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Fixed Point in Hilbert Spaces

Aradhana Sharma[1], Gauri Shanker Sao[2]
[1]*Deptt.Of Mathematics*
Govt.Bilasa Girls PG College Bilaspur
[2]*Deptt.Of Mathematics*
Govt.ERR PG Sc.College Bilaspur

Abstract: In the present paper ,we prove existence of fixed point and contraction mapping in Hilbert spaces by iretates.
Keywords And Phrases: Hilbert space, fixed point,Contraction,Cauchy sequence

I. Introduction

In recent years some fixed points of various type of compability mapping in Hilbert space and Banach spaces were obtained, among others by Browder[1],Browder and Petryshyn[2],Hicks and Huffman[3] , Jungck[4].

II. Preliminaries

2.1 **Norm** : A norm on X is a real-valued function ||.|| : X→R defined on X such that for any x, y ∈ X and for all λ∈K

(a) $||x|| = 0$ if and only if $x = 0$
(b) $||x+y|| \le ||x|| + ||y||$
(c) $||\lambda x|| = |\lambda|\ ||x||$

2.2 **Normed Linear Space** : It is a pair (X, ||.||) consisting of a linear space X and a norm ||.||. We shall abbreviate normed linear space as nls.

2.3 Cauchy Sequence : A Sequence $\{x_n\}$ in a normed linear space X is a Cauchy sequence if for any given $\varepsilon > 0$, there exist $n_0 \in N$ such that $||x_m - x_n|| < \varepsilon$ for $m, n \ge n_0$

2.4 **Convergence Condition In Nls** : A sequence $\{x_n\}$ in a nls X is said to be Convergent to $x \in X$ if for any given $\varepsilon > 0$, ∃ $n_0 \in N$ such that $||x_n - x|| < \varepsilon$ for $n \ge n_0$

2.5 **Completeness** : A nls X is said to be complete if for every Cauchy Sequence in X converges to an element of X.

2.6 **Banach Space** : A Banach Space (X, ||.||) is a complete nls.

2.7 **Inner Product Space** : Let X be a linear space over the scalar field C of complex numbers. An inner product on X is a function (. , .) : XxX → C which satisfies the following conditions

(a) $(x, y) = \overline{(y, x)}$ for $x, y \in X$
(b) $(\lambda x + \mu y, z) = \lambda (x, z) + \mu (y, z)$ for $\lambda, \mu \in C$, $x, y, z \in X$
(c) $(x, x) \ge 0$; x x) = 0 iff $x = 0$

2.8 Law Of Parallelogram: If x and y are any two elements of an inner product space X then $||x + y||^2 + ||x-y||^2 = 2||x||^2 + 2||y||^2$

2.9 **Hilbert Space** : An infinite dimensional inner product space which is complete for the norm induced by the inner product is called Hilbert Space.

III. Material And Method

Theorem:Let C be a closed subset of a Hilbert space H. T,g:C $\rightarrow$C are contraction and continuous map of C , then { $Tg^n x$} converges to Ta. If T,g satisfying the following condition

$||Tgx-Tgy||^2 \leq \alpha ||Tx-Ty||^2$ for $\alpha \in [0,1)$

Then g has a fixed point a.

IV. Result And Discussion

Proof of theorem: It can be proof in four Steps

Step 1: $Lim_{n\to\infty} \left\| Tg^{n+1}x - Tg^n x \right\|^2 = 0$

Since $\left\| Tg^{n+1}x - Tg^n x \right\|^2 \leq \alpha^n \left\| Tgx - Tx \right\|^2$

Therefore $\left\| Tg^{n+1}x - Tg^n x \right\|^2$ =0 as $\alpha \in [0,1)$

Step 2: { $Tg^n x$} is bounded sequence

Suppose { $Tg^n x$} is unbounded then there exist $\{n(k)\}_{k=1}^{\infty}$ s.t.n(1)=1 and for each $k \in N$,n(k+1) is ‘minimal’

So $\left\| Tg^{n(k+1)}x - Tg^{n(k)} x \right\|^2 > 1$ and $\left\| Tg^{m}x - Tg^{n(k)} x \right\|^2 \leq 1$,

for all m = n(k)+1,n(k)+2,……,n(k+1)+1

But $1 < \left\| Tg^{n(k+1)}x - Tg^{n(k)} x \right\|^2 \leq \left\| Tg^{n(k+1)}x - Tg^{n(k+1)-1} x \right\|^2 + \left\| Tg^{n(k+1)-1}x - Tg^{n(k)} x \right\|^2$

$\leq \left\| Tg^{n(k+1)}x - Tg^{n(k+1)-1} x \right\|^2 +1$

So $\left\| Tg^{n(k+1)}x - Tg^{n(k)} x \right\|^2 \rightarrow 1$ as $k \rightarrow \infty$

Therefore $\left\| Tg^{n(k+1)}x - Tg^{n(k)} x \right\|^2 \leq \alpha \left\| Tg^{n(k+1)-1}x - Tg^{n(k)-1} x \right\|^2$ contradiction

Step 3: $\{Tg^n x\}_{n=1}^{\infty}$ is a Cauchy sequence

Since $\left\| Tg^{m}x - Tg^{n} x \right\|^2 \leq \alpha^n \left\| Tg^{m-n}x - Tx \right\|^2$ for $\alpha \in [0,1)$

Therefore $\left\| Tg^{m}x - Tg^{n} x \right\|^2$ =0

Step 4: G has A Unique Fixed Point

V. Conclusion

As $\{Tg^n x\}_{n=1}^{\infty}$ is a Cauchy sequence ,then $Tg^n x$ =Ta.

So $\left\| Tg^{n+1}x - Tga \right\|^2 \leq \alpha \left\| Tg^{n}x - Ta \right\|^2$

$\rightarrow 0$ for $\alpha \in [0,1)$

Therefore Tga=Ta or ga=a ,hence g has a fixed point a.

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Dr.Aradhana Sharma is working as a Asst.Professor , Dept. of Mathematics at Govt.Bilasa Girls PG Sc.College,Bilaspur(C.G.).Her papers more than 22 were published in various esteemed reputable national /International journals. She is a member of var ious professional bodies.

Dr.Gauri Shanker Sao: He is working as a Asst.Professor & Head, Dept. of Mathematics at Govt.ERR PG Sc.College,Bilaspur(C.G.).His papers more than 69 were published in various esteemed reputable national /International journals. He is a member of var ious professional bodies. He published two books in mathematics and one book in fundamental of computer.