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Unification of Multi Dimensional Generating Relations through a Bilateral Relation for Generalized Hypergeometric Functions $H_{n;\lambda;(b_q)}^{\alpha;\mu;(a_p)}(x,w)$

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Abstract: *In 1993, Satyanarayana [8] has defined generalized hypergeometric functions through* $H_{n;\lambda;(b_q)}^{\alpha;\mu;(a_p)}(x,w)$ *by using difference operator technique. In this paper we established unification of several multilateral generating relations of certain special functions has been established through a bilateral generating relation for generalized hypergeometric functions,* $H_{n;\lambda;(b_q)}^{\alpha;\mu;(a_p)}(x,w)$.

I. Introduction

In the present paper an attempt has been made to develop various multidimensional bilateral generating relations of certain special functions e.g. Pseudo Laguerre polynomial, Sister celine's polynomials, Hermite, Laguerre, etc., through a bilateral generating relation for generalized hypergeometric functions $H_{n;\lambda;(b_q)}^{\alpha;\mu;(a_p)}(x,w)$. In Satyanarayana [8] defined and studied the generalized hypergeometric function $H_{n;\lambda;(b_q)}^{\alpha;\mu;(a_p)}(x,w)$ and also proved that

$$H_{n;\lambda;(b_q)}^{\alpha;\mu;(a_p)}(x,w) = \frac{(-\alpha)_n w^{-2n}}{n!(\frac{x}{w}-\mu-\alpha+1)_n(-\frac{x}{w}+\mu+\alpha)_n} F_{q:1;0}^{p:2;1}\left[\begin{matrix}(a_p):-n,\frac{x}{w}-\mu+1;-\frac{x}{w}+\lambda; \\ (b_q):1+\alpha-n;....;\end{matrix} w,w\right], \quad .... \quad (1.1)$$

where $F_{q;s;v}^{p;r;u}(x,y)$ is double hypergeometric functions (see Srivasatava and Karlesson [9, P.27(28)]). In particular, for

$$\lim_{w\to 0}\{e^x x^{2n} H_{n;\lambda;(a_p)}^{\alpha;\mu;(a_p)}(x,w)\} = L_n^{\alpha-n}(x), \quad .... \quad (1.2)$$

where $L_n^{\alpha-n}(x)$ is the pseudo Laguerre polynomials.

In this investigation the author require the generalized Lauricella function of several variables Srivastava and Manocha [10, P.64 (18)].

$$F_{C:D^1;D^{(2)};.....;D^{(n)}}^{A:B^1;B^{(2)};...;B^{(n)}}\left[\begin{matrix}[(a):\theta^1;\theta^{(2)};....;\theta^{(n)}]:[(b^1):\phi^1];....;[(b^{(n)}):\phi^{(n)}] \\ [(c):\varphi^1;\varphi^{(2)};....;\varphi^{(n)}]:[(d^1):\delta^1];....;[(d^{(n)}):\delta^{(n)}]\end{matrix} z_1,z_2,....,z_n\right]$$

$$= \sum_{m_1,m_2,...,m_n=0}^{\infty} \frac{\prod_{j=1}^{A}(a_j)_{m_1\theta_j^1+....+m_n\theta_j^{(n)}} \prod_{j=1}^{B^1}(b_j^1)_{m_1\phi_j^1}....\prod_{j=1}^{B^{(n)}}(b_j^{(n)})_{m_n\phi_j^{(n)}}}{\prod_{j=1}^{C}(c_j)_{m_1\varphi_j^1+....+m_n\varphi_j^{(n)}} \prod_{j=1}^{D^1}(d_j^1)_{m_1\delta_j^1}....\prod_{j=1}^{D^{(n)}}(d_j^{(n)})_{m_n\delta_j^{(n)}}} \frac{z_1^{m_1}z_2^{m_2}...z_n^{m_n}}{m_1!m_2!...m_n!} \quad .... \quad (1.3)$$

II. The Main Result

We prove the following main bilateral generating relation:

$$I=\sum_{n=0}^{\infty}\frac{[(\rho^{\sigma})]_n[(1-(d'))]_{(\mu')n}\ldots[1-(d^{M})]_{(\mu^{M})n}[(g^{1})]_{(\eta')n}\ldots[(g^{M})]_{(\eta^{M})n}}{[(\xi^{\ell})]_n[(1-(b'))]_{(\lambda')n}\ldots[1-(b^{M})]_{(\lambda^{M})n}[(h^{1})]_{(\theta')n}\ldots[(h^{M})]_{(\theta^{M})n}}H^{\alpha;\mu;(a_p)}_{n;\lambda;(b_q)}(x,w)$$

$$F^{A:B'+G'+E';\ldots;B^{M}+G^{M}+E^{M}}_{C:D'+H'+F';\ldots;D^{M}+H^{M}+F^{M}}\left[\begin{matrix}[(\delta_A):1;\ldots;1]:[(b^{1})-(\lambda^{1})n:t_1],[(g^{1})+(\eta^{1})n:r_1],\\ [(\gamma_C):1,\ldots,1]:[(d^{1})-(\mu^{1})n:\varphi_1],[(h^{(1)})+(\theta^{1})n:\phi_1],\end{matrix}\right.$$

$$\left.\begin{matrix}[(e^{1}):s_1];\ldots;[(b^{M})-(\lambda^{M1})n:t_M],[(g^{M})+(\eta^{M})n:r_M],[(e^{M}):s_M];\\ [(f^{1}):v_1];\ldots;[(d^{M})-(\mu^{M})n:\varphi_M],[(h^{M})+(\theta^{M})n:\phi_M],[(f^{M}):v_M];\end{matrix}\;z_1,z_2,\ldots,z_M\right]t^{n}$$

$$=F^{\sigma+p+D'+\ldots+D^{M}+G'+\ldots+G^{M}+A+1:1,1,1;E',\ldots,E^{M}}_{\ell+q+B'+\ldots+B^{M}+H'+\ldots+H^{M}+C+3:0,0,0,F',\ldots,F^{M}}\left[\begin{matrix}[(\rho^{\sigma}):1,1,0,0,\ldots,0],[-\alpha:1,1,0,\ldots,0],[(a_p):0,1,1,0,\ldots,0],\\ [(\xi^{\ell}):1,1,0,0,\ldots,0],[\frac{x}{w}-\mu-\alpha+1:1,1,0,\ldots,0],\end{matrix}\right.$$

$$[1-(d^{1}):\mu^{1},\mu^{1},0-\varphi_1,0,\ldots,0],\ldots,[1-(d^{M}):\mu^{M},\mu^{M},0,0,\ldots,0,-\varphi_M],[(g^{1}):\eta^{1},\eta^{1},0,r_1,0,\ldots,0],\ldots,[(g^{M}):\eta^{M},\eta^{M},0,\ldots,0,r_M],$$

$$[-\frac{x}{w}+\mu+\alpha:1,1,0,\ldots,0],[\alpha:1,1,.0,\ldots,0],[(b_q):0,1,1,0,\ldots,0],[1-(b^{1}):\lambda^{1},\lambda^{1},0,-t_1,0,\ldots,0],\ldots,[1-(b^{M}):\lambda^{M},\lambda^{M},0,0,\ldots,0,-t_M],$$

$$[(\delta_A):0,0,0,1,\ldots,1]:[\frac{x}{w}-\mu+1:0,1,0,\ldots,0],[-\frac{x}{w}+\lambda:0,0,1,0,\ldots,0],[\alpha:1,0,0,0,\ldots,0],$$

$$[(h^{1}):\theta^{1},\theta^{1},0,\phi_1 0,\ldots,0],\ldots,[(h^{M}):\theta^{M},\theta^{M},0,\ldots,0,\phi_M];[(\lambda_C):0,0,0,1,\ldots,1],$$

$$\left.\begin{matrix}[(e^{1}):0,0,0,s_1,0,\ldots,0],\ldots,[(e^{M}):0,0,\ldots,0,s_M]\\ [(f^{1}):0,0,0,v_1,0,\ldots,0],\ldots,[(f^{M}):0,0,0,\ldots,0,v_M]\end{matrix}\;tw^{-2},tw^{-1},-w,(-1)^{\varphi_1+l_1}z_1,(-1)^{\varphi_2+l_2}z_2,\ldots,(-1)^{\varphi_M+l_M}z_M\right]$$

…. (2.1)

where $F^{A:B^{1};B^{(2)};\ldots;B^{(n)}}_{C:D^{1};D^{(2)};\ldots;D^{(n)}}(z_1,z_2,\ldots,z_n)$ is Lauricella function of several variables Srivastava and Manocha [10,P.64(18)].

Prof of (2.1): To prove (2.1), we consider

$$I=\sum_{n=0}^{\infty}\frac{[(\rho^{\sigma})]_n[(1-(d'))]_{(\mu')_n}\ldots[1-(d^{M})]_{(\mu^{M})_n}[(g^{1})]_{(\eta')n}\ldots[(g^{M})]_{(\eta^{M})n}}{[(\xi^{\ell})]_n[(1-(b'))]_{(\lambda')_n}\ldots[1-(b^{M})]_{(\lambda^{M})_n}[(h^{1})]_{(\theta')n}\ldots[(h^{M})]_{(\theta^{M})n}}H^{\alpha;\mu;(a_p)}_{n;\lambda;(b_q)}(x,w)$$

$$F^{A:B'+G'+E';\ldots;B^{M}+G^{M}+E^{M}}_{C:D'+H'+F';\ldots;D^{M}+H^{M}+F^{M}}\left[\begin{matrix}[(\delta_A):1;\ldots;1]:[(b^{1})-(\lambda^{1})n:t_1],[(g^{1})+(\eta^{1})n:r_1],[(e^{1}):s_1];\ldots;\\ [(\gamma_C):1,\ldots,1]:[(d^{1})-(\mu^{1})n:\varphi_1],[(h^{(1)})+(\theta^{1})n:\phi_1],[(f^{1}):v_1];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(b^{M})-(\lambda^{M1})n:t_M],[(g^{M})+(\eta^{M})n:r_M],[(e^{M}):s_M];\\ [(d^{M})-(\mu^{M})n:\varphi_M],[(h^{M})+(\theta^{M})n:\phi_M],[(f^{M}):v_M];\end{matrix}\;z_1,z_2,\ldots,z_M\right]t^{n}$$

By using (1.1) and (1.3) and writing (n+k) for n, we obtain

$$I=\sum_{n=0}^{\infty}\sum_{k=0}^{n}\sum_{l_1=0}^{\infty}\cdots\sum_{l_M=0}^{\infty}\frac{[(\rho^{\sigma})]_{n+k}(-\alpha)_{n+k}[(1-(d'))]_{(\mu')(n+k)-\varphi_1 l_1}\ldots[1-(d^{M})]_{(\mu^{M})(n+k)-\varphi_M l_M}[(g^{1})]_{(\eta')(n+k)+r_1 l_1}}{[(\xi^{\ell})]_{n+k}(\alpha)_{n+k}[(b_q]_{k+r}(-\frac{x}{w}+\mu+\alpha)_{n+k}(\frac{x}{w}-\mu-\alpha+1)_{n+k}}$$

$$\frac{\ldots\ldots[(g^{M})]_{(\eta^{M})(n+k)+r_M l_M}[(\delta_A)]_{l_1+l_2+\ldots+l_M}[(a_p]_{k+r}[(e^{1})]_{s_1 l_1}\ldots[(e^{M})]_{s_M l_M}(\alpha)_n(\frac{x}{w}-\mu+1)_k}{[(1-(b'))]_{(\lambda')(n+k)-t_1 l_1}\ldots[1-(b^{M})]_{(\lambda^{M})(n+k)-t_M l_M},[(h^{1})]_{(\theta')(n+k)+\phi_1 l_1},\ldots,[(h^{M})]_{(\theta^{M})(n+k)+\phi_M l_M}[(\gamma_C)]_{l_1+l_2+\ldots+l_M}}$$

$$\frac{(-\frac{x}{w}+\lambda)_r}{[(f^1)]_{v_1 l_1}...[(f^M)]_{v_M l_M} n!k!r!l_1!....l_M!}(w)^{-2n-2k}(tw)^n(-w)^k z_1^{l_1}....z_M^{l_M} t^M (-1)^{k+\varphi_1 l_1+....+\varphi_M l_M+t_1 l_1+....+t_M l_M}$$

from this we get the result (2.1).

III. Applications

(i). By writing $A=C,(\delta_A)=(\gamma_C),(B^M)=1,(G^M)=1,(E^M)=u,(F^M)=v+2,(D^M)=0=(H^M)$
$(b_1^M)=0,(\lambda_1^M)=1=(g_1^M)=(\eta_1^M),(e_u^M)=(a_u^M),(f_v^M)=(b_v^M),(f_{v+1}^M)=1,(f_{v+2}^M)=\frac{1}{2},$
$(t_M)=(r_M)=1,(s_M)=(v_M)=1$ in (2.1), we obtain a multidimensional generating relation for generalized hypergeometric function $H_{n;\lambda;(b_q)}^{\alpha;\mu;(a_p)}(x,w)$ and the Sister celine's polynomials ([6, P.290(2)]).

$$\sum_{n=0}^{\infty}\frac{[(\rho^\sigma)]_n}{[(\xi^\ell)]_n}H_{n;\lambda;(b_q)}^{\alpha;\mu;(a_p)}(x,w)f_n((a_u^1);(b_v^1);z_1).f_n((a_u^2);(b_v^2);z_2)....f_n((a_u^M);(b_v^M);z_M)t^n$$

$$=F_{\ell+q+M+3:0,0,0,v+2,...,v+2}^{\sigma+p+M+1:1,1,1;u,...,u}\left[\begin{matrix}[(\rho^\sigma):1,1,0,0,...,0],[-\alpha:1,1,0...,0],[(a_p):0,1,1,0,...,0],\\ [(\xi^\ell):1,1,0,0,...,0],[\alpha:1,1,0,..,0],[(b_q):0,1,1,0,...,0],\end{matrix}\right.$$

$$[1:1,1,01,0,...,0],...,[1:1,1,0,0,...,0],[\alpha:1,0,...,0],[\frac{x}{w}-\mu+1:0,1,0,...,0],[-\frac{x}{w}+\lambda:0,0,1,0,...,0],$$

$$[1:1,1,0,-1,0...,0],...,[1:1,1,0,0,...,0,-1],[\frac{x}{w}-\mu-\alpha+1:1,1,0...,0],[\frac{x}{w}+\mu+\alpha:1,1,0,...,0],$$

$$\left.\begin{matrix}[(a_u^1):1],...,[(a_u^M):1]\\ [(b_v^1):1],[1:1],[\frac{1}{2}:1];....;[(b_v^M):1],[1:1],[\frac{1}{2}:1]\end{matrix}\; t,-wt,w,(-1)^{\varphi_1+l_1}z_1,(-1)^{\varphi_2+l_2}z_2,....,(-1)^{\varphi_M+l_M}z_M\right]$$

$$\quad .... \quad (3.1)$$

(ii). By taking $A=C,(\delta_A)=(\gamma_C),(B^M)=1,(G^M)=(E^M)=(D^M)=0=(H^M)=(F^M)=0$
$(b_1^M)=0,\ (\lambda_1^M)=1,(t_M)=2,z_M=\frac{-1}{4X_M^2}$ and $T=\frac{t}{\prod_{j=1}^{M}(2x_j)}$

in (2.1), we have a multilateral generating relation for generalized hypergeometric function $H_{n;\lambda;(b_q)}^{\alpha;\mu;(a_p)}(x,w)$ and the Hermite polynomial

$$\sum_{n=0}^{\infty}\frac{[(\rho^\sigma)]_n}{[(\xi^\ell)]_n(n!)^M}H_{n;\lambda;(b_q)}^{\alpha;\mu;(a_p)}(x,w)H_n(x_1).H_n(x_2)....H_n(x_M)T^M$$

$$=F_{\ell+q+3:0,0,0;0;...;0}^{\sigma+p+M+1:1,1,1;0,...,0}\left[\begin{matrix}[(\rho^\sigma):1,1,0,0,...,0],[-\alpha:1,1,0,...,0],[(a_p):0,1,1,0,...,0],[1:1,1,0,-2,0,...,0],...,\\ [(\xi^\ell):1,1,0,0,...,0],[\frac{x}{w}-\mu-\alpha+1:1,1,0,...,0],[-\frac{x}{w}+\mu+\alpha:1,1,.0,....,0],\end{matrix}\right.$$

$$[1{:}1,1,0,0,...,0,-2],[\frac{x}{w}-\mu+1{:}0,1,0,...,0],[-\frac{x}{w}+\lambda{:}0,0,1,0,...,0],[\alpha{:}1,0,0,...,0]$$
$$[\alpha{:}1,1,0,...,0],[(b_q){:}0,1,1,0,...,0]$$
$$\left. T\prod_{j=1}^{M}(2x_j),-wT\prod_{j=1}^{M}(2x_j),w,\frac{-1}{4x_1^2},\frac{-1}{4x_2^2},...,\frac{-1}{4x_n^2}\right] \quad .... \quad (3.2)$$

(iii). By writing $A=C,(\delta_A)=(\gamma_C),(B^M)=1,(G^M)=0=(E^M)=(F^M),(D^M)=1,(H^M)=0,$
$(b_1^M)=1+(\alpha_M),(d_1^M)=1+(\alpha_M),(\lambda_1^M)=(\mu^M)-1,(\mu_1^M)=(\mu^M),t_M=1=(\varphi_M)$and $t=(-1)^M T$
$(s_M)=(v_M)=1$

in (2.1), we obtain a multilateral generating relation for generalized hypergeometric function and the modified Laguerre polynomials

$$\sum_{n=0}^{\infty}\frac{[(\rho^{\sigma})]_n(n!)^M}{[(\xi^{\ell})]_n}H_{n;\lambda;(b_q)}^{\alpha;\mu;(a_p)}(x,w)L_n^{\alpha_1-\mu_n^1}(z_1).L_n^{\alpha_2-\mu_n^2}(z_2)....L_n^{\alpha_M-\mu_n^M}(z_n)T^n=$$
$$=e^{z_1+z_2+...+z_M}$$
$$F_{\ell+q+M+3:0,1,0;0;...;0}^{\sigma+p+M+1:1,1,1;0,...,0}\left[\begin{matrix}[(\rho^{\sigma}):1,1,0,0,...,0]:[(a_p):0,1,1,0,...,0],[-\alpha_1:\mu^1,\mu^1 0,-1,0,...,0],...,[-\alpha_M:\mu^M,\mu^M,0,0,...,0,-1],\\ [(\xi^{\ell}):1,1,0,0,...,0]:[(b_q):0,1,1,0,...,0],[-\alpha_1:\mu^1-1,\mu^1-1,0,-1,0,...,0],...,[-\alpha_M:\mu^M-1,\mu^M-1,0,...,0,-1],\end{matrix}\right.$$
$$\left.\begin{matrix}[-\alpha:1,1,0,...,0],[\alpha:1,0,...,0],[\frac{x}{w}-\mu+1:0,1,0,...,0],[-\frac{x}{w}+\lambda:0,0,1,0,...,0]\\ [\frac{x}{w}-\mu-\alpha+1:1,1,0,...,0],[-\frac{x}{w}+\mu+\alpha:1,1,0,...,0],[\alpha:1,1,0,0,...,0],,\end{matrix}\;(-1)^M T,(-1)^M wT,w,-z_1,-z_2,...,-z_M\right] \quad .... \quad (3.3)$$

(iv). By writing $p=q,(a_j)=(b_j),(j=1,2,3,...,p)$ and applying limit as $w\to 0$ in (2.1), (3.1), (3.2) and (3.3), we obtain a bilateral generating relations and multidimensional generating relations for Laguerre polynomial and Sister celine's polynomials, Hermite Polynomials, modified Laguerre polynomial:

$$\sum_{n=0}^{\infty}\frac{[(\rho^{\sigma})]_n[(1-(d'))]_{(\mu^1)_n}...[1-(d^M)]_{(\mu^M)_n}[(g^1)]_{(\eta^1)n}...[(g^M)]_{(\eta^M)n}}{[(\xi^{\ell})]_n[(1-(b'))]_{(\lambda^1)_n}...[1-(b^M)]_{(\lambda^M)_n}[(h^1)]_{(\theta^1)n}...[(h^M)]_{(\theta^M)n}}L_n^{\alpha-n}(x)$$

$$F_{C:D^1+H^1+F^1;.....;D^M+H^M+F^M}^{A:B^1+G^1+E^1;...;B^M+G^M+E^M}\left[\begin{matrix}[(\delta_A):1;...;1]:[(b^1)-(\lambda^1)n:t_1],[(g^1)+(\eta^1)n:r_1],[(e^1):s_1];...;\\ [(\gamma_C):1,...,1]:[(d^1)-(\mu^1)n:\varphi_1],[(h^{(1)})+(\theta^1)n:\phi_1],[(f^1):v_1];...;\end{matrix}\right.$$
$$\left.\begin{matrix}[(b^M)-(\lambda^{M1})n:t_M],[(g^M)+(\eta^M)n:r_M],[(e^M):s_M];\\ [(d^M)-(\mu^M)n:\varphi_M],[(h^M)+(\theta^M)n:\phi_M],[(f^M):v_M];\end{matrix}\; z_1,z_2,....,z_M\right]t^n$$

$$\frac{1}{e^x x^{2n}}=F_{\ell+B^1+...+B^M+H^1+...+H^M+C+1:0,0,0,F^1,...,F^M}^{\sigma+D^1+...+D^M+G^1+...+G^M+A+1:1,0,0;E^1,...,E^M}\left[\begin{matrix}[(\rho^{\sigma}):1,1,0,0,...,0]:[1-(d^1):\mu^1,\mu^1,0-\varphi_1,0,...,0],...,\\ [(\xi^{\ell}):1,1,0,0,...,0]:[1-(b^1):\lambda^1,\lambda^1,0,-t_1,0...,0],...,\end{matrix}\right.$$
$$\begin{matrix}[(1-(d^M):\mu^M,\mu^M,0,0,...,0,-\varphi_M],[(g^1):\eta^1,\eta^1,0,r_1,0,...,0],...,[(g^M):\eta^M,\eta^M,0,...,0,r_M],\\ [1-(b^M):\lambda^M,\lambda^M,0,0,...,0,-t_M],[(h^1):\theta^1,\theta^1,0,\phi_1 0,...,0],[(h^M):\theta^M,\theta^M,0,...,0,\phi_M];\end{matrix}$$
$$\begin{matrix}[(\delta_A):0,0,0,1,...,1],[-\alpha:1,1,0,...,0],[\alpha:1,0,....,0],[(e^1):0,0,0,s_1,0,...,0],...,[(e^M):0,0,..,0,s_M]\\ [(\lambda_C):0,0,0,1,...,1],[\alpha:1,1,0,0,...,0],[(f^1):0,0,0,v_1,0,...,0],....,[(f^M):0,0,0,...,0,v_M]\end{matrix}$$

$$t,-xt,(-1)^{\varphi_1+l_1} z_1,(-1)^{\varphi_2+l_2} z_2,\ldots,(-1)^{\varphi_M+l_M} z_M \Bigg] \qquad \ldots \quad (3.4)$$

where $L_n^{\alpha-n}(x)$ is pseudo Laguerre polynomial [6].

$$\sum_{n=0}^{\infty} \frac{[(\rho^{\sigma})]_n}{[(\xi^{\ell})]_n} L_n^{\alpha-n}(x) f_n((a_u^1);(b_v^1);z_1).f_n((a_u^2);(b_v^2);z_2)\ldots f_n((a_u^M);(b_v^M);z_M)t^n$$

$$= \frac{1}{e^x x^{2n}} F^{\sigma+M:1,0,0;u,\ldots,u}_{\ell+M:0,1,0,v+2,\ldots,v+2}\left[\begin{matrix}[(\rho^{\sigma}):1,1,0,0,\ldots,0]:[1:1,1,01,0,\ldots,0],\ldots,[1:1,1,0,0,\ldots,0],\\ [(\xi^{\ell}):1,1,0,0,\ldots,0]:[1:1,1,0,-1,0\ldots,0],\ldots,[1:1,1,0,0,\ldots,0,-1];\end{matrix}\right.$$

$$\left.\begin{matrix}[-\alpha:1,1,0,\ldots,0],[\alpha:1,0,\ldots,0],[(a_u^1):1],\ldots,[(a_u^M):1];\\ [\alpha:1,1,0,0,\ldots,0],[(b_v^1):1],[1:1],[\tfrac{1}{2}:1];\ldots;[(b_v^M):1],[1:1],[\tfrac{1}{2}:1]\end{matrix}\; t,-xt,(-1)^{\varphi_1+l_1}z_1,(-1)^{\varphi_2+l_2}z_2,\ldots,(-1)^{\varphi_M+l_M}z_M\right] \qquad \ldots \quad (3.5)$$

where $L_n^{\alpha-n}(x)$ is pseudo Laguerre polynomial [6].

$$\sum_{n=0}^{\infty} \frac{[(\rho^{\sigma})]_n}{[(\xi^{\ell})]_n (n!)^M} L_n^{\alpha-n}(x) H_n(x_1).H_n(x_2)\ldots H_n(x_M)T^n$$

$$= \frac{1}{e^x x^{2n}} F^{\sigma+M:1,0,;0,\ldots,0}_{\ell:0,1,0;0;\ldots;0}\left[\begin{matrix}[(\rho^{\sigma}):1,1,0,0,\ldots,0]:[1:1,1,0,-2,0,\ldots,0],\ldots,[1:1,1,0,0,\ldots,0,-2],[-\alpha:1,1,0,\ldots,0],[\alpha:1,0,\ldots,0]\\ [(\xi^{\ell}):1,1,0,0,\ldots,0]:[\alpha:1,1,0,0,\ldots,0]\end{matrix}\right.$$

$$\left. T\prod_{j=1}^{M}(2x_j),-xT\prod_{j=1}^{M}(2x_j),\frac{-1}{4x_1^2},\frac{-1}{4x_2^2},\ldots,\frac{-1}{4x_n^2}\right] \qquad \ldots \quad (3.6)$$

where $L_n^{\alpha-n}(x)$ is pseudo Laguerre polynomial [6].

$$\sum_{n=0}^{\infty} \frac{[(\rho^{\sigma})]_n (n!)^M}{[(\xi^{\ell})]_n} L_n^{\alpha-n}(x) L_n^{\alpha_1-\mu_n^1}(z_1).L_n^{\alpha_2-\mu_2^2}(z_2)\ldots L_n^{\alpha_M-\mu_n^M}(z_n)T^n$$

$$= \frac{e^{z_1+z_2+\ldots+z_M}}{e^x x^{2n}}$$

$$F^{\sigma+M+1:1,0,0;0,\ldots,0}_{\ell+M+1:0,0;0;\ldots;0}\left[\begin{matrix}[(\rho^{\sigma}):1,1,0,0,\ldots,0]:[-\alpha_1:\mu^1,\mu^1 0,-1,0,\ldots,0],\ldots,[-\alpha_M:\mu^M,\mu^M,0,0,\ldots,0,-1],\\ [(\xi^{\ell}):1,1,0,0,\ldots,0]:[-\alpha_1:\mu^1-1,\mu^1-1,0,-1,0,\ldots,0],\ldots,[-\alpha_M:\mu^M-1,\mu^M-1,0,\ldots,0,-1],\end{matrix}\right.$$

$$\left.\begin{matrix}[-\alpha:1,1,0,\ldots,0],[\alpha:1,0,0,\ldots,0]\\ [\alpha:1,1,0,0,\ldots,0],\end{matrix}(-1)^M T,(-1)^M xT,-z_1,-z_2,\ldots,-z_M\right] \qquad \ldots \quad (3.7)$$

where $L_n^{\alpha-n}(x)$ is pseudo Laguerre polynomial [6].

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