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Inverse Accurate Independent Domination In Fuzzy Graphs

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Abstract: *In this paper we discuss the concepts of inverse accurate domination and inverse accurate independent domination in fuzzy graph.We determine the inverse accurate domination number γ_{fa}^{-1} (G) and the inverse accurate independent domination number γ_{fai}^{-1} (G) for several classes of fuzzy graph and obtain bounds for the same.We also obtain Nordhaus-Gaddum type result for these parameters.*
Index Terms: *Fuzzy Graph, Fuzzy Independent Dominating set, Fuzzy Accurate Independent Dominating set, Fuzzy Accurate Independent Domination Number. Fuzzy inverse Accurate Independent Dominating set, Fuzzy inverse Accurate Independent Domination Number*.

I. Introduction

The study of domination set in graphs was begun by ore and Berge. A.Somasundram and S.Somasundram discussed domination in Fuzzy graphs .V. R. Kulli and M .B .Kattimani introduced the concept of Accurate domination in graphs. V.R.Kulli and S.C. Sigarkanti discussed the inverse domination in graphs. In this paper, we discuss the inverse accurate domination and inverse accurate independent domination number of fuzzy graphs and establish the relationship with other parameter which is also investigated.

II. Preliminaries

Definition:2.1

A ***fuzzy graph*** G = (σ,μ) is a pair of function $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$, where $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all u ,v ∈ V.

Definition:2.2

The ***order*** *p* and ***size*** *q* of the fuzzy graph G = (σ , μ) are define by $p = \sum_{v \in V} \sigma(v)$ and $q = \sum_{u,v \in E} \mu(u,v)$.

Definition:2.3

The ***complement*** of a fuzzy graph G = (σ , μ) is a fuzzy graph G = (σ,μ) where σ = σ and $\mu^c(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v)$ for all u ,v in V.

Definition:2.4

The **fuzzy cardinality** of a fuzzy subset D of V is $|D|_f = \sum_{v \in D} \sigma(v)$.

Definition:2.5

An edge e = {u , v} of a fuzzy graph is called an **effective edge** if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$.

Definition:2.6

The **effective degree** of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by $d_E(u)$.

Definition:2.7

The Minimum **effective degree** $\delta_E(G) = \min\{d_E(u) / u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{d_E(u) / u \in V(G)\}$.

Definition:2.8

Let V be a point is connected fuzzy graph G.The **diameter** is defined by diam(G)=max{e(v) / v∈V(G)}.

Definition:2.9

In a fuzzy tree,a vertex of degree one is referred to as a **leaf.**

Definition:2.10

The **vertex independence number** $\beta_0(G)$ of G is the maximum cardinality among the independent sets of vertices.

Definition:2.11

The **edge independence number** $\beta_1(G)$ of G is the maximum cardinality among the independent sets of edges.

Definition:2.12

For any graph G is a complete subgraph of G is called a Clique of G. The number of vertices in a largest Clique of G is called the Clique number $\omega(G)$ of G.

Definition:2.13

If $\mu(u, v) = 0$ for every $v \in V$ then u is called **isolated node.** A set $S \subseteq V$ in a fuzzy graph G is said to be **independent** if $\mu(u, v) < \sigma(u) \Lambda \sigma(v)$ for all $u, v \in S$.

Definition:2.14

A dominating set is called an **independent dominating set** if D is independent. An independent dominating set S of a fuzzy graph G is said to be a **maximal independent dominating set** if there is no independent dominating set S^1 of G such that $S^1 \subset S$. An independent dominating set S of a fuzzy graph G is said to be a **maximum independent dominating set** if there is no independent dominating set S^1 of G such that $|S^1| > |S|$.The minimum scalar cardinality of an maximum independent dominating set of G is called the **independent domination number** of G and is denoted by i(G).

Definition:2.15

Let x , y $\in$V. We say that x **dominates** y in G if $\mu(u, v) = \sigma(u) \Lambda \sigma(v)$. A subset S of V is called a **dominating set** in G if for every $v \notin S$, there exists $u \in S$ such that u dominates v. The minimum cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma(G)$.

Definition:2.16

Let $G = (\sigma,\mu)$ be a fuzzy graph. A subset D of V is said to be **fuzzy dominating set** of G if for every $v \in V-D$ there exists $u \in D$ such that (u, v) is a strong arc.

Definition:2.17

A dominating set D of a graph G is called **minimal dominating set** of G if for every node $v \in D$, $D-\{v\}$ is not a dominating set of the domination number $\gamma(G)$ is the minimum cardinalities taken over all minimal dominating sets of G.

Definition:2.18

A dominating set D of a graph G is an **accurate dominating set**, if V−D has no dominating set of cardinality $|D|$. The accurate domination number $\gamma_a(G)$ of G is the minimum cardinality of an accurate dominating set.

Definition:2.19

A dominating set D of a fuzzy graph G is an **fuzzy accurate dominating set**, if V−D has no dominating set of cardinality $|D|$. The fuzzy accurate domination number $\gamma_{fa}(G)$ of G is the minimum cardinality of an accurate dominating set.

Definition:2.20

An independent dominating set D of a graph G is an **accurate independent dominating set** if V−D has no independent dominating set of cardinality $|D|$. The accurate independent domination number $i_a(G)$ of G is the minimum cardinality of an accurate dominating set of G.

Definition:2.21

An independent dominating set D of a fuzzy graph G is an **fuzzy accurate independent dominating set** if V−D has no independent dominating set of cardinality $|D|$. The fuzzy accurate independent domination number $i_{fa}(G)$ of G is the minimum cardinality of an accurate independent dominating set of G.

Definition:2.22

Let S be a minimum dominating set of G. If V-S contains a dominating set S^1 of G, then S^1 is called an **inverse dominating set** with respect to S.The inverse domination number γ^{-1} (G) of G is the minimum cardinality of an inverse dominating set of G.

Definition:2.23

An **wounded spider** is obtained by subdividing atmost n-1 edges of a star $K_{1,n}$ for $n \geq 0$.

III. Prior Results

Theorem:3.1

Let G be a connected fuzzy graph with $P \geq 3$ and $\delta(G) = 1$.Let $L \subseteq V$ be the set of all degree one vertices and S = N(L), then $\gamma_f(G) + \gamma_f(\bar{G}) = P$. iff the following two condition holds

i) V-S is independent.

ii) For every vertex $x \in V-(S \cup L)$, every stem in N(x) is adjacent to atleast two leaves.

Theorem:3.2

If G and $\bar{G}$ have no isolate vertices then $\gamma_f(G) + \gamma_f(\bar{G}) \leq \lfloor P/2 \rfloor + 2$.

Theorem:3.3

For any fuzzy tree T of order P ≥ 2, γ_f^{-1} (T) ≥ $\frac{P+1}{3}$.

Theorem: 3.4

Let x be a dominating vertex of a fuzzy graph G. Then γ_f^{-1} (G) = γ_f (G-x).

Proof:

Since x is a dominating vertex of a fuzzy graph G,{ x} is a γ_f set of G. Hence any γ_f^{-1} set of G lies in G-{x } and is a minimum dominating set of G-{x}.Therefore γ_f^{-1} (G) = γ_f (G-x).

Theorem : 3.5

For any fuzzy tree T, γ_f (T) = P- Δ(G) iff T is a Wounded spider.

Theorem: 3.6

For any fuzzy graph G without isolated vertices then γ_f^{-1} (G) ≤ β_0(G)

Theorem: 3.7

For any connected fuzzy graph G then p-q ≤ γ_f^{-1} (G) ≤ p - $\lceil \Delta\ (G)/2 \rceil$

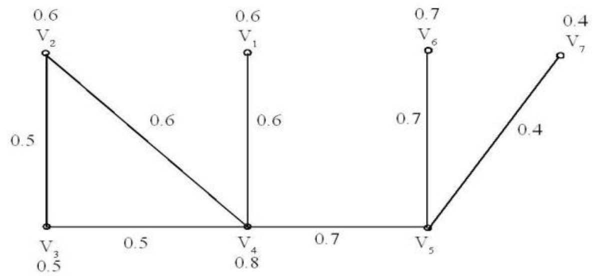
IV. Main Results

4. Inverse Accurate Domination In Fuzzy Graphs

Definition: 4.1

Let G = (V, E) be a fuzzy graph. Let D be a minimum accurate dominating set of G. If V-D contains an accurate dominating set D_1 of G then D_1 is called an inverse accurate dominating set with respect to D. The inverse accurate domination number γ_{fa}^{-1} (G) of G is the minimum cardinality of an inverse accurate dominating set of G.

Example:4.1



Proposition :4.1

If a fuzzy graph G has a γ_f^{-1} set then γfa(G)=1.5 , γfa-1(G)=2.3-------(1) and this bound is sharp.

Proof:

Clearly every inverse accurate dominating set of a fuzzy graph G is a accurate dominating set. Thus (1) holds.

Proposition:4.2

If a fuzzy graph G has a γ_f^{-1} set then γ_{fa} (G) + γ_{fa}^{-1} (G) ≤ p-------(2) and this bound is sharp.

Proof:

Equation (2) follows from the definition of γ_{fa}^{-1} (G).

Proposition:4.3

For any fuzzy graph G, p-q ≤ γ_{fa}^{-1} (G) ≤ q.

Proof:

For any fuzzy graph G, we have p-q ≤ γ_{fa} (G) ≤ γ_{fa}^{-1} (G). Also γ_{fa}^{-1} (G) ≤ p- γ_{fa} (G). Thus γ_{fa}^{-1}(G) ≤ q.

Theorem:4.1

For any connected fuzzy graph G, γ_{fa}^{-1} (G) + diam (G) ≤ p+ γ_{fa} (G)-1.

Proof:

Let V (G) contains atleast two vertices u and v such that dist(u, v) forms a diametral path in G. clearly dist(u,v) = diam(G). Let F_1={v_1,v_2,……..v_k} ⊆ V(G) be the set of vertices which are adjacent to all end vertices in G. Suppose J={u_1,u_2,……u_n} be the set of vertices such that dist(v_i,u_j)≥1,for all 1≤i≤k, 1≤j≤n. Then $F_1 \cup J^1$ where $J^1 \subseteq$ J forms a γ_{fa} set of G. Now in G, suppose D={v_1,v_2,……..v_m} ⊆ $F_1 \cup J^1$ be the γ_{fa} set of G. Then the complementary set V(G)-D contains the vertex set $D^1 \subseteq$ V(G) – D, which covers all the vertices in G. clearly, D^1 forms a inverse accurate dominating set of G and it follow that │ D^1 │+diam(G)≤ p+│ $F_1 \cup J^1$│- 1. Hence γ_{fa}^{-1} (G) + diam(G) ≤ p + γ_{fa} (G)-1.

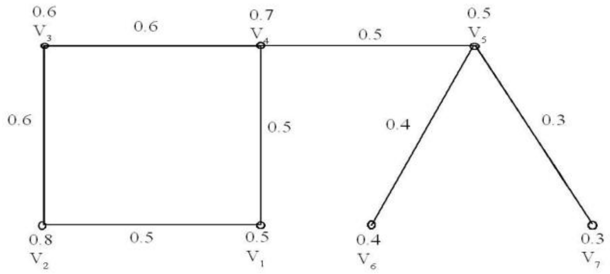
V. Inverse Accurate Independent Domination In Fuzzy Graphs

Definition: 5.1

Let G = (V, E) be a fuzzy graph. Let D be a minimum accurate independent dominating set of G.If V-D contains an accurate independent dominating set D_1 of G then D_1 is called an inverse accurate independent dominating set with respect to D. The inverse accurate independent domination number γ_{fai}^{-1} (G) of G is the minimum cardinality of an inverse accurate independent dominating set of G.

Example:5.1

γfai(G)=1.3 , γfai-1(G)=1.8



Theorem: 5.1

If every non end vertex of a fuzzy tree is adjacent to atleast one end vertex, then $\gamma_{fai}^{-1}(T) \leq \lceil \frac{p-m}{2} \rceil + 1$. Where m is the end vertices in T.

proof:

Let F={$v_1, v_2, \ldots, v_m$ } be the set of all end vertices in fuzzy tree T such that the vertex $|F| = m$ and $F_1 \in N(F)$.Suppose, vertex $D \subseteq F_1$ is a γ_{fai} set of G. Then $D^1 \subseteq V(G) - D$ forms a inverse accurate independent dominating set of G .since every fuzzy tree T contains atleast one non end vertex. it follows that $|D^1| \leq \lceil \frac{p-m}{2} \rceil$ +1.Therefore $\gamma_{fai}^{-1}(T) \leq \lceil \frac{p-m}{2} \rceil + 1$.

Definition:5.2

The upper inverse accurate independent domination number $\Gamma\gamma_{fai}^{-1}(G)$ of a fuzzy graph G is the maximum cardinality of an inverse accurate independent dominating set of G.

Proposition: 5.1

If a fuzzy graph G has a $\gamma_f^{-1}(G)$ set then $\gamma_{fa}(G) \leq i(G) \leq \gamma_{fai}(G) \leq \gamma_{fai}^{-1}(G) \leq \Gamma\gamma_{fai}^{-1}(G)$.

Theorem:5.2

Let G be a connected fuzzy graph with p vertices, q edges and $\gamma_{fai}(G) = \gamma_{fai}^{-1}(G)$.
Then $\gamma_{fai}^{-1}(G) \geq 1/3(2p\text{-}q)$.

proof:

Let D be a γ_{fai}- set and D^1 be a γ_{fai}^{-1} set of G. Every vertex in $V\text{-}(D \cup D^1)$ has atleast one neighbor in D and one neighbor in D^1. Every vertex in D has a neighbor in D^1 ,and every vertex in D^1 has a neighbor in D. Thus the number of edges in G is atleast $2(V\text{-}(D \cup D^1)) + |D^1|$. ie, $m \geq 2(n\text{-} 2\gamma_{fai}^{-1}) + \gamma_{fai}^{-1}$ and so $m \geq 2p\text{-}3\gamma_{fai}^{-1}(G)$. Therefore $\gamma_{fai}^{-1}(G) \geq 1/3(2p\text{-}q)$.

Corollary: 5.1

Let T be a fuzzy tree with p vertices then $\gamma_{fai}^{-1}(T) \geq \frac{P+1}{3}$.

proof:

Note that T has q=p-1 edges. Applying the previous theorem we get $\gamma_{fai}^{-1}(T) \geq \frac{P+1}{3}$.

Theorem: 5.3

For any fuzzy graph G then $q \leq \lfloor 1/2(p\text{-} \gamma_{fai}^{-1}(G))(p\text{-} \gamma_{fai}^{-1}(G)+2) \rfloor$.

Theorem : 5.4

For any fuzzy graph G then $\gamma_{fai}^{-1}(G) \leq p+1\text{-}\sqrt{1+2q}$.

Theorem : 5.5

For any fuzzy graph G, $\gamma_f^{-1}(G) \leq \min\{ \gamma_{fa}^{-1}(G), \gamma_{fai}^{-1}(G)\}$.

proof:

since every inverse accurate dominating set and every inverse accurate independent dominating set of G are the inverse dominating sets of G, we have $\gamma_f^{-1}(G) \leq \gamma_{fa}^{-1}(G)$ and $\gamma_f^{-1}(G) \leq \gamma_{fai}^{-1}(G)$ and hence $\gamma_f^{-1}(G) \leq \min\{ \gamma_{fa}^{-1}(G), \gamma_{fai}^{-1}(G)\}$.

Theorem: 5.6

Let T be a fuzzy tree in which all the vertices are either pendant vertices or their supports. Then T is a wounded spider iff $\gamma_{fai}^{-1}(T) = \Delta_E(T)$.

Proof:

Let T be a fuzzy tree with p vertices and whose vertices are either pendant vertices or their supports and let $\gamma_{fai}^{-1}(T) = \Delta_E(T)$. Let L be the set of all pendant vertices of T and S be its neighbor set.Then $|S| \leq |L|$. By the assumption of T, V (T) = $S \cup L$. Hence S is a γ_{fai} set of T and L is a γ_{fai}^{-1} set of T. since $|S| + |L| = p$. we get $\gamma_{fai}(T) + \gamma_{fai}^{-1}(T) = p$. This implies that $\gamma_{fai}(T) = p - \Delta_E(T)$ .By theorem 2 .5 T is wounded spider.

Conversely assume that T is a wounded spider and so $\gamma_{fai}(T) = p- \Delta_E(T)$ (Theorem 2.5). As discussed above we have V (T) = $S \cup L$ and so by theorem 2.1, $\gamma_{fai}(T) + \gamma_{fai}^{-1}(T) = p$. Hence $\gamma_{fai}^{-1}(T) = \Delta_E(T)$.

Corollary : 5.2

Let G be a wounded spider on p vertices. Then $\gamma_{fai}(G) = \gamma_{fai}^{-1}(G)$ iff $\Delta_E(T) = p/2$.

Proof:

Assume that $\gamma_{fai}(G) = \gamma_{fai}^{-1}(G)$. By Theorem 4.6 for a wounded spider G, we have $\gamma_{fai}(G) + \gamma_{fai}^{-1}(G) = p$ and $\gamma_{fai}^{-1}(G) = \Delta_E(G)$. Hence $\Delta_E(G) = p/2$ on the reverse assume that $\Delta_E(G) = p/2$. We have $\gamma_{fai}^{-1}(G) = \Delta_E(G)$. Then by Theorem 2.5 $\gamma_{fai}(G) = p- \Delta_E(G)$. Hence $\gamma_{fai}(G) = \gamma_{fai}^{-1}(G)$

Nordhaus-Gaddum Type Result:

Theorem: 5.7

For any fuzzy graph G such that both G and $\bar{G}$ have no isolates.Then

i) $\gamma_{fai}^{-1}(G) + \gamma_{fai}^{-1}(\bar{G}) \leq 2(p-1)$.

ii) $\gamma_{fai}^{-1}(G)\ \gamma_{fai}^{-1}(\bar{G}) \leq (n-1)^2$

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