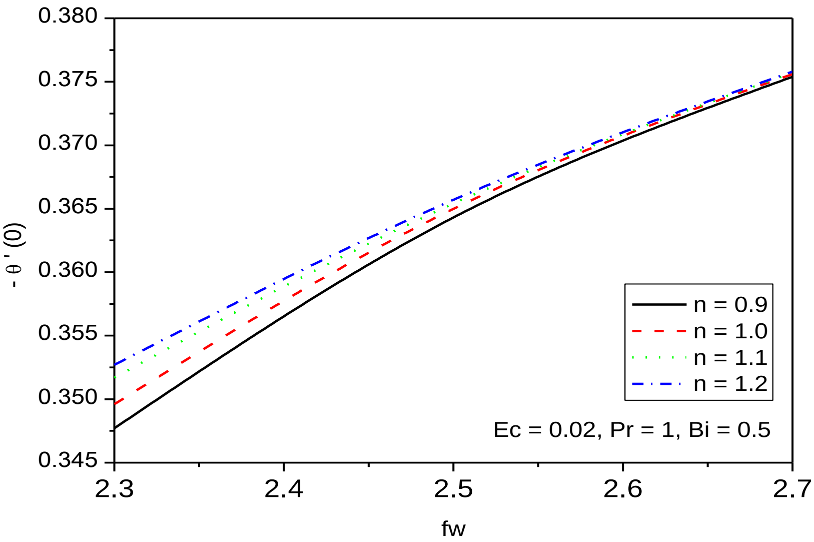

Fig.9 Nusselt number for different values of f_w and *n*.

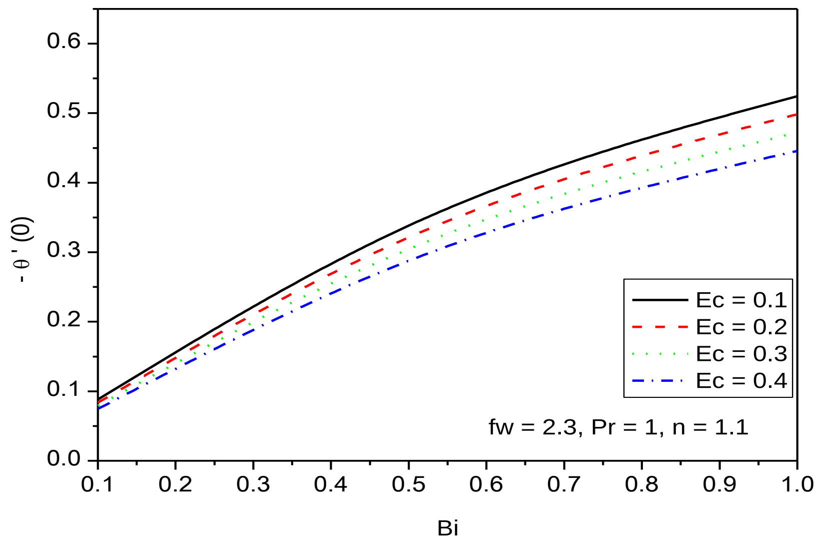

Fig.10 Nusselt number for various values of *Bi* and *Ec*.

Table 1. Comparison for the values of $[f''(0)]^n, -\theta'(0)$ for the values of n and λ when Pr=1, Ec=Bi=0.

n	f_w	$[f''(0)]^n$		$-\theta'(0)$	
		Present Results	Yacob and Ishak (2014)	Present Results	Yacob and Ishak (2014)
0.6	3.3	1.974939	1.9198	2.037115	2.0307
0.7	3.0	1.694985	1.6783	1.801927	1.7989
0.9	2.5	1.210824	1.2070	1.386086	1.3851
1.0	2.3	0.985765	0.9840	1.195510	1.1946
1.1	2.2	0.911617	0.9116	1.111739	1.1112
1.2	2.1	0.825308	0.8253	1.021205	1.0203
1.3	2.0	0.720385	0.7204	0.917866	0.9156
1.4	1.93	0.678034	0.6780	0.862276	0.8594

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