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Boundary Domination in Middle Graphs

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Abstract: ***Let*** *$G = (V,E)$ be a graph. A subset S of V is called a boundary dominating set of G if every vertex in $V - S$ is vertex boundary dominated by some vertex in S. The minimum taken over all boundary dominating sets of G is called the boundary domination number of G and is denoted by $\gamma_b(G)$. In this paper, we introduce the boundary domination and the boundary domatic number* $d_b(G)$ *in Middle graph.*

Keywords: *Boundary dominating set, Boundary domination number, Boundary domatic number, Middle graph.*
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I. Introduction

Domination in graphs has become an important area of research in graph theory, as evidenced by the many results contained in the two books by Haynes, Hedetniemi and Slater (1998) [3]. All graphs in this paper will be finite and undirected, without loops and multiply edges. As usual $n = |V|$ and $m = |E|$ denote the number of vertices and edges of a graph G, respectively. In general, we use $N(v)$ and $N[v]$ to denote the open and closed neighbourhood of a vertex v, respectively. A subset D of vertices in a graph G is a dominating set if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G.

Middle graph $M(G)$ of a graph G is defined with the vertex set $V(G) \cup E(G)$, in which two elements are adjacent if and only if either both are adjacent edges in G or one of the elements is a vertex and the other one is an edge incident to the vertex in G.

Let G be a simple graph $G = (V,E)$ with vertex set $V(G) = \{v_1, v_2, \dots, v_n\}$. For $i \neq j$, a vertex v_i is a boundary vertex of v_j if $d(v_j, v_t) \leq d(v_j, v_i)$ for al $v_t \in N(v_i)$ (see [1, 2]).

A vertex v is called a boundary neighbor of u if v is a nearest boundary of u. If $u \in V$, then the boundary neighbourhood of u denoted by $N_b(u)$ is defined as

$N_b(u) = \{v \in V : d(u,w) \leq d(u,v)$ for all $w \in N(u)\}$. The cardinality of $N_b(u)$ is denoted by $deg_b(u)$ in G. The maximum and minimum boundary degree of a vertex in G are denoted respectively by $\Delta_b(G)$ and $\delta_b(G)$. That is $\Delta_b(G) = \max_{u \in V}|N_b(u)|$, $\delta_b(G) = \min_{u \in V}|N_b(u)|$. A vertex u boundary dominate a vertex v if v is a boundary neighbor of u.

A subset B of $V(G)$ is called a boundary dominating set if every vertex of $V - B$ is boundary dominated by some vertex of B. The minimum taken over all boundary dominating sets of a graph G is called the boundary domination number of G and is denoted by $\gamma_b(G)$.[4]. We need the following theorems.

Theorem.1.1. [4]
(1) For any complete graph K_n, $\gamma_b(K_n) = 1$.
(2) For the complete bipartite graph $K_{m,n}$ with $2 \leq m \leq n$, $\gamma_b((K_{m,n}) = 2$.

The definition of boundary domination motivated us to introduce the boundary domination in Middle graph of G.

II. Main Results

Boundary Domination number of middle graphs are obtained in this section. Let $G = (V, E)$ be a graph with vertices $v_1, v_2, \ldots, v_n$. Let $e_i = v_i v_{i+1}$ for $1 \le i \le n-1$ be the edges of G. The middle graph of G is $M(G)$. Here $V(M(G)) = \{v_i e_i ; 1 \le i \le n\}$ and $E(M(G)) = \{e_i\, e_{i+1}, v_i\, e_i, e_i\, v_{i+1} ; 1 \le i \le n-1\}$.

Definition2.1. A vertex $v \in V(M(G))$ is said to be a boundary of u if $d(v,w) \le d(v,u)$ for all $w \in N_b(v)$. A vertex v is a boundary neighbor of a vertex u if v is a nearest boundary of u. Two vertices v and u are boundary adjacent if v adjacent to u and there exist another vertex w adjacent to both v and u.

Definition2.2. A subset S of vertices is called a boundary dominating set if every vertex of $V(M(G)) - S$ is boundary dominated by some vertex of S. The minimum taken over all boundary dominating sets of a middle graph of G is called the boundary domination number of $M(G)$ and is denoted by $\gamma_b(M(G))$.

A boundary dominating set S is minimal if for any vertex $v \in S, S - \{v\}$ is not boundary dominating set of $M(G)$. A subset S of $V(M(G))$ is called boundary independent set, if for any $v \in S$, $v \in N_b(w)$, for all $w \in S - \{v\}$. If a vertex $v \in V(M(G))$ be such that $N_b(v) = \varphi$ then j is in any boundary dominating set. Such vertices are called boundary-isolated. The minimum boundary dominating set denoted by $\gamma_b(M(G)) - set$. For a real number x; $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x and $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

Example2.3. In Figure 1, $\{v_1, v_4\}, \{v_2, v_3\}, \{v_5, v_5\}$ are the minimum boundary dominating set of G, $\gamma_b(G) = 2$, $\{v_2, e_2, v_3\}, \{v_2, v_3, v_6\}$, $\{e_4, e_5, e_6\}, \{e_2, e_4, e_6\}$ are the minimum boundary dominating sets of M(G), and $\gamma_b(M(G)) = 3$.

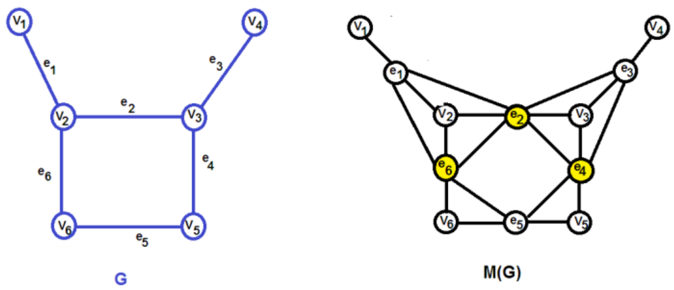

Figure 1: G and M (G)

Observation2.4. For any graph G, we have $\gamma_b(G) \le \gamma_b(M(G))$.

2.1 Boundary Domination Number

By the definition of Middle graph, we have $V(M(G) = V(G) \cup E(G)$, and

1) $|V[M(P_n)]| = 2n - 1, \Delta_b(M(P_n)) = 4$ and $\delta_b(M(P_n)) = 2$.
2) $|V[M(C_n)]| = 2n, \Delta_b(M(C_n)) = \delta_b(M(C_n)) = 4$.

3) $|V[M(K_{1,n})]| = 2n+1$, $\Delta_b(M(K_{1,n})) = 2n-1$ and $\delta_b(M(K_{1,n})) = n-1$.
4) $|V[M(K_n)]| = \frac{n(n+1)}{2}$, $\Delta_b(M(K_n)) = \frac{n(n-1)}{2}$ and $\delta_b(M(K_n)) = \frac{(n-1)(n-2)}{2}$.
5) $|V[M(W_n)]| = 3n+1$, $\Delta_b(M(W_n)) = 2n$ and $\delta_b(M(W_n)) = 2n-3$.
6) $|V[M(K_{m,n})]| = m+n+mn$, $\Delta_b(M(K_{m,n})) = mn$ and $\delta_b(M(K_{m,n})) = mn-1$.
7) $|V[M(B_{m,n})]| = 2(m+n)+3$, $\Delta_b(M(B_{m,n})) = m+n+1$ and $\delta_b(M(B_{m,n})) = m+n$.

We now proceed to compute $\gamma_b(M(G))$ for some standard graphs. It can be easily verified that

(1) For the complete bipartite graph $K_{m,n}$ with $2 \le m \le n$, $\gamma_b(M(K_{m,n})) = 3$.
(2) For the wheel W_n on n vertices, $\gamma_b(M(W_n)) = 3$.
(3) For the bistar graph $B_{m,n}$ with $2 \le m \le n$, $\gamma_b(M(B_{m,n})) = 3$.

Theorem2.5. For any path P_n, $n \ge 3$, $\gamma_b(M(P_n)) = \left\lfloor \frac{n}{2} \right\rfloor + 1$.

Proof. It is easy to observe that $\gamma_b(M(P_2)) = 2$, Let P_n be a path on n vertices where $n \ge 3$, $V(P_n) = \{v_1, v_2, \ldots, v_n\}$, $e_i = v_i v_{i+1}$; $1 \le i \le n-1$ and let $u_i \in V(M(P_n))$ be the vertex corresponding to e_i. Since $d(v_1, u_1) < d(v_1, v_2)$, then let $S = \{v_i \in V(M(P_n)) : i = 2j+1\}$. If n is odd, then S is a boundary dominating set of $M(P_n)$. If n is even, then $S \cup \{u_{n-1}\}$ is a boundary dominating set of $M(P_n)$, and the boundary dominating set of $M(P_n)$ is

$$S = \begin{cases} \{v_1, v_3, \ldots, v_{n-1}, u_{n-1}\} & \text{if } n = 2k \\ \{v_1, v_3, \ldots, v_n\} & \text{if } n = 2k+1. \end{cases}$$

and

$$|S| = \begin{cases} \frac{n}{2} + 1 & \text{if } n = 2k \\ \frac{n-1}{2} + 1 & \text{if } n = 2k+1 \end{cases}$$

Hence $\gamma_b(M(P_n)) = \left\lfloor \frac{n}{2} \right\rfloor + 1$.

Theorem2.6. For any cycle graph C_n, $n > 3$, $\gamma_b(M(C_n)) = \left\lceil \frac{n}{2} \right\rceil$

Proof. It is easy to observe that $\gamma_b(M(C_3)) = 3$, let C_n be a cycle on n vertices where $n \ge 4$, $V(C_n) = \{v_1, v_2, \ldots, v_n, v_1\}$ and $e_i = v_i v_{i+1}$; $1 \le i \le n-1$, $e_n = v_n v_1$ and let $u_i \in V(M(C_n))$ be the vertex corresponding to e_i. Since $d(v_1, u_n) < d(v_1, v_2) = d(v_1, v_n)$, then let $S = \{v_i \in V(M(C_n)) : i = 2j+1\}$. If n is even, then S is a boundary dominating set of $M(C_n)$. If n is odd, then $S \cup \{u_n\}$ is boundary dominating set of $M(C_n)$. Therefore the minimum boundary dominating set of $M(C_n)$ is

$$S = \begin{cases} \{v_1, v_3, \ldots, v_{n-1}\} & \text{if } n = 2k, \\ \{v_1, v_3, \ldots, v_n\} & \text{if } n = 2k+1. \end{cases}$$

and

$$|S| = \begin{cases} \frac{n-1-1}{2} + 1 & \text{if } n = 2k, \\ \frac{n-1}{2} + 1 & \text{if } n = 2k+1. \end{cases}$$

$$= \begin{cases} \frac{n}{2} & \text{if } n = 2k, \\ \frac{n+1}{2} & \text{if } n = 2k+1. \end{cases}$$

Hence $\gamma_b(M(C_n)) = \left\lfloor \frac{n}{2} \right\rfloor$.

Theorem2.7. For any star graph $K_{1,n}$, $\gamma_b(M(K_{1,n})) = 2$.

Proof. Let $G \cong K_{1,n}$ and let $u_i \in V(M(K_{1,n}))$ be the vertex corresponding to e_i, since $d(v,w) \leq d(v,t)$ for all $w \in N(v)$ and $N_b(v) = \{v \in V : d(v,w) \leq d(v,t)$ for all $w \in N(v)\} = V(G) - \{v,u\}$ and $\Delta_b(v) = |N_b(v)| = |V(G) - \{v,u\}| = L - 2$, we have $\Delta_b(M(K_{1,n})) = 2n - L = L - 2$, then there exist only one vertex say $u \notin N_b(v)$, $u \in N(v)$ and u is boundary dominating itself, therefore $S = \{v,u\}$ and $|S| = 2$. Hence $\gamma_b(M(K_{1,n})) = 2$.

Theorem2.8. For any complete graph K_n , $\gamma_b(M(K_n)) = 3$.

Proof. Let $G \cong M(K_n)$ and let $u_i \in V(G)$ be the vertex corresponding to e_i. Suppose $v_i \in V(G)$, since $d(v_i,v_j) \leq d(v_i,v_t)$ for all $j \neq t$, then there exist $\frac{n(n-1)}{2} \in N_b(v_i)$ and $(n-1) \in N(v_i)$ If a vertex $u_i \in V(G)$ is adjacent v_i then there exist $\frac{(n-1)(n-2)}{2} \in N_b(u_i)$ and $2(n-1) \in N(u_i)$. Now we take a vertex $v_j \in S$ such that $i \neq j$, then there exist $\frac{n(n-1)}{2} \in N_b(v_j)$ and $(n-1) \in N(v_i)$ also there exists are boundary vertices are common between v_i, u_i and v_j which this number is boundary vertices of u_i, therefore only $(n-2)$ are boundary neighbor vertices of v_i also v_j and $|N_b(v_i) \cup N_b(v_j) \cup N_b(e_i) \cup \{v_i, e_i, v_j\}| = 2(n-2) + \frac{(n-1)(n-2)}{2} + 3 = \frac{n(n+1)}{2} = V(G)$. Therefore the boundary dominating set is $S = \{v_i, e_i, v_j\}$. Which contained any path P_3 in $M(G)$. Hence $\gamma_b(M(K_n)) = 3$.

Proposition 2.9. Let u be a vertex of a middle graph of G. Then $V(M(G)) - N_b(u)$ is a boundary dominating set for $M(G)$.

Theorem 2.10. If G is a connected graph of order $n \geq 3$ and $M(G)$ is a middle graph of G of order $l \geq 5$, then $\gamma_b(M(G)) \leq l - \Delta_b(M(G))$.

Proof. Let u be a vertex of a middle graph of G. Then by the above proposition, $V(M(G)) - N_b(u)$ is a boundary dominating set for $M(G)$ and $|N_b(u)| = \Delta_b(u)$. But $|N_b(u)| \geq 1$. Thus $\gamma_b(M(G)) \leq l - 1$. Suppose $\gamma_b(M(G)) = l - 1$, then there exists a unique vertex u^* in $M(G)$ such that u^* is a boundary neighbour of every vertex of $V(M(G)) - \{u^*\}$, this is a contradiction to the fact that in a graph there exist at least two boundary vertices. Thus $\gamma_b(M(G)) \leq l - 2$. Hence $\gamma_b(M(G)) \leq l - \Delta_b(M(G))$.

Theorem 2.11. If G is a connected graph of order $n \geq 3$ and M(G) its a middle graph of order $l \geq 5$, then $\gamma_b(M(G)) \geq \left\lceil \frac{l}{1+\Delta_b} \right\rceil$.

Proof. We have four cases:

Case 1: If $G \cong P_n$, since $l = |V[M(P_n)]| = 2n - 1$, and $\Delta_b(M(P_n)) = 4$, then $\left\lceil \frac{l}{1+\Delta_b} \right\rceil = \left\lceil \frac{2n-1}{5} \right\rceil \leq \left\lfloor \frac{n}{2} \right\rfloor + 1 = \gamma_b(M(P_n))$.

Case 2: If $G \cong C_n$, since $l = |V[M(C_n)]| = 2n$, and $\Delta_b(M(C_n)) = 4$, then $\left\lceil \frac{l}{1+\Delta_b} \right\rceil = \left\lceil \frac{2n}{5} \right\rceil \leq \left\lceil \frac{n}{2} \right\rceil = \gamma_b(M(C_n))$.

Case 3: If $G \cong K_n$, since $l = |V[M(K_n)]| = \frac{n(n+1)}{2}$, and $\Delta_b(M(K_n)) = \frac{n(n-1)}{2}$, then $\left\lceil \frac{l}{1+\Delta_b} \right\rceil = \left\lceil \frac{\frac{n(n+1)}{2}}{\frac{n(n-1)}{2}+1} \right\rceil \leq 1 + \left\lceil \frac{2n}{n(n-1)} \right\rceil \leq 3 = \gamma_b(M(K_n))$.

Case 4: If $G \cong K_{1,n}$, since $l = |V[M(K_{1,n})]| = 2n+1$, and $\Delta_b(M(K_{1,n})) = 2n-1$, then $\left\lceil \frac{l}{1+\Delta_b} \right\rceil = \left\lceil \frac{2n+1}{2n} \right\rceil \leq 1 + \left\lceil \frac{1}{2n} \right\rceil \leq 2 = \gamma_b(M(K_{1,n}))$.

From the Theorems 2.10 and 2.11, we obtained the upper and lower bounds of the boundary domination of the middle graphs.

Observation 2.12. For any graph G, we have, $\left\lceil \frac{l}{1+\Delta_b} \right\rceil \leq \gamma_b(M(G)) \leq l - \Delta_b(M(G))$.

Theorem 2.13. For any graph G and $M(G)$ its middle graph of order l,

$$\gamma(M(G)) + \gamma_b(M(G)) \leq l + 1.$$

Proof. Let $v \in V(M(G))$, then $N(v) \cup N_b(v) \cup \{v\} = V(M(G))$, $|N(v)| + |N_b(v)| + 1 = l$ and $\Delta(M(G)) + \Delta_b(M(G)) + 1 = l$. But we have $\gamma(M(G)) \leq l - \Delta(M(G))$ and $\gamma_b(M(G)) \leq l - \Delta_b(M(G))$. Therefore

$\gamma(M(G)) + \gamma_b(M(G)) \leq 2l - \Delta(M(G)) + \Delta_b(M(G)) = 2l - l + 1 = l + 1$.

Hence $\gamma(M(G)) + \gamma_b(M(G)) \leq l + 1$.

2.2 Boundary Domatic Number

Definition 2.14. Let $G = (V, E)$ be a connected graph. The maximal order of partition of the vertices V into boundary dominating sets is called the boundary domatic number of G and denoted by $d_b(G)$.;

Example 2.15. $\{\{v_1, v_4\}, \{v_2, v_3\}, \{v_5, v_6\}\}$ is a boundary domatic partition of G, and $\{\{v_1, v_2, v_3\}, \{e_2, e_4, e_6\}\}$ is a boundary domatic partition of $M(G)$, therefore $d_b(G) = 3$ and $d_b(M(G)) = 2$.

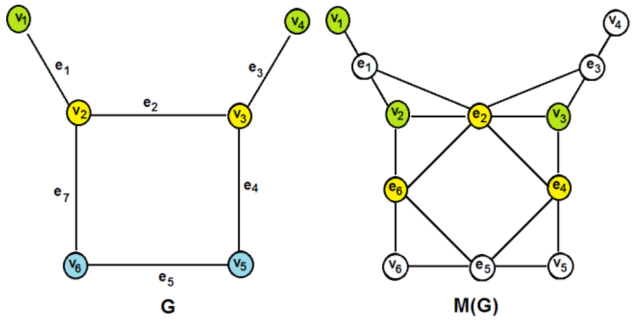

Figure 2: G and M (G)

Observation2.16. For any graph G, we have $d_b(M(G)) \leq d_b(G)$.

We first determine the boundary domatic number of middle graphs of some standard graphs, we observe that:

1. $d_b(M(B_{m,n})) = 1$.
2. $d_b(M(K_{m,n})) = m + n - 1$.

3. $d_b(M(P_n)) = \begin{cases} 1 & \text{if } n = 2 \text{ or } 9, \\ 2 & \text{otherwise.} \end{cases}$

4. $d_b(M(W_n)) = \begin{cases} 2 & \text{if } n = 3, \\ n & \text{otherwise.} \end{cases}$

5. $d_b(M(C_n)) = \begin{cases} 2 & \text{if } n = 3 \text{ or } 4, \\ 3 & \text{if } n = 2k+1; k \geq 2, \\ 4 & \text{if } n = 2k; k \geq 3. \end{cases}$

Remark2.17. Since $\gamma_b(K_n) = 1$, then $d_b(K_n) \leq \left\lfloor \frac{n}{\gamma_b(K_n)} \right\rfloor \leq n.$

Theorem2.18. For a complete graph K_n,

$$d_b(M(K_n)) = \begin{cases} 2 & \text{if } n = 3 \text{ or } 4, \\ n & \text{otherwise.} \end{cases}$$

□

Proof. It is easy to observe that $d_b(M(K_3)) = d_b(M(K_4)) = 2$, let K_n be a complete graph with $n \geq 5$ such that its middle graph of order $l = \frac{n(n+1)}{2}$, since $d_b(M(G)) \leq d_b(G)$ and $d_b(K_n) \leq n$, it follows that $d_b(M(K_n)) \leq d_b(K_n) \leq n$. To prove the reverse inequality, from the Theorem 2.8 we have the boundary domination number of $M(K_n)$ is 3, then $V(M(K_n))$ contains n disjoint paths of order 3. Clearly $\{S_1, S_2, \dots, S_n\}$ is a boundary domatic partition of $M(K_n)$, so that $d_b(M(K_n)) \geq n$, therefore $d_b(M(K_n)) = n$. Hence

$$d_b(M(K_n)) = \begin{cases} 2 & \text{if } n = 3 \text{ or } 4, \\ n & \text{otherwise.} \end{cases}$$

Theorem2.19. For a star graph $K_{1,n}$, $d_b(M(K_{1,n})) = n.$

Proof. Let $K_{1,n}$ be a star graph with $n \geq 2$ such that its middle graph of order $l = 2n+1$, since $\gamma_b(M(K_{1,n})) = 2$, it follows that $d_b(M(K_{1,n})) \leq \left\lfloor \frac{l}{\gamma_b(M(K_{1,n}))} \right\rfloor \leq \left\lfloor \frac{2n+1}{2} \right\rfloor \leq n$. Since $d(v_1, u_1) < d(v_1, v_2)$, $\Delta_b(v) = \Delta_b\left(M\left(K_{1,n}\right)\right) = 2n - 1$ and $\delta_b(M(K_{1,n})) = n - 1$. Then let $S_i = \{v_i, u_i : 1 \leq i \leq n\}$ be the boundary dominating sets of $M(K_{1,n})$. Clearly $\{S_1, S_2, \dots, S_n\}$ is a boundary domatic partition of $M(K_{1,n})$, so that $d_b(M(K_{1,n})) \geq n$, Hence $d_b(M(K_{1,n})) = n$.

Proposition 2.20. For any connected graph G of order n and $M(G)$ its middle graph with l vertices,

1. $d_b(M(G)) \leq \left\lfloor \frac{l}{\gamma_b(M(G))} \right\rfloor$
2. $d_b(M(G)) \leq \delta_b(M(G)) + 1$

Theorem 2.21. For any connected graph G, and $M(G)$ its middle graph, then

(i) $\gamma_b(M(G)) + d_b(M(G)) \leq n + 2$ and equality holds if and only if G is isomorphic to $K_{1,n}$.

(ii) $\gamma_b(M(G)) + d_b(M(G)) \leq n$ and equality holds if and only if G is isomorphic to W_3.

Proof. (i) Since $d_b \leq \delta_b + 1$ and $\gamma_b \leq l - \Delta_b$, we have $\gamma_b + d_b \leq 2n + 1 - 2n + 1 + n - 1 + 1 = n + 2$. Further $\gamma_b + d_b = n + 2$ if and only if $d_b = \delta_b + 1$ and $\gamma_b = l - \Delta_b$. We claim $\gamma_b = 2$ If $\gamma_b \geq 3$, then $d_b \leq \left\lfloor \frac{n}{\gamma_b} \right\rfloor \leq \frac{n}{3}$. Since $\gamma_b + d_b = n + 2$, we have $\gamma_b \geq \frac{2n}{3}$. It follows that $d_b = 2$ so that $\gamma_b = n$. Which is a contradiction. Hence $\gamma_b = 2$ and $d_b = n$ so that $G \cong K_{1,n}$.

(ii) Similarly as (i).

Theorem 2.22. For any connected graph G, and $M(G)$ its middle graph, then

(i) $\gamma_b(M(G)) + d_b(M(G)) \leq \delta_b(M(G))$ and equality holds if and only if G is isomorphic to $K_{m,n}$ or K_n with $n > 6$.

(ii) $\gamma_b(M(G)) + d_b(M(G)) \leq n$ and equality holds if and only if G is isomorphic to $B_{m,n}$ or P_n.

Proof. (i)

Case 1: Let $G \cong K_{m,n}$ with $4 \leq m \leq n \leq 5$, since $d_b(M(G)) \leq d_b(G) \leq \left\lfloor \frac{m+n}{\gamma_b} \right\rfloor \leq \left\lfloor \frac{m+n}{2} \right\rfloor \leq m + n$, and $\gamma_b\left(M(K_{m,n})\right) \leq l - \Delta_b \leq mn + m + n - mn = m + n$, we have $\gamma_b + d_b \leq 2m + 2n \leq mn - 1 = \delta_b$. Further equality holds if and only if $\gamma_b = 3$ and $d_b = m + n - 1$, and it follows that $G \cong K_{m,n}$.

Case2: Let $G \cong K_n$ with $n > 6$, since $d_b(M(G)) \leq d_b(G) \leq \left\lfloor \frac{n}{\gamma_b} \right\rfloor \leq n$, and $\gamma_b(M(G)) \leq l - \Delta_b \leq \frac{n(n+1)}{2} - \frac{n(n-1)}{2} = n$, we have $\gamma_b + d_b \leq 2n \leq \frac{(n-1)(n-2)}{2} = \delta_b$. Further equality holds if and only if $\gamma_b = 3$ and $d_b = n$, and it follows from the Theorem 2.7 that $G \cong K_n$.

Proof. (ii) Similarly as (i).

III. Conclusion

In this paper we computed the exact value of the boundary domination number and the boundary domatic number for middle graphs of paths, cycles, complete graphs, complete bipartite graphs, star graphs, bistar graphs, wheel graphs and some special graphs. In addition, we found some upper and lower bounds for boundary domination number and boundary domatic number for middle graph of graph.

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