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Some Domination Parameters on Interval Valued Intuitionistic Fuzzy Graphs

[1]S. Yahya Mohamed and [2] M. Saral Mary
[1]*Asst. Professor of Mathematics,P.G and Research Department of Mathematics, Govt. Arts College, Trichy-22.*
[2]*M.Phil Scholar, P.G and Research Department of Mathematics, Govt. Arts College, Trichy-22, India*

Abstract: *In this paper, we define Some domination parameters on IVIFG and derive some relations and then verify by using Interval valued intuitionistic fuzzy graphs and discussed some properties. Also, we introduced Immoderate semi-complete IVIFG and its properties.*
Keywords: Interval valued intuitionistic fuzzy set, Interval valued intuitionistic fuzzy graph, Domination on IVIFG, Split domination on IVIFG, global domination set on IVIFG, Semi-complete *IVIFG.*
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I. Introduction

The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers. Fuzzy set (FS) was introduced by Zadeh [14] in 1965, as a generalization of crisp sets. The concept of intuitionistic fuzzy sets (IFSs), as a generalization of fuzzy set was introduced by K. Atanassov [1] and defined new operations on intuitionistic fuzzy graphs. Later, K. Attannasov and G. Gargov [3] introduced the Interval valued intuitionistic fuzzy sets (IVIFSs) theory, as a generalization of both interval valued fuzzy sets(IVFSs) and intuitionistic fuzzy sets (IFSs) . Hence, many scholars applied IFS and IVIFS to decision analysis and pattern recognition widely.

By introducing the degree of membership $M_A(x)$, the degree of non-membership $N_A(x)$ and the degree of hesitancy $H_A(x)$, the IFS theory and IVIFS theory established. According to the IVIFS definition, $M_A(x)$, $N_A(x)$ and $H_A(x)$ are intervals, where $M_A(x)$ denotes the range of support party, $N_A(x)$ denotes the range of opposition party and $H_A(x)$ denotes the range of absent party. Atannasov's IVIFS is based on point estimation, which means that these intervals can be regarded as estimation result of an experiment. K. Atanassov [2] in 1994 introduced some operations over interval valued fuzzy sets (IVFSs). In 2000, Some Operations on intuitionistic fuzzy sets were proposed by De S. K., Biswas. R and Roy A. R [13]. In 2006, some operators defined over Interval-valued intuitionistic fuzzy sets were proposed by N. K. Jana and M. Pal [9]. In 2008, properties of some IFS operators and operations were proposed by Q. Liu, C. Ma and X. Zhou [8]. In 2011, Intuitionistic fuzzy sets: Some new results were proposed by R. K. Verma and B. D. Sharma [10]. In 2014, An Interval-valued Intuitionistic Fuzzy Weighted Entropy (IVIFWE) Method for Selection of Vendor was proposed by D. Ezhilmaran and S. Sudharsan [6]. In 2014, M. Ismayil[7] et.al. introduced the notion of strong IVIFG and discussed some operations. Recently, Yahya Mohamed and Jahir Hussain[14] discussed about constant IVIFG.

In this paper, we define Some domination parameters on IVIFG and derive some relations among them. Also, we introduced semi-complete IVIFG, Immoderate IVIFG with examples.

Cardinality of an intuitionistic fuzzy set was defined by Liu and Yu in [9]. Following the same line, we have defined cardinality of an interval-valued IFG.

II. Preliminary

Definition 2.1: An intuitionistic fuzzy graph is of the form G = (V, E), where

(i) $V=\{v_1,v_2,\dots,v_n\}$ such that μ_1: V→[0,1]and γ_1: V →[0,1] denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$ for every $v_i \in V$, (i = 1,2, ……. n),

(ii) $E \subseteq V \times V$ where μ_2: V x V →[0,1] and γ_2: V x V →[0,1] are such that $\mu_2(v_i, v_j) \le \min[\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \le \max[\gamma_1(v_i), \gamma_1(v_j)]$ and $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ for every $(v_i, v_j) \in E$, i, j = 1,2, …, n.

Definition 2.2: An interval valued intuitionistic fuzzy graph (IVIFG) with underlying set V is defined to be a pair G = (μ, γ), Where

(i) μ_1 : V→D[0,1]and γ_1 : V →D[0,1] denote the degree of membership and non-membership of the element $v_i \in V$, respectively, and $0 \le \mu_1^R(v_i) + \gamma_1^R(v_i) \le 1$ for every $v_i \in V$, (i = 1,2, …, n),

(ii) The function μ_2: $E \subseteq V \times V$ →D[0,1] and γ_2: $E \subseteq V \times V$ →D[0,1] where

$\mu_2^L(v_i,v_j) \le \min[\mu_1^L(v_i),\mu_1^L(v_j)]$ and $\mu_2^R(v_i,v_j) \le \min[\mu_1^R(v_i),\mu_1^R(v_j)]$
$\gamma_2^L(v_i,v_j) \le Max[\gamma_1^L(v_i),\gamma_1^L(v_j)]$ and $\gamma_2^R(v_i,v_j) \le Max[\gamma_1^R(v_i),\gamma_1^R(v_j)]$ such that
$0 \le \mu_2^R(v_i,v_j) + \gamma_2^R(v_i,v_j) \le 1$ for every $(v_i , v_j) \in E$, $i, j = 1,2, \ldots, n$.

Definition 2.3: An IVIFG is called Strong Interval valued intuitionistic fuzzy graph (SIVIFG) if $\mu_2^R(v_i,v_j) = \min[\mu_1^R(v_i),\mu_1^R(v_j)]$ $\mu_2^L(v_i,v_j) = \min[\mu_1^L(v_i),\mu_1^L(v_j)]$ and
$\gamma_2^L(v_i,v_j) = Max[\gamma_1^L(v_i),\gamma_1^L(v_j)]$, $\gamma_2^R(v_i,v_j) = Max[\gamma_1^R(v_i),\gamma_1^R(v_j)]$

Definition 2.4: Let G = (V, E) be an IVIFG. Then the degree of a vertex v is defined by $d(v) = (d_\mu(v), d_\gamma(v))$ where $d_\mu(v) = (\sum_{u \ne v} \mu_2^L(v,\ u), \sum_{u \ne v} \mu_2^R(v,\ u))$ and $d_\gamma(v) = (\sum_{u \ne v} \gamma_2^L(v,u), \sum_{u \ne v} \gamma_2^R(v,u))$

Definition 2.5: An edge e = (x, y) of an IVIFG G is called an effective edge if
$\mu_2^R(x,y) = \min[\mu_1^R(x),\mu_1^R(y)]$, $\mu_2^L(x,y) = \min[\mu_1^L(x),\mu_1^L(y)]$ and
$\gamma_2^L(x,y) = Max[\gamma_1^L(x),\gamma_1^L(y)]$, $\gamma_2^R(x,y) = Max[\gamma_1^R(x),\gamma_1^R(y)]$

Definition 2.6: An Interval valued Intuitionistic fuzzy graph is complete, if
$\mu_2^R(v_i,v_j) = \min[\mu_1^R(v_i),\mu_1^R(v_j)]$ $\mu_2^L(v_i,v_j) = \min[\mu_1^L(v_i),\mu_1^L(v_j)]$ and
$\gamma_2^L(v_i,v_j) = Max[\gamma_1^L(v_i),\gamma_1^L(v_j)]$, $\gamma_2^R(v_i,v_j) = Max[\gamma_1^R(v_i),\gamma_1^R(v_j)]$ for all $(v_i , v_j) \in V$.

Definition 2.7 : The complement of an IVIFG G = (V, E) is denoted by $\overline{G} = (\overline{V}, \overline{E})$ and is defined as
(i) $[\overline{\mu_1^L}(v), \overline{\mu_1^R}(v)] = [\mu_1^L(v), \mu_1^R(v)]$ and $[\overline{\gamma_1^L}(v), \overline{\gamma_1^R}(v)] = [\gamma_1^L(v), \gamma_1^R(v)]$

(ii) $[\overline{\mu_2^L}(u,v), \overline{\mu_2^R}(v)] = [Min(\mu_1^L(u),\mu_1^L(v)] - \mu_2^L(u,v), Min(\mu_1^R(u),\mu_1^R(v)] - \mu_2^R(u,v)]$ and
$[\overline{\gamma_2^L}(u,v), \overline{\gamma_2^R}(v)] = [Max(\gamma_1^L(u),\gamma_1^L(v)] - \gamma_2^L(u,v), Max(\gamma_1^R(u),\gamma_1^R(v)] - \gamma_2^R(u,v)]$
provided lower interval value less than or equal to upper intervalvalue, for all u, v in V

Example 2.8:

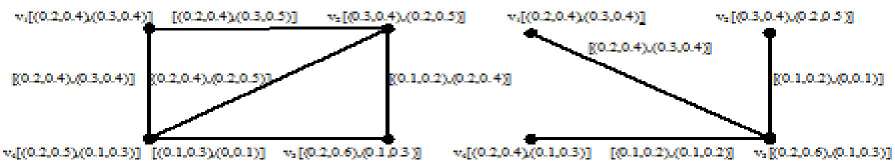

Fig -1: Interval Valued Intuitionistic Fuzzy Graph (IVIFG) and its complement

Definition 2.9: Let G be an IVIFG, then the cardinality of G is defined to be

$$|G| = \sum_{x \in V} \frac{1 + \mu_1^L(x) - \mu_1^R(x) + 1 + \gamma_1^L(x) - \gamma_1^R(x)}{2} + \sum_{xy \in E} \frac{1 + \mu_2^L(x,y) - \mu_2^R(x,y) + 1 + \gamma_2^L(x,y) - \gamma_2^R(x,y)}{2}$$

Definition 2.10: The vertex cardinality of IVFG G is defined by

$$|V| = \sum_{x \in V} \frac{1 + \mu_1^L(x) - \mu_1^R(x) + 1 + \gamma_1^L(x) - \gamma_1^R(x)}{2} = p$$

and the edge cardinality of IVFG G is defined by

$$|E| = \sum_{xy \in E} \frac{1 + \mu_2^L(x,y) - \mu_2^R(x,y) + 1 + \gamma_2^L(x,y) - \gamma_2^R(x,y)}{2} = q.$$

The vertex cardinality of IFG is called the order of G and denoted by O(G). The edge cardinality of G is called the size of G, denoted by S(G).

III. Domination On Interval Valued Intuitionistic Fuzzy Graph

Definition 3.1: Let G= (V, E) be an IVIFG. Let u, v ∈ V, we say that u dominated v in G if there exist a strong arc between them. A subset D ⊆V is said to be dominating set of IVIFG G if for every v ∈ V-D, there exist u in D such that u dominated v. The minimum scalar cardinality taken over all dominating sets of IVIFG is called domination number of IVIFG and is denoted by γ. The maximum scalar cardinality of a minimal domination set is called upper domination number of IVIFG and is denoted by the symbol Γ .

Definition 3.2: A dominating set D⊆V of IVIFG G is said to be a connected dominating set of IVIFG G if the subgraph <D> induced by D is connected. The minimum cardinality taken over all minimal connected dominating sets of IVIFG is called connected domination number of IVIFG G and it is denoted by $\gamma_c(G)$.

Definition 3.3: An independent set of an IVIFG G = (V, E) is a subset S of V such that no two vertices of S are adjacent in G.

Example 3.4

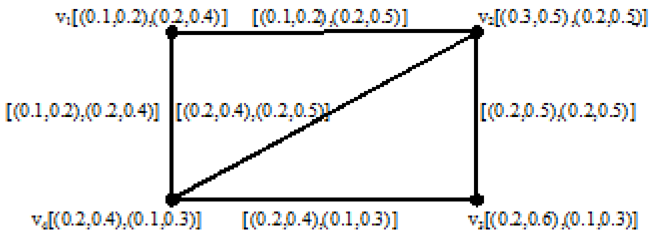

Fig-2: Purely Semi Complete IVIFG

Here, $|v_1| = 0.85$, $|v_2| = 0.75$, $|v_3| = 0.75$, $|v_4| = 0.8$ and The minimum domination sets D = {v2} , {v4} and therefore, The domination number $\gamma(G) = 0.75$ and $\Gamma(G) = 0.8$

Definition 3.5: A dominating set D of an Interval Valued Intuitionistic fuzzy graph G = (V,E) is a split dominating set if the induced fuzzy IVIF sub graph H = $(\langle V-D\rangle, V', E')$ is disconnected. The minimum cardinality of a IVIF split dominating set is called a split domination number of IVIFG and is denoted by $\gamma_s(G)$.

In fig-2, The dominating set in IVIFG is D = $\{v_2,v_4\}$and V-D =$\{v_1,v_3\}$for every v ϵ V-D. That is V-D is induced Intuitionistic fuzzy sub graph and is disconnected. The split domination number of IVIFG, $\gamma_s(G)$ = 1.55.

Definition 3.6: Let G = (V,E) be an IVIFG. An Interval valued Intuitionistic fuzzy dominating set S⊆V is said to be global Inter valued Intuitionistic fuzzy dominating set of G if S is also a inter valued Intuitionistic fuzzy dominating set of $\bar{G}$.

The minimum cardinality of global Interval valued Intuitionistic fuzzy dominating sets is global Interval valued Intuitionistic fuzzy domination number and is denoted by $\gamma_g(G)$.

In fig-2, minimum γ_g-sets is $\{v_1,v_2,v_4\}$ and $\{v_2,v_3,v_4\}$. Therefore $\gamma_g(G) = 2.3$

Theorem 3.7: Let G = (V,E) be an IVIFG without isolated vertices and D is a minimal dominating set. Then V- D is a dominating set of G.

Proof: Let D be a minimal dominating set. Let v be any vertex of D. Since G has no isolated vertices, there is a vertex d ϵ N(v). v must be dominated by at least one vertex in D-{v}.That is D-{v}is a dominating set. It follows that d ϵ V-D. Thus every vertex in D is dominated by at least one vertex in V-D, and V-D is a dominating set of IVIFG.

Theorem 3.8: A dominating set D of a Interval valued Intuitionistic fuzzy graph G is a split dominating set if and only if there exists two Intuitionistic fuzzy vertices u ,v ϵ V-D such that every u-v path contains an intuitionistic fuzzy vertex of D.

Proof: Let D be a split dominating set of an Interval valued Intuitionistic fuzzy graph G. Then induced Intuitionistic fuzzy sub graph V-D is disconnected. Hence there exists two vertices u ,υ ϵ V-D such that every u-v path contain a vertex of D in IVIFG.

Suppose there exists two vertices $\mathcal{U},\mathcal{V}$ ∈ V-D such that every u-v path contains a IVIF vertex of D. Let D be a dominating set then induced subgraph V-D is connected (or) disconnected. If it is connected then there exists u ,v two vertices in V-D such that some u - v path does not contain a fuzzy vertex of D, which is a contradiction. Hence V-D is disconnected, which implies D is a split dominating set of IVIFG G.

Theorem 3.9: A split domination set D of Interval valued Intuitionistic fuzzy graph G is minimal iff for each vertex v ϵ D one of the following conditions holds,

(i) There exists a vertex u ϵ V-D such that N(u) ∩ D = {v}

(ii) v is an isolate in $\langle D \rangle$

(iii) $\langle V - D' \rangle$ is connected.

Proof: Suppose that D is minimal and there exists a vertex v ϵ D such that v does not satisfy any of the above conditions (i),there exists a vertex u ϵV-D such that N(u)∩D ≠{v} and by condition(ii), v is not an isolate vertex of the induced subgraph $\langle D \rangle$.

Let D'=D-{v},then D is a split dominating set of IVIFG, which satisfies above two conditions. Hence the induced subgraph $\langle V - D' \rangle$ is disconnected. This contradicts the third condition. Which implies a vertex v is in D. Therefore D is minimal split dominating set of IVIFG, which satisfies one of the above conditions.

Theorem 3.10: For any Interval valued Intuitionistic fuzzy graph G = (V,E) with IVIF end vertex, then $\gamma_s(G) \geq \gamma(G)$, Furthermore, there exists a split dominating set of IVIFG G containing some vertices adjacent to IVIF end vertices.

Proof: Let v be a end vertex of Interval valued Intuitionistic fuzzy graph G, then there exists IVIF end vertex ‘w’ such that strong edge between v and w. Let D be a dominating set of IVIFG G. Suppose that w ϵ D then D is a split dominating set of IVIFG G.

Repeating this process for all such cut vertices adjacent to IVIF end vertices, we obtain a split dominating set of IVIFG G containing some vertices adjacent to the end vertices.

Example 3.11:

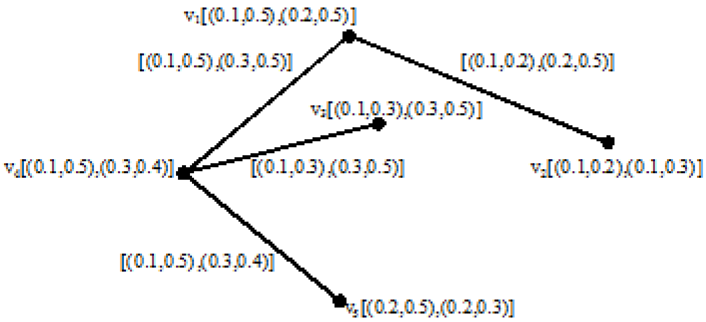

Fig – 3: IVIFG

Let D = {v_1,v_4}. Therefore, D is a dominating set with $\gamma(G) = 1.4$

Let D = {v_2,v_4}, V-D = {v_1,v_3,v_5} which is disconnected. Hence, D is a split dominating set of IVIFG with $\gamma_s(G) = 1.6$.

Hence $\gamma_s(G) \geq \gamma(G)$.

Theorem 3.12: Let G = (V,E) be the IVIFG and the Interval valued Intuitionistic fuzzy dominating set S of G is global IVIF dominating set iff for each v ϵ V-S, there exists uϵ S such that u is not adjacent to v.

Proof: Let S is global dominating set and also dominating set. Suppose u is adjacent to v, then we get S is not a dominating set. Which is contradiction. That is u is not adjacent to v.

Conversely, for each v ϵ V-S and u is not adjacent to v then the set S is dominating both G and $\bar{G}$. That is S is global Interval valued intuitionistic fuzzy dominating set.

Theorem 3.13: Let G = (V,E) be strongly connected IVIFG then G satisfy the following condition $\gamma_c(G) \leq \gamma_g(G)$.

Proof: Since γ_c set also dominating set and induced Interval valued intuitionistic fuzzy subgraph is connected then the G^c may be disconnected and it is less than or equal to γ_g set.

That is $\gamma_c(G) \leq \gamma_g(G)$.

In fig-2, Let D = {v_2 }, V-D = {v_1,v_3, v_4} which is connected dominating set. Therefore, $\gamma_c(G) = 0.75$. And let D_1 = {v_1,v_3,v_4}, D_2 = {v_2,v_3,v_4} which is global dominating set .The minimum γ_g set is {v_1,v_3,v_4}. Therefore, $\gamma_g(G) = 2.3$.

Hence $\gamma_c(G) \leq \gamma_g(G)$.

Definition 3.14 : An IVIFG G is said to be semi-complete if there exist a unique strong path of length 2 between any two vertices. That is any two vertices there is a common neighbor.

An IVIFG G is said to be purely semi-complete IVIFG if and only if G is semi-complete but not complete.

Definition 3.15: A semi-complete IVIFG G is said to be Immoderate semi-complete IVIFG (I.S.C. IVIFG) if and only if there is atleast one edge of G whose removal from G does not affect the semi-complete property.

A semi-complete IVIFG G is Immoderate semi-complete IVIFG iff there is an edge uv of G such that from u to any vertex of N(v) – {u} and v to N(u) – {v} there are at least two paths with two edges.

Example 3.16: Fig-2 is a semi-complete and also purely semi-complete IVIFG, but not a Immoderate semi-complete IVIFG. Also, Fig-3 is not even a semi-complete IVIFG.

Theorem 3.17: Let G be a purely semi-complete IVIFG, then

(i) Any subset V^1 of the vertex set V such that for any distinct pair of non-adjacent vertices in G there is a shortest path whose internal vertices are from V^1 is a domination set of IVIFG.

(ii) If the set S contain more than two vertices the S is total domination set of IVIFG.

Proof: Since any semi-complete IVIFG is connected, we have a set S which is non-empty. IF S having single vertex the every vertex of G is adjacent to that vertex. Thus S is a dominating set of IVIFG.

Assume that S contains more than two vertices. Let u ϵ V and v ϵ S-{u}. If u is adjacent to v then total dominating set of IVIFG. Otherwise since G is semi-complete IVIFG, there is a w ϵ S such that {u, w, v} is a shortest path in G. Therefore u is adjacent to w ϵ S. That is, S is total dominating set in IVIFG.

IV. Conclusion

In this paper, we defined some domination parameters on IVIFG and discussed some properties on it. The complement of IVIFG is defined and found some conditions on complement of IVIFG. Also we discussed semi complete IVIFG, purely semi-complete IVIFG and Immoderate IVIFG with examples. In future we going to derive some other domination parameters using semi complete IVIFG.

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