IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 1 Ver. V (Jan. - Feb. 2016), PP 24-31
www.iosrjournals.org

Isomorphism On Interval-Valued Fuzzy Graphs

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Abstract: *The present paper is focusing on the order, size and degree of the nodes of the interval-valued fuzzy graphs, which are isomorphic. Isomorphism between Interval-valued fuzzy graphs is proved to be an equivalence relation, whereas the Weak isomorphism is proved to be a partial order. Few properties of interval valued fuzzy graphs, which are self complementary, are discussed.*

Key Words: *'Interval-valued fuzzy graph', 'Isomorphism', 'weak isomorphism', 'co-weak isomorphism', 'Complement'.*

I. Introduction

The notion of ''Interval-valued fuzzy sets'' was introduced by 'Zadeh' [27] In 1975,as an extension of 'fuzzy sets' [26] in which the values of the membership degrees are instead of the numbers, the intervals of numbers. The 'Interval-valued fuzzy sets' supply more satisfactory description of ambiguity than traditional fuzzy sets. Hence, the applications of 'Interval-valued fuzzy sets' are very much important to use in fuzzy control. The fuzzy control is one of the computationally most exhaustive part of defuzzification [15]. As 'Interval-valued fuzzy sets' are widely studied and used, we describe briefly the work on approximate reasoning by 'Gorzalczany' [10, 11], medical diagnosis by 'Roy' and 'Biswas' [22], multi valued logic by 'Turksen' [25] and intelligent control by 'Mendel' [15].

'Rosenfeld' [23] introduced the generalization of 'fuzzy graph theory' as 'Euler's graph theory'. 'Rosenfeld' considered the fuzzy relations between fuzzy sets and improved the structure of 'fuzzy graphs' by obtaining analogs of several graph theoretical concepts. Later, some remarks on fuzzy graphs were given by 'Bhattacharya' [5], and 'Mordeson' and 'Peng' [19] introduced some operations on fuzzy graphs . He has defined the complement of a fuzzy graph and further studied by 'Sunitha' and 'Vijayakumar' [24]. the concept of M-strong fuzzy graphs was introduced by 'Bhutani' [7] and 'Rosenfeld'[8]. The concept of strong arcs in fuzzy graphs was discussed in. The definition of interval-valued graph was given by 'Hongmei' and 'Lianhua' [12].

In this present paper, We study 'isomorphism' between the two interval-valued fuzzy graphs is an equivalence relation and 'weak isomorphism' between them is a partial order. We introduce the notion of interval-valued fuzzy complete graphs and present some properties of self complementary and self weak complementary interval-valued fuzzy complete graphs.
The definitions and terminologies that we used in this paper are standard.

II. Preliminaries

" graph is an ordered pair 'G* = (V, E)', where 'V' is the set of vertices of 'G*' and 'E' is the set of edges of 'G*'. Two vertices 'x' and 'y' in a graph 'G*' are said to be adjacent in G* if {xy} is in an edge of 'G*'. For simplicity an edge {xy} will be denoted by xy."

"S***imple graph*** is a graph without loops and multiple edges."

" C***omplete graph*** is a simple graph in which every pair of distinct vertices is connected by an edge".

The complete graph on n vertices and n(n – 1)/2 edges. We will consider only graphs with the finite number of vertices and edges.

" ***Complementary graph*** $\overline{G\ *}$ of a simple graph G* we mean a graph having the same vertices as G* and such that two vertices are adjacent in $\overline{G\ *}$ if and only if they are not adjacent in G*."

' I**somorphism** of graphs G_1^* and G_2^* is a bijection between the vertex sets of G_1^* and G_2^* such that any two vertices v_1 and v_2 of G_1 are adjacent in G_1 if and only if $f(v_1)$ and $f(v_2)$ are adjacent in G_2. Isomorphic graphs are denoted by $G_1^* \cong G_2^*$."

III. Homomorphism & Isomorphism- Basic Properties

Definition 3.1 : [7] : "The interval-valued fuzzy set A in V is defined by

$$A = \{(x, [\mu_A^-(x), \mu_A^+(x)]) : x \in V\},$$

where $\mu_A^-(x)$ and $\mu_A^+(x)$ are fuzzy subsets of V such that $\mu_A^-(x) \le \mu_A^+(x)$ for all $x \in V$." For any two interval-valued sets $A = [\mu_A^-(x), \mu_A^+(x)]$ and $B = [\mu_B^-(x), \mu_B^+(x)]$ in V we define:

- $A \cup B = \{(x, \max(\mu_A^-(x), \mu_B^-(x)), \max(\mu_A^+(x)\mu_B^+(x))) : x \in V\}$,
- $A \cap B = \{(x, \min(\mu_A^-(x), \mu_B^-(x)), \min(\mu_A^+(x)\mu_B^+(x))) : x \in V\}$.

"***Interval-valued fuzzy relation*** B on a set E of the graph G* = (V, E) , is such that

$$\mu_B^-(xy) \le \min(\mu_A^-(x), \mu_A^-(y))$$

$$\mu_B^+(xy) \le \min(\mu_A^+(x), \mu_A^+(y)) \quad \text{for all } xy \in E \text{ ."}$$

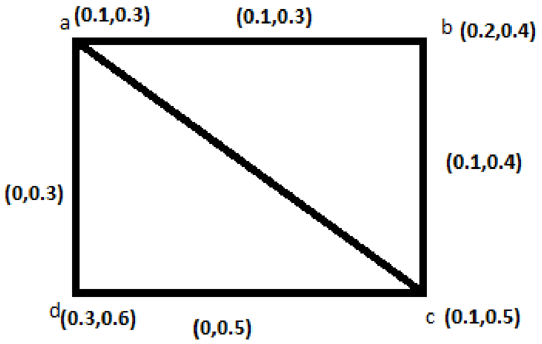
"***A strong interval valued fuzzy graph*** **o**f a graph G* =(V,E) is such that $\mu_B^-(xy) = \min(\mu_A^-(x), \mu_A^-(y))$, $\mu_B^+(xy) = \min(\mu_A^+(x), \mu_A^+(y))$"

Example 3.2: Let G : (V,E) be an Interval-valued fuzzy graph with the underlying set S={a,b,c,d}, such that

$\mu_{A^-(a)} = 0.1, \mu_{A^+(a)} = 0.3, \mu_{A^-(b)} = 0.2, \mu_{A+(b)} = 0.4, \mu_{A^-(c)} = 0.1, \mu_{A^+(c)} = 0.5, \mu_{A^-(d)} = 0.3, \mu_{A^+(d)} = 0.6$

$\mu_{B^-(a,b)} = 0.1, \mu_{B^+(a,b)} = 0.3, \mu_{B^-(b,c)} = 0.1, \mu_{B^+(b,c)} = 0.4, \mu_{B^-(c,d)} = 0, \mu_{B^+(c,d)} = 0.5,$

$\mu_{B^-(d,a)} = 0, \mu_{B+(d,a)} = 0.3$



Definition 3.2 :[7] : "For a given Interval-valued fuzzy graph G (V,E) with the underlying set S, the order of G is defined and denoted by O(G)= $\sum_{x \in V} \frac{1 + \mu_{A^-(x)} + \mu_{A^+(x)}}{2}$ and

the size of G is denoted and defined as S(G) = $\sum_{x,y \in V} \frac{1 + \mu_{B^-(x,y)} + \mu_{B^+(x,y)}}{2}$"

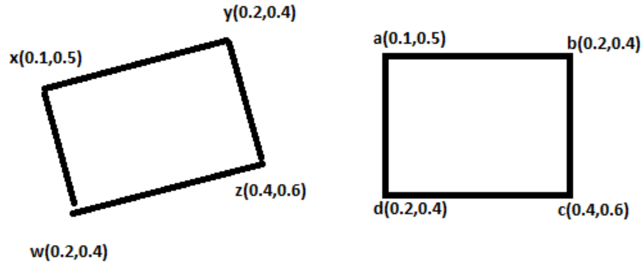
Definition 3.3.[16] : "A homomorphism of fuzzy graphs h : G→ G' is a map h: S→ S^1 which satisfies $\sigma(x) \le \sigma'(h(x)) \forall x \in S$ and $\mu(x, y) \le \mu'(h(x), h(y)) \forall x, y \in S$"

Definition 3.4. [16] : "A weak Isomorphism h:G→ G' is a map h: S→ S^1 which is a bijective homomorphism that satisfies $\sigma(x) = \sigma'(h(x)) \forall x \in S$"

Definition 3.5 : [16] : "A homomorphism h: G→ G' is a map h: S→ S^1 between two interval-valued fuzzy graphs, which satisfies $\mu_{A^-(x)} \le \mu_{A^-(h(x))} \forall x \in S$ and $\mu_{B^-(xy)} \le \mu_{B^-(h(xy))}, \mu_{B^+(xy)} \le \mu_{B^+h((xy))} \forall x, y \in S$"

Definition 3.6:[16] : "A weak Isomorphism h: G→ G' is a map h: S→ S^1, which is bijective homomorphism that satisfies, $\mu_{A^-(x)} = \mu_{A^-(h(x))} \forall x \in S$, $\mu_{A+(x)} = \mu_{A+(h(x))} \forall x \in S$"

Example : 3.7:



Definition :3.8[16]: "The Co-Weak Isomorphism between the two interval-valued fuzzy graphs h : G→ G' is a map h: S→ S^1, which is a bijective homomorphism that satisfies

$$\mu_{B^-(xy)} = \mu_{B^-(h(xy))}, \mu_{B^+(xy)} = \mu_{B^+h((xy))} \forall x, y \in S"$$

Definition 3.9[16] : "The Isomorphism between the two interval-valued fuzzy graphs h : G→ G' is a map h: S → S^1, which is a bijective homomorphism that satisfies

$$\mu_{A^-(x)} = \mu_{A^-(h(x))} \forall x \in S, \quad \mu_{A+(x)} = \mu_{A+(h(x))} \forall x \in S \text{ and}$$

$$\mu_{B^-(xy)} = \mu_{B^-(h(xy))}, \mu_{B^+(xy)} = \mu_{B^+h((xy))} \forall x, y \in S"$$

Definition :3.10[4] : "The complement of an interval-valued fuzzy graph G=(A,B) of G^*=(V,E) is an interval-valued fuzzy graph

$\overline{G} = (\overline{A}, \overline{B})$ on G^*, where $\overline{A} = A = [\mu_A^-, \mu_A^+]$ and $\overline{B} = [\overline{\mu_B^-}, \overline{\mu_B^+}]$ is defined by

$$\overline{\mu_B^-}(xy) = \begin{cases} 0, \text{if } \mu_B^-(xy) > 0. \\ \min(\mu_A^-(x), \mu_A^-(y), \text{if } \mu_B^-(xy) = 0 \end{cases}$$

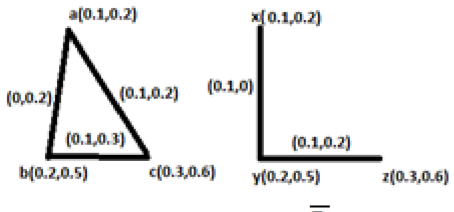
$$\mu_B^+(xy) = \begin{cases} 0, \text{if } \mu_B^+(xy) > 0 \\ \min(\mu_A^+(x), \mu_A^+(y), \text{if } \mu_B^+(xy) = 0 \end{cases}$$

OR

$$\overline{\mu_B^-}(xy) = \min(\mu_A^-(x), \mu_A^-(y)) - \mu_B^-(xy))$$

$$\overline{\mu_B^+}(xy) = \min \ (\mu^+(x), \mu_A^+(y)) - \mu_B^+(xy))"$$

Example 3.11 :



G $\qquad$ $\overline{G}$

Definition 3.12 [4] "An interval valued fuzzy graph is self complementary, if $\overline{\overline{G}} = G"$

Example 3.13: Consider a graph G^* = (V,E) such that V=$\{a, b, c\}$, E=$\{ab, bc\}$, then an interval valued fuzzy graph G = (A,B), where A={a(0.1,0.3),b(0.2,0.4),c(0.3,0.5)} and B={ab(0.1,0.3),bc(0.2,0.4)} is self complementary.

Solution: $\overline{\mu_B^-}(ab) = 0, \overline{\mu_B^+}(ab) = 0, \overline{\mu_B^-}(bc) = 0, \overline{\mu_B^+}(bc) = 0$ (by definition)

$$\overline{\overline{\mu_B^-}}(ab) = 0.1 = \mu_B^-(ab), \overline{\overline{\mu_B^+}}(ab) = 0.3 = \mu_B^+(ab),$$

$$\overline{\overline{\mu_B^-}}(bc) = 0.2 = \mu_B^-(bc), \overline{\overline{\mu_B^+}}(bc) = 0.4 = \mu_B^+(bc)$$

Theorem 3.14: For any two Isomorphic Interval-valued fuzzy graphs, their order and size are the same.

***Proof* :** If h: G→ G' is a map h: S→ S^1 is an isomorphism between two interval-valued fuzzy graphs with the underlying sets S and S^1 respectively., then

$\mu_{A^-(x)} = \mu_{A^-(h(x))} \forall x \in S$, $\mu_{A+(x)} = \mu_{A+(h(x))} \forall x \in S$ and

$\mu_{B^-(xy)} = \mu_{B^-(h(xy))}, \mu_{B^+(xy)} = \mu_{B^+h((xy))} \forall x, y \in S$

Order of G =O(G)= $\sum_{x\in V} \frac{1+\mu_{A^-(x)}+\mu_{A^+(x)}}{2} = \sum_{x\in V} \frac{1+\mu_{A^-(h(x))}+\mu_{A^+(h(x))}}{2}$ =O(G')

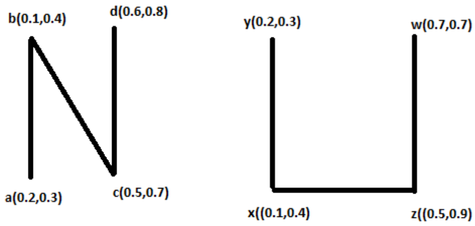
the size of G is denoted and defined as S(G) = $\sum_{x,y\in V} \frac{1+\mu_{B^-(x,y)}+\mu_{B^+(x,y)}}{2}$

$= \sum_{x,y\in V} \frac{1+\mu_{B^-(h(x,y))}+\mu_{B^+(h(x,y))}}{2}$

=S(G')

Corollary(3.15) : The converse of the above theorem need not be true

***Example (3.16)* :**Consider the Interval-valued fuzzy graphs G and G' with underlying sets S and S',



where S={a,b,c,d} and S'={x,y,z,w} such that

$\mu_{A^-(a)} = 0.2$, $\mu_{A^+(a)}$ =0.3 $\mu_{A^-(b)} = 0.1$, $\mu_{A^+(b)} = 0.4$, $\mu_{A^-(c)} = 0.5$, $\mu_{A^+(c)} = 0.7$,

$\mu_{A^-(d)} = 0.6$, $\mu_{A^+(d)} = 0.8$,

Here O(G) = O(G')=3.8

$\mu_{B^-(ab)}$ =0.1, $\mu_{B^+(ab)} = 0.3$, $\mu_{B^-(bc)} = 0.1$, $\mu_{B^+(bc)} = 0.4$, $\mu_{B^-(cd)} = 0.5$, $\mu_{B^+(cd)} = 0.7$

$\mu_{B^-(xy)} = 0.1$, $\mu_{B^+(xy)} = 0.3$, $\mu_{B^-(xz)} = 0.1$, $\mu_{B^+(xz)} = 0.4$, $\mu_{B^-(zw)} = 0.5$, $\mu_{B^+(zw)} = 0.7$

Here S(G)=S(G')=2.55

But G is not isomorphic to G'

***Theorem 3.17*:** Isomorphism between two Interval-valued fuzzy graphs is an Equivalence relation.

***Proof*:** Let G: (A,B) , G':(A_1,B_1) , G'' : (A_2,B_2) be the two Interval valued fuzzy graphs with underlying sets S, S' , S'' respectively.

Reflexive : Consider the identity map h: S→ S such that h(x)=x $\forall x \in S$, which is bijective map satisfying

$\mu_{A^-(x)} = \mu_{A^-(h(x))} \forall x \in S$, $\mu_{A+(x)} = \mu_{A+(h(x))} \forall x \in S$ ....(1)

and $\mu_{B^-(x,y)} = \mu_{B^-(h((x),h(y))} \forall x, y \in S \because h(x) = x \forall x \in S$ ......By(1)

Hence the Isomorphism is Reflexive.

***Symmetric* :**Let h: S→ S^1 be an isomorphism of G onto G', i.e $G \cong G'$

then 'h' is a bijective map defined by $h(x) = x' \quad \forall x \in S$, which satisfy

$\mu_{A^-(x)} = \mu_{A_1^-(h(x))}$, $\mu_{A^+(x)} = \mu_{A_1^+(h(x))} \quad \forall x \in S$,

$\mu_{B^-(x,y)} = \mu_{B_1^-(h((x),h(y))}$ $\quad \mu_{B^+(x,y)} = \mu_{B_1^+(h((x),h(y))}$ $\quad \forall x, y \in S$, As 'h' is bijective $\exists h^{-1}: S' \to S$ such that $h^{-1}(x') = x \forall x' \in S'$

Then, $\mu_{A_1^-(h^{-1}(x'))} = \mu_{A^-(x)}$, $\mu_{A_1^+(h^{-1}(x'))} = \mu_{A^+(x)}$

similarly, $\mu_{B_1^-((h^{-1}(x'),h^{-1}(y'))} = \mu_{B^-(x,y)}$, $\mu_{B_1^+((h^{-1}(x')h^{-1}(y')} = \mu_{B^+(x,y)}$

$$\therefore G' \cong G$$

Hence the Isomorphism is Symmetric.

Transitivity **:** Let h: $S \to S'$ and g: $S' \to S''$ be the isomorphisms from the Interval-valued fuzzy graphs G onto G' and then from G' to G'' respectively.

Then the composition of the mapping "goh" is 1-1 and onto from $S \to S''$ such that goh(x)=g(h(x)) $\forall x \in S$,

h: $S \to S'$ is an Isomorphism defined by h(x)= x' $\forall x \in S$, then

$$\left.\begin{array}{l} \mu_{A^-(x)} = \mu_{A_1^-(h(x))}, \mu_{A^+(x)} = \mu_{A_1^+(h(x))} \\ \mu_{A^-(y)} = \mu_{A_1^-(h(y))}, \mu_{A^+(y)} = \mu_{A_1^+(h(y))} \\ \mu_{B^-(x,y)} = \mu_{B_1^-(h(x)h(y))} \\ \mu_{B^+(x,y)} = \mu_{B_1^+(h(x)h(y))} \end{array}\right\} \text{.........}(1)$$

Now, g: $S' \to S''$ is an Isomorphism defined by g(x')= x'' $\forall x' \in S'$,

i.e.,

$$\left.\begin{array}{l} \mu_{A_1^-(x')} = \mu_{A_2^-(g(x'))}, \mu_{A_1^+(x')} = \mu_{A_2^+(g(x'))} \\ \mu_{A_1^-(y')} = \mu_{A_2^-(g(y'))}, \mu_{A_1^+(y')} = \mu_{A_2^+(g(y'))} \\ \mu_{B_1^-(x',y')} = \mu_{B_2^-(g(x'),g(y'))} \\ \mu_{B_1^+(x',y')} = \mu_{B_2^+(g(x'),g(y'))} \end{array}\right\} \text{.........}(2)$$

Now form (1) and (2) we get,

$$\mu_{A^-(x)} = \mu_{A_1^-(h(x))} = \mu_{A_1^-(x')} = \mu_{A_2^-(g(x'))} = \mu_{A_2^-(x'')}$$

$$\mu_{A^+(x)} = \mu_{A_1^+(h(x))} = \mu_{A_1^+(x')} = \mu_{A_2^+(g(x'))} = \mu_{A_2^+(x'')}$$

$$\mu_{B^-(x,y)} = \mu_{B_1^-(h(x),h(y))} = \mu_{B_1^-(x',y')} = \mu_{B_2^-(g(x'),g(y'))} = \mu_{B_2^-(x'',y'')}$$

$$\mu_{B^+(x,y)} = \mu_{B_1^+(h(x),h(y))} = \mu_{B_1^+(x',y')} = \mu_{B_2^+(g(x'),g(y'))} = \mu_{B_2^+(x'',y'')}$$

Hence the Isomorphism is transitive.

And hence Isomorphism between the Interval-valued fuzzy graphs is an Equivalence Relation.

Theorem (3.18): The weak Isomorphism between two interval valued fuzzy graphs is a Partial Order.

Proof: Reflexive : Consider the identity map h: S→ S such that h(x)=x $\forall x \in S$, which is bijective map satisfying $\mu_{A^-(x)} = \mu_{A^-(h(x))} \forall x \in S$, $\mu_{A+(x)} = \mu_{A+(h(x))} \forall x \in S$ ……..(1)

and $\mu_{B^-(x,y)} = \mu_{B^-(h((x),h(y))} \forall x, y \in S \because h(x) = x \forall x \in S$ ……By(1)

Hence the weak Isomorphism is Reflexive.

Anti Symmetry : Assume that the two interval-valued fuzzy graphs G and G' are such that G $\cong G'$ and $G' \cong G$.

Consider the weak Isomorphism h: S→ S^1 , which is a bijective map such that

h(x)= x' $\forall x \in S$ satisfying the condition

$$\left.\begin{array}{l}\mu_{A^-(x)}=\mu_{A_1^-(h(x))}=\mu_{A_1^-(x')},\mu_{A^+(x)}=\mu_{A_1^+(h(x))}=\mu_{A_1^+(x')}\\ \mu_{B^-(x,y)}\le\mu_{B_1^-(h(x)h(y))}=\mu_{B_1^-(x',y')}\Rightarrow\mu_{B^-(x,y)}\le\mu_{B_1^-(x',y')}\\ \mu_{B^+(x,y)}\le\mu_{B_1^+(h(x)h(y))}=\mu_{B_1^+(x',y')}\Rightarrow\mu_{B^+(x,y)}\le\mu_{B_1^+(x',y')}\end{array}\right\}\text{.........}(2)$$

Similarly, Consider the weak isomorphism g: $S^1 \to S$, which is bijective map such that g(x')= x $\forall x' \in S'$

satisfying the condition $$\left.\begin{array}{l}\mu_{A_1^-(x')}=\mu_{A^-(g(x'))}=\mu_{A^-(x)},\mu_{A_1^+(x')}=\mu_{A^+(g(x'))}=\mu_{A^+(x)}\\ \mu_{B_1^-(x',y')}\le\mu_{B_1^-(g(x')g(y'))}=\mu_{B^-(x,y)}\Rightarrow\mu_{B_1^-(x',y')}\le\mu_{B^-(x,y)}\\ \mu_{B_1^+(x',y')}\le\mu_{B_1^+(g(x')g(y'))}=\mu_{B^+(x,y)}\Rightarrow\mu_{B_1^+(x',y')}\le\mu_{B^+(x,y)}\end{array}\right\}\text{.........}(3)$$

From (2) and (3) it's clear that If G $\cong G'$ and $G' \cong G$ then G= G'

Hence the Weak isomorphism is anti symmetric

Transitivity **:** Assume that G $\cong G'$ and $G' \cong G''$

Let h: $S \to S'$ is a weak Isomorphism defined by h(x)= x' $\forall x \in S$, and

g: $S' \to S''$ is a weak Isomorphism defined by g(x')= x'' $\forall x' \in S'$,

then

$$\left.\begin{array}{l}\mu_{A^-(x)}=\mu_{A_1^-(h(x))},\mu_{A^+(x)}=\mu_{A_1^+(h(x))}\\ \mu_{A^-(y)}=\mu_{A_1^-(h(y))},\mu_{A^+(y)}=\mu_{A_1^+(h(y))}\\ \mu_{B^-(x,y)}\le\mu_{B_1^-(h(x)h(y))}\\ \mu_{B^+(x,y)}\le\mu_{B_1^+(h(x)h(y))}\end{array}\right\}\text{.........}(1)$$

Now,

$$\left.\begin{array}{l}\mu_{A_1^-(x')}=\mu_{A_2^-(g(x'))},\mu_{A_1^+(x')}=\mu_{A_2^+(g(x'))}\\ \mu_{A_1^-(y')}=\mu_{A_2^-(g(y'))},\mu_{A_1^+(y')}=\mu_{A_2^+(g(y'))}\\ \mu_{B_1^-(x',y')}\le\mu_{B_2^-(g(x'),g(y'))}\\ \mu_{B_1^+(x',y')}\le\mu_{B_2^+(g(x'),g(y'))}\end{array}\right\}\text{.........}(2)$$

Now form (1) and (2) we get,

$$\mu_{A^-(x)}=\mu_{A_1^-(h(x))}=\mu_{A_1^-(x')}=\mu_{A_2^-(g(x'))}=\mu_{A_2^-(x'')}$$

$$\mu_{A^+(x)}=\mu_{A_1^+(h(x))}=\mu_{A_1^+(x')}=\mu_{A_2^+(g(x'))}=\mu_{A_2^+(x'')}$$

$$\mu_{B^-(x,y)}\le\mu_{B_1^-(h(x),h(y))}=\mu_{B_1^-(x',y')}=\mu_{B_2^-(g(x'),g(y'))}=\mu_{B_2^-(x'',y'')}$$

$$\therefore\ \mu_{B^-(x,y)}\le\mu_{B_2^-(x'',y'')}$$

$$\mu_{B^+(x,y)}=\mu_{B_1^+(h(x),h(y))}=\mu_{B_1^+(x',y')}=\mu_{B_2^+(g(x'),g(y'))}=\mu_{B_2^+(x'',y'')}$$

$$\therefore\ \mu_{B^+(x,y)}\le\mu_{B_2^+(x'',y'')}$$

Hence there is a weak Isomorphism from G to G''

And hence the weak isomorphism is transitive.

Hence the weak isomorphism of the interval-valued fuzzy graphs is a partial order.

3.19 Theorem **:** Two Interval-valued fuzzy graphs are isomorphic iff their complements are isomorphic.

Proof : Let the two interval-valued fuzzy graphs **G=**(A,B) and G'=(A_1,B_1) and assume that $G \cong G'$

i.e h: $S \to S'$ is an Isomorphism ,defined by h(x)= x' $\forall x \in S$,

i.e., 'h' is a bijective from S to S' and satisfies the following

$$\left.\begin{array}{l} \mu_{A^-(x)} = \mu_{A_1^-(h(x))} = \mu_{A_1^-(x')}, \mu_{A^+(x)} = \mu_{A_1^+(h(x))} = \mu_{A_1^+(x')} \\ \mu_{B^-(x,y)} = \mu_{B_1^-(h(x)h(y))} = \mu_{B_1^-(x',y')} \\ \mu_{B^+(x,y)} = \mu_{B_1^+(h(x)h(y))} = \mu_{B_1^+(x',y')} \end{array}\right\}$$

Now, to show that $\overline{G} \cong \overline{G'}$, we have $\overline{A} = A$ therefore h is a bijection from $\overline{S}$ to $\overline{S'}$

And $\overline{\mu}_{B^-(x,y)} = \min[\mu_{A^-(x)}, \mu_{A^-(y)}] - \mu_{B^-(x,y)}$ =min[$\mu_{A_1^-(h(x))}, \mu_{A_1^-(h(y))}$]- $\mu_{B_1^-(h(x)h(y))}$

$= \overline{\mu}_{B_1^-(h(x)h(y))}$ $(\because G \cong G')$

Also $\overline{\mu}_{B^+(x,y)}$ = min [$\mu_{A^+(x)}, \mu_{A^+(y)}$] - $\mu_{B^+(x,y)}$ =min[$\mu_{A_1^+(h(x))}, \mu_{A_1^+(h(y))}$] $-$ $\mu_{B_1^+(h(x)h(y))}$

$= \overline{\mu}_{B_1^+(h(x)h(y))}$ $(\because G \cong G')$

Conversely assume that $\overline{G} \cong \overline{G'}$

we have $\overline{A} = A$ therefore h is a bijection from $\overline{S}$ to $\overline{S'}$ and

$\overline{\mu}_{A^-(x)} = \overline{\mu}_{A_1^-(h(x))} = \overline{\mu}_{A_1^-(x')}, \overline{\mu}_{A^+(x)} = \overline{\mu}_{A_1^+(h(x))} = \overline{\mu}_{A_1^+(x')}$ …….(1)

$\overline{\mu}_{B^-(x,y)} = \overline{\mu}_{B_1^-(h(x)h(y))}, \overline{\mu}_{B^+(x,y)} = \overline{\mu}_{B_1^+(h(x)h(y))}$

To show that $G \cong G'$,

As $\overline{A} = A$, h : $S \to S'$ is a bijection

And $\mu_{B^-(x,y)} = \min[\mu_{A^-(x)}, \mu_{A^-(y)}] - \overline{\mu}_{B^-(x,y)}$ =min[$\overline{\mu}_{A^-(x)}, \overline{\mu}_{A^-(y)}$]- $\overline{\mu}_{B^-(x,y)} = \overline{\mu}_{B_1^-(h(x),h(y))}$

$\mu_{B^+(x,y)} = \min[\mu_{A^+(x)}, \mu_{A^+(y)}] - \overline{\mu}_{B^+(x,y)} = \min[\overline{\mu}_{A^+(x)}, \overline{\mu}_{A^+(y)}] - \overline{\mu}_{B^+(x,y)} = \overline{\mu}_{B_1^+((h(x),h(y))}$

Hence $G \cong G'$

IV. Conclusion

It is well known that 'Interval-valued fuzzy sets' constitute a generalization of the notion of fuzzy sets. The interval-valued fuzzy models give more flexibility and compatibility to the system as compared to the classical and fuzzy models. So, we have introduced interval-valued fuzzy graphs and have presented several properties in this paper. The further study of interval-valued fuzzy graphs may also be extended with the following projects.

- Data base theory
- Expert systems
- Neural Networks
- Shortest paths in networks

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