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Unsteady Free Convective Flow along a Vertical Porous Plate with Variable Viscosity and Thermal Conductivity

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Abstract : *The effects of temperature dependent viscosity and thermal conductivity on a two dimensional unsteady laminar MHD free convective flow over a vertical plate immersed in a porous medium with viscous dissipation and heat generation have been studied in the current work. The governing boundary layer equations are converted into a system of coupled nonlinear ordinary partial differential equations by using non-dimensional parameters and then solved numerically using finite difference method. Effects of variable viscosity, variable thermal conductivity, and the other parameters engaged in the study on the velocity, temperature and concentration profiles are investigated graphically. Skin friction and Nusselt number profiles have been also illustrated graphically.*

Keywords: *MHD, Porous plate, Variable viscosity, Variable thermal conductivity, viscous dissipation, Heat and Mass transfer.*

I. Introduction

Magneto hydrodynamic (MHD) is the branch of continuum mechanics which deals with the flow of electrically conducting fluids in electric and magnetic fields. Fluid flow problems of free convective, heat and mass transfer through a porous medium had been given attention due to its applications in many engineering problems such as nuclear reactor design, geothermal systems, petroleum engineering applications, evaporation of the surface of a water body, control of pollutants in ground water, energy transfer in a wet cooling tower, food processing cooler, and the problem of heat and mass transfer flow of a laminar boundary layer over a stretching sheet in a impregnated porous medium has an important application in the metallurgy and chemical engineering fields. Postelnicu (2004) investigated the influence of magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous medium with Soret and Dufour effects [1]. Then viscous flow for a non-linearly stretching sheet with chemical reaction and magnetic field has been carried out by Raptis and Perdikis (2006). Further Alam et al. (2007) took into account the viscous dissipation effects on MHD natural convection boundary layer over a sphere of an electrically conducting fluid in the presence of heat generation [2]. Salem and El-Aziz (2008) presented the effects of hall currents and chemical reactions on hydro magnetic flow of a stretching vertical surface with internal heat generation [3].

The Soret effect with the influence of variable viscosity and natural convection from a melting vertical surface in a non-Darcy porous medium saturated with Newtonian fluid of variable viscosity has discussed by Kairi and Murthy (2013) [4]. Seddeek and Salama (2007) carried out the effects of temperature dependent viscosity and thermal conductivity with variable suction on MHD convective heat transfer past a vertical moving porous plate with variable suction [5]. Anghel et al. (2000) studied Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium [6]. A numerical study on unsteady natural convection of air with variable viscosity over an isothermal vertical has investigated Rani and Kim (2010). After that Hazarika and Konch (2014) illustrated the effects of variable viscosity and thermal conductivity on MHD free convective flow along a vertical porous plate with viscous dissipation for steady case [8]. Recently effects of variable viscosity, Dufour, Soret, and thermal conductivity on free convective heat and mass transfer of non-Darcian flow past porous flat surface has been described by Animasuanand Oyem (2014) [9].

In all the aforementioned analysis the combined effects of temperature dependent viscosity and thermal conductivity with viscous dissipation and heat generation for the unsteady case have not been studied. That's why in the present study an attempt has been made to incorporate the unsteady free convective flow along a vertical porous plate with variable viscosity and thermal conductivity. Explicit finite difference method has used for solving the dimensionless governing equations.

II. Mathematical Formulation

Consider an unsteady two-dimensional heat transfer flow of a viscous incompressible electrically conducting fluid along a vertical stretching permeable sheet in a porous medium with viscous dissipation and heat generation. The flow is taken along an x' -direction, which is along the plate in the upward direction while the *y*-axis is taken to be normal to the plate. The velocity component u' and v' are taken to be x' and y' direction respectively. B_0 is a transverse magnetic field which is applied in the y' directions and it generates magnetic force in x'-directions. The governing mass, momentum, energy and concentration equations take the following forms,

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \frac{1}{\rho}\frac{\partial}{\partial y'}\left(\mu\frac{\partial u'}{\partial y'}\right) - \vartheta\frac{u'}{K} - \frac{\sigma B_0^2 u'}{\rho} \tag{2}$$

$$\frac{\partial T'}{\partial t'} + u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \frac{1}{\rho C_p}\frac{\partial}{\partial y'}\left(\kappa\frac{\partial T'}{\partial y'}\right) + \frac{\mu}{\rho C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 + \frac{Q_0}{\rho C_p}(T' - T'_\infty) \tag{3}$$

$$\frac{\partial C'}{\partial t'} + u'\frac{\partial C'}{\partial x'} + v'\frac{\partial C'}{\partial y'} = D_m\frac{\partial^2 C'}{\partial y'^2} \tag{4}$$

The corresponding boundary conditions can be written as

$$\left.\begin{aligned} &u' = U_w, v' = V_w, T' = T'_W, C' = C'_w \quad at\ y' = 0 \\ &u' \to 0, T' \to T'_\infty, C' \to C'_\infty \qquad at\ y' \to \infty \end{aligned}\right\} \tag{5}$$

Where, ρ is the density in the stream, μ is the viscosity of the fluid, g is the acceleration due to gravity, C_p is specific heat at constant pressure, β is the volumetric coefficient of thermal expansion and $\beta*$ is the volumetric coefficient of expansion with concentration, k is the permeability constant, σ is the electrical conductivity, κ is the thermal conductivity of the fluid, Q_0 is the volumetric rate of heat generation and D_m is the molecular diffusivity of the concentration.

Let, the viscosity as well as the thermal conductivity varies as a linear function of temperature.

$\mu(T') = \mu^*\left(1 + \lambda(T' - T'_\infty)\right)$ and $\kappa(T') = \kappa^*\left(1 + \xi(T' - T'_\infty)\right)$ (Rani and Kim, 2010)

Let γ and ε denote the non-dimensional viscosity and thermal conductivity parameter and be given by $\gamma = \lambda(T'_w - T'_\infty)$ and $\varepsilon = \xi(T'_w - T'_\infty)$. Then the fluid viscosity and thermal conductivity takes the following form, $\mu = \mu^*(1 + \gamma\theta)$ and $\kappa = \kappa^*(1 + \varepsilon\theta)$

It is convenient to employ the following dimensionless variables for making the governing equations non-dimensional

$$\begin{gathered} U = \frac{u'}{U_0},\ V = \frac{v'}{U_0},\ X = \frac{x'U_0}{\vartheta},\ Y = \frac{y'U_0}{\vartheta},\ S_c = \frac{\vartheta}{D},\ M = \frac{\sigma B_0^2\vartheta}{\rho U_0^2},\ Q = \frac{\vartheta Q_0}{\rho C_p U_0^2} \\ x' = \frac{X\vartheta}{U_0},\ y' = \frac{Y\vartheta}{U_0},\ \tau = \frac{t'U_0^2}{\vartheta},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\ k = \frac{U_0^2 K'}{\vartheta^2},\ P_r = \frac{\rho\vartheta C_p}{\kappa} \\ G_r = \frac{g\beta\vartheta(T'_w - T'_\infty)}{U_0^3},\ G_m = \frac{g\beta*\vartheta(C'_w - C'_\infty)}{U_0^3},\ E_c = \frac{U_0^2}{C_p(T'_w - T'_\infty)} \end{gathered} \tag{6}$$

With the help of equation (6), the equations (1) to (4) reduced the following non-dimensional form

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{7}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = G_r\theta + G_m C + \left(\frac{\gamma}{1 + \gamma\theta}\right)\frac{\partial \theta}{\partial Y}\frac{\partial U}{\partial Y} + \frac{\partial^2 U}{\partial Y^2} - \frac{U}{k} - MU \tag{8}$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{P_r}\left(\frac{\partial^2\theta}{\partial Y^2} + \left(\frac{\varepsilon}{1 + \varepsilon\theta}\right)\left(\frac{\partial \theta}{\partial Y}\right)^2\right) + E_c\left(\frac{\partial U}{\partial Y}\right)^2 + Q\theta \tag{9}$$

$$\frac{\partial C}{\partial \tau}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{S_C}\frac{\partial^2 C}{\partial Y^2} \tag{10}$$

The dimensionless boundary conditions is as follows,

$$U=1,\ V=1,\ \theta=1,\ C=1 \quad \text{at} \quad Y=0$$
$$U\to 0,\ \theta\to 0,\ C\to 0 \quad \text{as} \quad Y\to\infty \tag{11}$$

Skin friction and Nusselt number:

The non-dimensional skin-friction coefficient is generally defined as follows, $\tau=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ and the non-dimensional Nusselt number and is defined as follows, $N_u=-\mu\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$.

III. Numerical Solution

For simplicity, the explicit finite difference method has been used to solve from equations 7 to 10 subject to the conditions given by 11. To obtain the finite difference equations the region of the flow is divided into a grid or mesh of lines parallel to *X* and *Y* axis is taken along the plate. Here the plate of height $X_{max}(=20)$ i.e. X varies from 0 to 20 and regard $Y_{max}(=50)$ as corresponding to $Y\to\infty$ i.e. *Y* varies from 0 to 50. There are *m*=100 and *n*=300 grid spacing in the *X* and *Y* directions respectively. It is assumed that ΔX and ΔY are constant mesh sizes along *X* and *Y* directions respectively and taken as follows,

$\Delta X=0.20(0\le x\le 20)$ And $\Delta Y=0.25(0\le x\le 50)$ with the smaller time-step, Δt = 0.005.

Using the explicit finite difference approximation, the following appropriate set of finite difference are obtained as

$$\frac{U_{i,j}-U_{i-1,j}}{\Delta X}+\frac{V_{i,j}-V_{i,j-1}}{\Delta Y}=0 \tag{12}$$

$$\begin{aligned}\frac{U'_{i,j}-U_{i,j}}{\Delta\tau}+U_{i,j}\frac{U_{i,j}-U_{i-1,j}}{\Delta X}+V_{i,j}\frac{V_{i,j}-V_{i,j-1}}{\Delta Y}&=G_r\theta_{i,j}+G_mC_{i,j}+\left(\frac{\gamma}{1+\gamma\theta_{i,j}}\right)\\ \left(\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta Y}\right)\left(\frac{U_{i,j+1}-U_{i,j}}{\Delta Y}\right)&+\left(\frac{U_{i,j+1}-2U_{i,j}+U_{i,j-1}}{(\Delta Y)^2}\right)-\frac{U_{i,j}}{k}-MU_{i,j}\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\theta'_{i,j}-\theta_{i,j}}{\Delta\tau}+U_{i,j}\frac{\theta_{i,j}-\theta_{i-1,j}}{\Delta X}+V_{i,j}\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta Y}&=\frac{1}{P_r}\left(\left(\frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i,j-1}}{(\Delta Y)^2}\right)+\left(\frac{\varepsilon}{1+\varepsilon\theta_{i,j}}\right)\left(\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta Y}\right)^2\right)\\ &+E_c\left(\frac{U_{i,j+1}-U_{i,j}}{\Delta Y}\right)^2+Q\theta_{i,j}\end{aligned} \tag{14}$$

$$\frac{C'_{i,j}-C_{i,j}}{\Delta\tau}+U_{i,j}\frac{C_{i,j}-C_{i-1,j}}{\Delta X}+V_{i,j}\frac{C_{i,j+1}-C_{i,j}}{\Delta Y}=\frac{1}{S_C}\left(\frac{C_{i,j+1}-2C_{i,j}+C_{i,j-1}}{(\Delta Y)^2}\right) \tag{15}$$

Then the initial and boundary condition in finite difference scheme takes the following form as,

$U^n_{i,0}=1,\ V^n_{i,0}=1,\ \theta^n_{i,0}=1,\ C^n_{i,0}=1$ at $Y=0$. Then, $U^n_{i,L}=0,\ \theta^n_{i,L}=0,\ C^n_{i,L}=0$

Where, L corresponds to ∞. Here *i* and *j* designate the grid points with *X* and *Y* coordinates respectively and the superscript *n* represent a value of time $\tau=n\Delta\tau$ where, n=0, 1, 2……..

IV. Results And Discussion

A theoretical research is carried out and results are presented in this paper on unsteady free convective flow along a vertical porous plate with variable viscosity and thermal conductivity. Viscosity of the fluid together with the thermal conductivity is assumed to vary as a linear function of temperature. Explicit finite difference method is employed to solve the governing equations. Default values of thermo physical parameters are specified as follows, Magnetic parameter $M=1$, thermal Grashof number $G_r=10$, mass Grashof number $G_m=5$, permeability parameter $k=0.5$, viscosity variation parameter $\varepsilon=0.5$, Prandtl number $P_r=0.71$ for air, thermal conductivity parameter $\gamma=0.5$, Eckert number $E_c=0.01$, Heat generation parameter $Q=0.05$ and Schmidt number $S_c=0.60$ for water vapor. Figure (1) reveals the effects of magnetic parameter (*M*) on velocity distribution. It is clear from the graph that velocity decreases for increasing of magnetic parameter in air. It is due to the fact that the presence of transverse magnetic field produces a resistive force called Lorentz force, which leads to slow down the motion of fluid. The velocity variation for thermal Grashof number (G_r) is presented in the figure (2). The increase in the values of thermal Grashof number has the tendency to raise the thermal buoyancy effect.

This gives rise to an increase in the fluid flow. Figure (3) and (4) display the effects of viscosity variation parameter (γ) and thermal conductivity parameter (ε) on velocity field. It is noticed from the figure that velocity increases with the increase of both viscosity variation parameter and thermal conductivity parameter. The influence of permeability parameter (k) over velocity profiles has been showed in the figure (5). It is noticed that permeability parameter increases, the velocity increases. This is due to the fact that the presence of porous medium increases the resistance of flow. The effect of several parameters like as Prandtl number, thermal conductivity, Eckert number and heat generation on temperature profiles are presented in the figure (6) to (9). Figure (6) exhibit the effect of Prandtl number (P_r) over temperature profiles. It is found from the graph that, increasing Prandtl number decelerates the temperature throughout the boundary layer regime. In figure (7), (8) and (9) depicts the temperature distribution for thermal conductivity parameter, Eckert number and heat absorption parameter.

It is clear from the figure that temperature increases for the increase of thermal conductivity parameter, Eckert number and heat absorption parameter. It is marked from the figure (10) that concentration decreases with increasing the Schmidt number. The skin friction and Nusselt number profiles are illustrated in the figure (11) to (14). From the figure, it is obvious that skin friction increases for the increasing of viscosity variation parameter (γ). On the other hand skin friction decreases for the increasing of magnetic parameter (M). In case of figure (13) and (14), Nusselt number profiles decreases for the increase of Eckert number (E_c) and thermal conductivity parameter (ε).

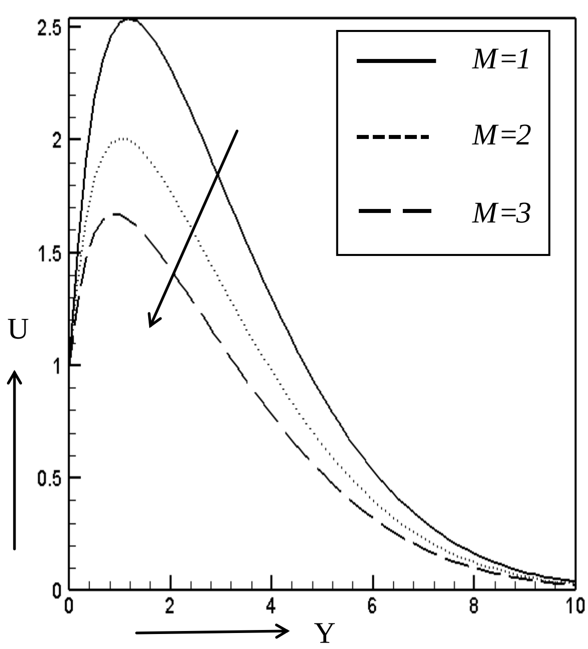

Figure 1: Velocity profile for magnetic parameter M against Y when $G_r=10$, $G_m=5$, $k=0.5$, $S_c=0.60$, $P_r=0.71$, $\varepsilon=0.5$, $\gamma=0.5$, $E_c=0.01$ and $Q=0.05$.

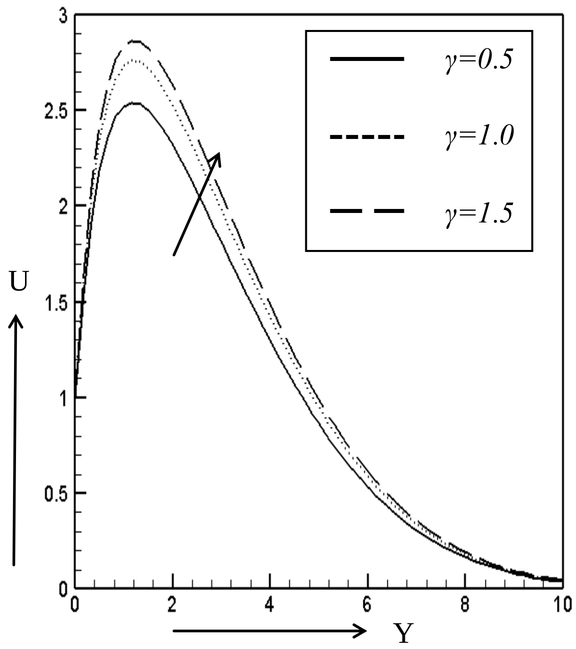

Figure 3: Velocity profile for viscosity variation parameter γ against Y when $G_r=10$, $G_m=5$, $k=0.5$, $S_c=0.60$, $P_r=0.71$, $\varepsilon=0.5$, $M=1$, $E_c=0.01$ and $Q=0.05$.

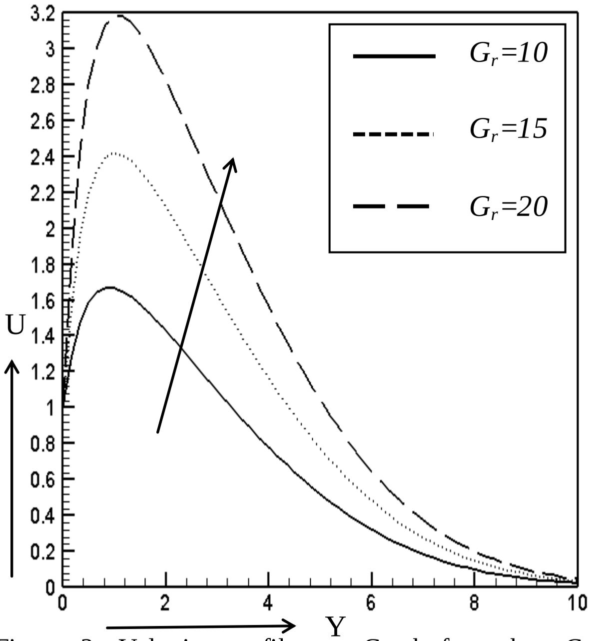

Figure 2: Velocity profile Grashof number G_r against Y when $M=1$, $G_m=5$, $k=0.5$, $S_c=0.60$, $P_r=0.71$, $\varepsilon=0.5$, $\gamma=0.5$, $E_c=0.01$ and $Q=0.05$.

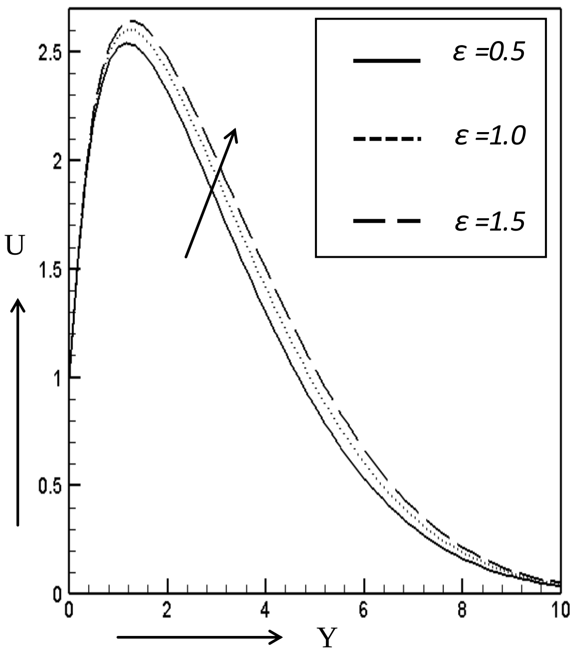

Figure 4: Velocity profile for thermal conductivity parameter ε against Y when $G_r=10$, $G_m=5$, $k=0.5$, $S_c=0.60$, $P_r=0.71$, $M=1$, $\gamma=0.5$, $E_c=0.01$ and $Q=0.05$.

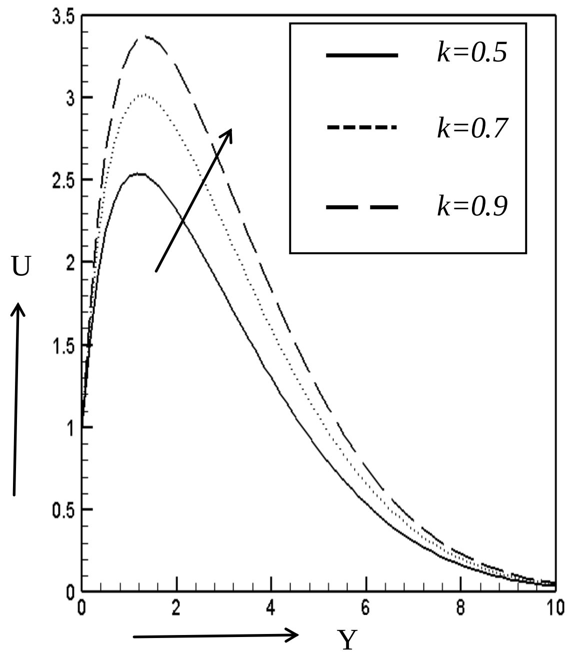

Figure 5: Velocity profile for permeability parameter k against YwhenGr=10, Gm=5, M=1, Sc=0.60, Pr=0.71, ε=0.5, γ=0.5, Ec=0.01 and Q=0.05.

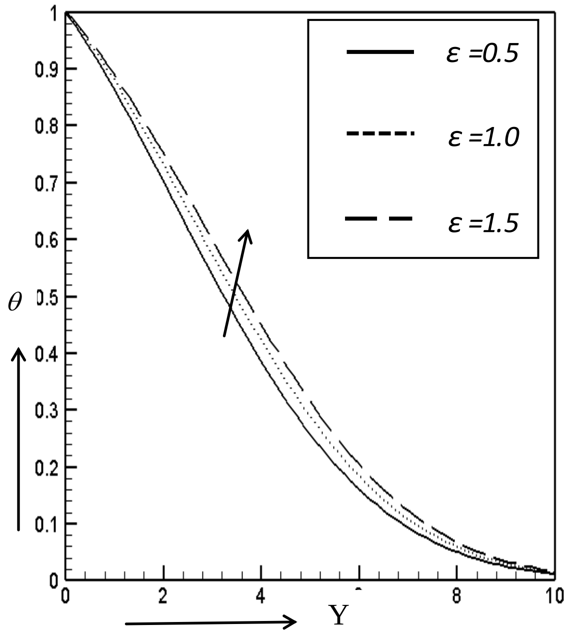

Figure 7: Temperature profile for thermal conductivity parameter ε against YwhenGr=10, Gm=5, k=0.5, Sc=0.60, M=1, Pr=0.5, γ=0.5, Ec=0.01 and Q=0.05.

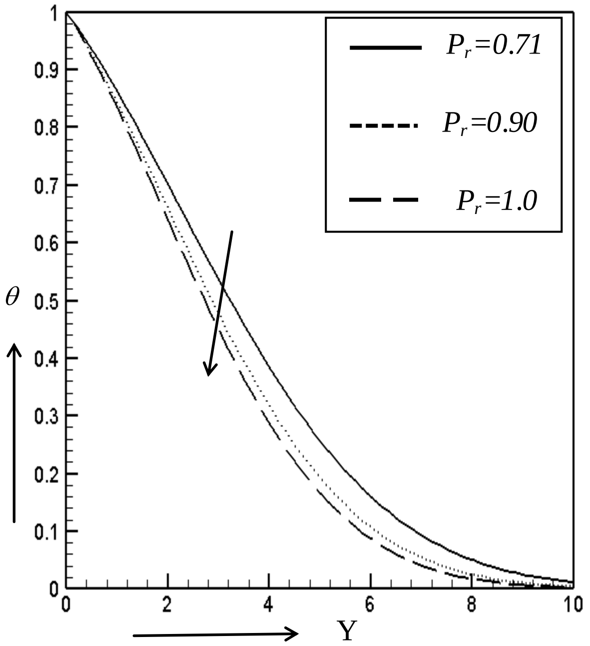

Figure 6: Temperature profile forPrandtl numberPragainst Ywhen Gr=10, Gm=5, k=0.5, Sc=0.60, M=1, ε=0.5, γ=0.5, Ec=0.01 and Q=0.05.

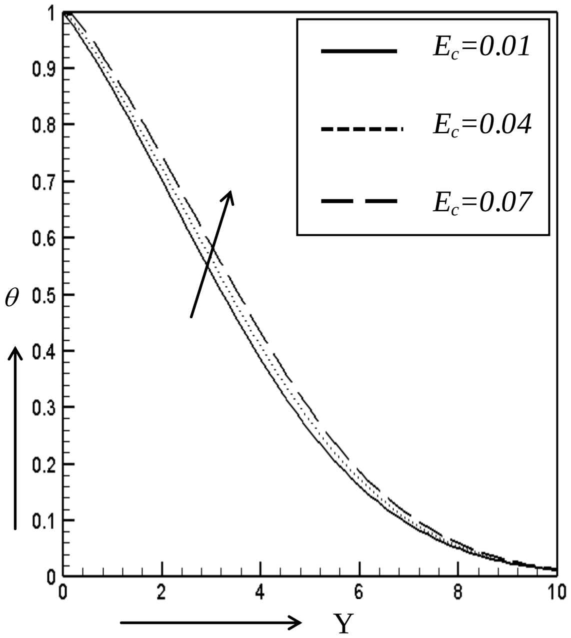

Figure 8: Temperature profile for Eckert number Ecagainst YwhenGr=10, Gm=5, k=0.5, Sc=0.60, Pr=0.71, ε=0.5, γ=0.5, M=1 and Q=0.05.

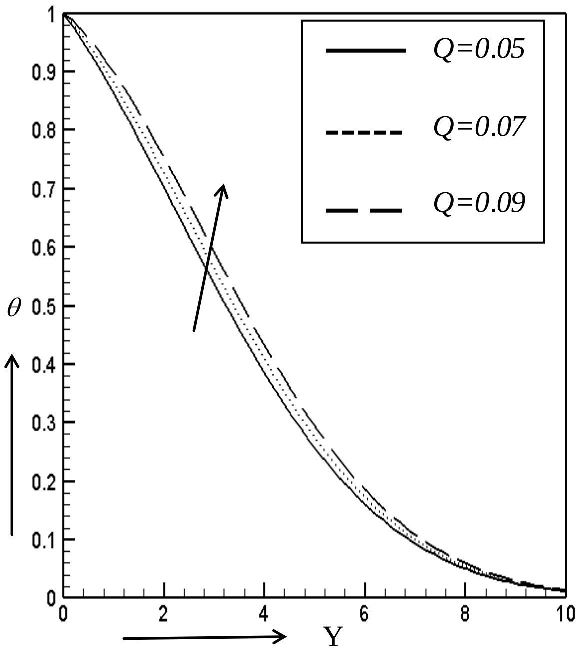

Figure 9: Temperature profile for heat absorption parameter Qagainst YwhenGr=10, Gm=5, k=0.5, Sc=0.60, Pr=0.71, ε=0.5, γ=0.5, Ec=0.01 and M=1.

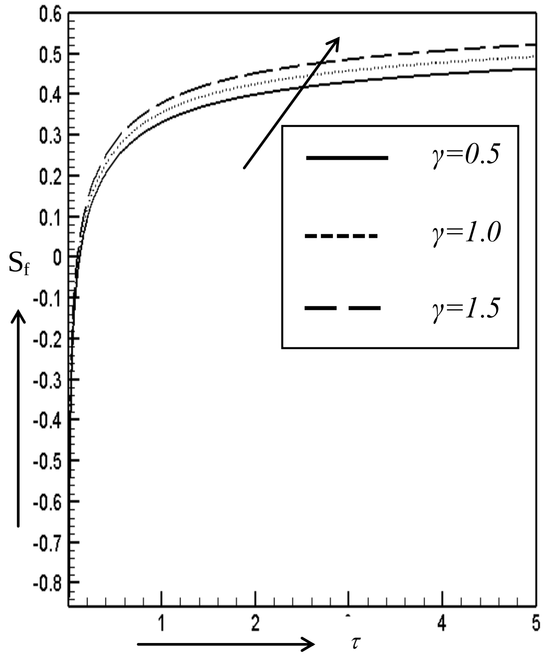

Figure 11: Skin friction profile for thermal conductivity parameterγagainst τ whenGr=10, Gm=5, k=0.5, Sc=0.60, Pr=0.71, ε=0.5, M=1, Ec=0.01 and Q=0.05.

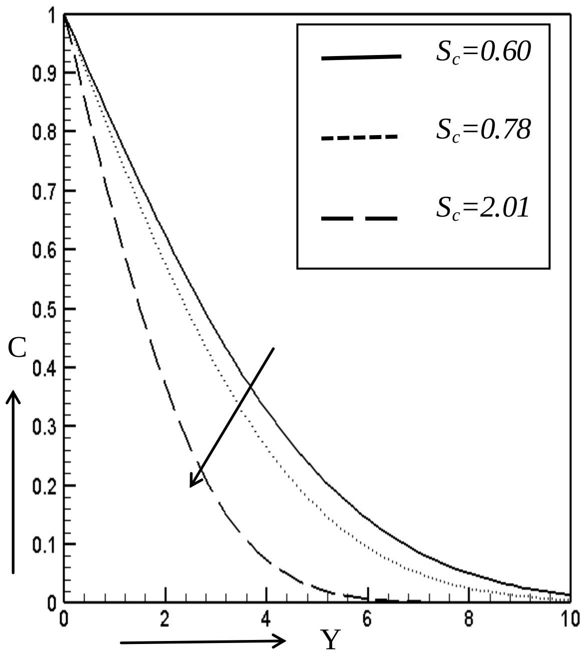

Figure 10:Concentration profile forSchimdt numberScagainst Ywhen Gr=10, Gm=5, k=0.5, M=1, Pr=0.71, ε=0.5, γ=0.5, Ec=0.01 and Q=0.05.

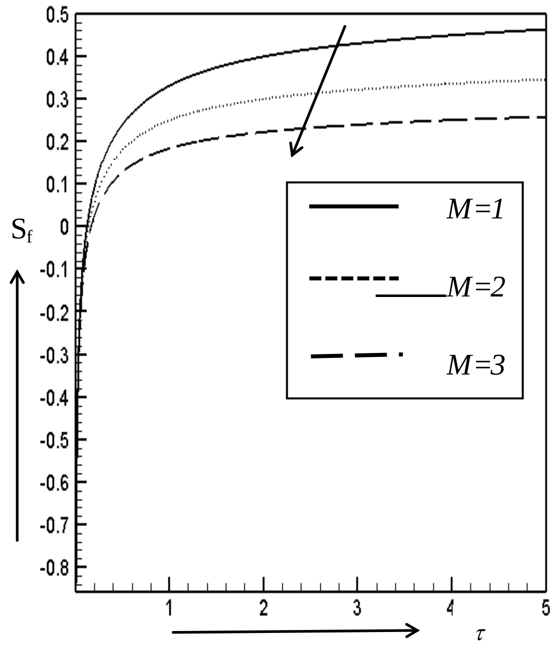

Figure 12: Skin friction profile for magnetic parameterMagainst τ when Gr=10, Gm=5, k=0.5, Sc=0.60, Pr=0.71, ε=0.5, γ=0.5, Ec=0.01 and Q=0.05.

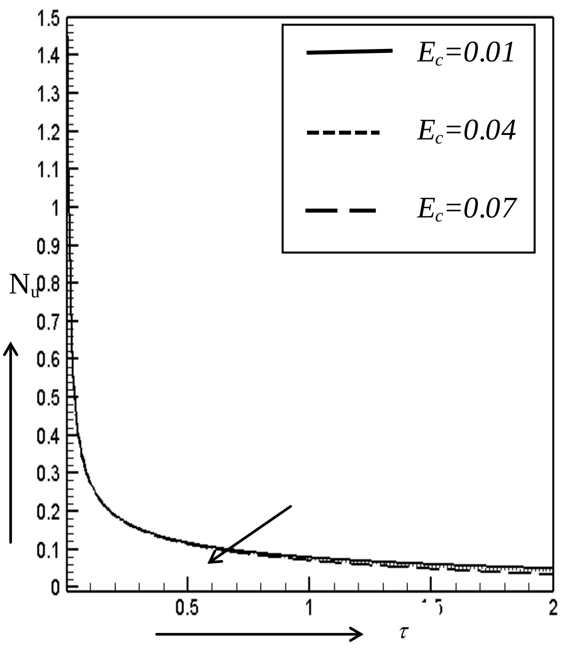

Figure 13: Nusselt number profile for Eckert number E_c against τ when $G_r=10$, $G_m=5$, $k=0.5$, $S_c=0.60$, $P_r=0.71$, $\varepsilon=0.5$, $\gamma=0.5$, $M=1$ and $Q=0.05$.

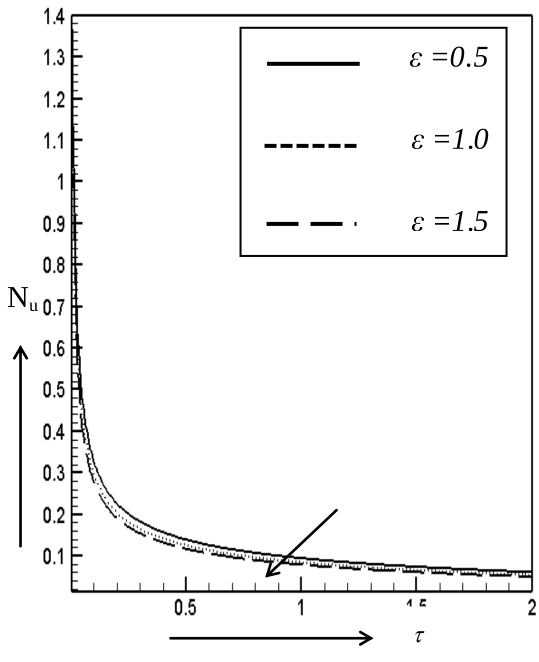

Figure 14: Nusselt number profile for thermal conductivity parameter ε against τ when $G_r=10$, $G_m=5$, $k=0.5$, $S_c=0.60$, $M=1$, $P_r=0.71$, $\gamma=0.5$, $E_c=0.01$ and $Q=0.05$.

V. Conclusion

From the above discussion it can be concluded that, velocity profiles increases with the increasing value of viscosity variation parameter (γ), thermal Grashof number (G_r), permeability parameter (k), and thermal conductivity parameter (ε). On the other hand velocity profiles decreases for the increasing value of magnetic parameter (M). Temperature profile increases for the increasing of thermal conductivity parameter (ε), Eckert number (E_c) and heat generation parameter (Q). In addition to temperature decreases for the increasing value of Prandtl number (P_r). Concentration decreases for the increasing value of Schmidt number (S_c). Skin friction increases with the increasing of viscosity variation parameter (γ) and decreases for the increase of magnetic parameter (M). Nusselt number decreases for the increasing of thermal conductivity parameter (ε) and Eckert number (E_c).

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