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"A Two Warehouse Inventory Model with Weibull Deterioration Rate and Time Dependent Demand Rate and Holding Cost"

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Abstract: *In this paper, we have analysed a two-warehouse inventory model for deteriorating items with quadratic demand with time varying holding cost. In the model considered here, shortages are allowed and partially backlogged. The backlogging rate is assumed to be dependent on the length of waiting for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. Numerical example is provided to illustrate the model and sensitivity analysis is also carried out for the parameters.*

I. Introduction

Inventory is a quantity or store of goods that is held for some purpose or use. Inventory management is the overseeing and controlling of the ordering, storage and use of components that a company will use in the production of the item. The objective of inventory management is to provide uninterrupted production, sales or customer service levels at minimum cost. In the inventory management, to decide where to stock the goods produced by a production system plays an important role. Every company, in general, has its own warehouse (OW) with a fixed capacity. If the quantity exceeds the capacity of OW then these quantities should be stored in another rented warehouse (RW). The customers are served first from RW and then from OW. The basic two-warehouse inventory model was, first, introduced by Hartley [1]. Researchers have developed inventory models by assuming constant demand rate for the items like electronic goods, vegetables, food stuffs, fashionable clothes etc., since the demand rate is always fluctuating and introducing new products will attract more in demand. Attempting the phenomenon of time-varying demand pattern in the deteriorating inventory models yields very much real time application. In the last three decades, the models for inventory replenishment policies involving time-varying demand patterns have received the attention of several researchers. Dave and Patel [2] first considered the inventory model for deteriorating items with time-varying demand.

Mishra and Singh [3] studied an inventory model with time dependent demand of deteriorating items considering partial backlogging. Vinodkumar Mishra [4] proposed an inventory model of constant demand with Weibull rate of deterioration. He incorporated variable holding cost considering shortages and salvage value. Venkateswarlu and Mohan [5] proposed an EOQ model with 2-parameter Weibull deterioration, time dependent quadratic demand and salvage value. Silver et al. [6] explained inventory management and production planning and scheduling. The term, deterioration is defined as damage, spoilage and dryness of items. Research in the field of deterioration began with the work of Whitin [7] where he considered fashion goods deterioration at the end of a prescribed storage period. Deteriorating items with linear trend in demand was formulated by Chakraborty and Choudhuri [8]. Many researchers like Giri and Chaudhuri [9], Hariga [10], Khanra and Choudhuri [11], Ghosh and Chaudhuri [12] and others discussed about deterioration and different types of demand. Chang and Dye [13] explored an EOQ model for deteriorating items with time varying demand and partial backlogging. Goyal and Giri [14] presented recent trends in modelling of deteriorating inventory. Teng and Chang [15] defined economic production quantity models for deteriorating items with price and stock dependent demand. Hariga [10] defined optimal EOQ models for deteriorating items with time – varying demand. In practice, some customers would like to wait for backlogging during the shortage period, but the others would not. Consequently, the opportunity cost due to lost sales should be considered.

Recently, Kirtan Parmar and U. B. Gothi [16] have developed an order level lot size inventory model for deteriorating items under quadratic demand with time dependent inventory holding cost and partial backlogging. Devyani Chatterji and U. B. Gothi [17] have developed an EOQ model for deteriorating items under two and three parameter Weibull distribution and constant inventory holding cost with partially backlogged shortages. Ankit Bhojak and U. B. Gothi [18] have developed an EOQ model for with time dependent demand and Weibull distributed deterioration.

Ajay Singh Yadav and Anupam Swami [19] have developed a two-warehouses inventory model in which they have assumed exponential demand. They have taken different inventory holding costs in both OW and RW and profit maximization technique is used.

In this paper, an effort has been made to develop a two-warehouse inventory model with quadratically increasing demand and time varying deterioration. Weibull distribution with three parameters is regarded as the deterioration rate. Backlogging rate is assumed to be a fixed fraction of demand rate during the shortage period. Here an attempt has been made to model the situation where the demand rate is a time-quadratic function and the items deteriorate at a constant rate in the interval[0, μ] and at a three parameter Weibull deterioration rate in the interval [μ, t_2] with partial backlogged shortages. A numerical example, graphical illustration and sensitivity analysis are used to illustrate the model.

II. Notations

We use the following notations for the mathematical model

1. $I_1(t)$: Inventory level for the rented warehouse (RW).
2. $I_2(t)$: Inventory level for the owned warehouse (OW).
3. $I_3(t)$: Inventory level for the backorder.
4. w : The capacity of the owned warehouse.
5. R(t) : Demand rate.
6. θ(t) : Rate of deterioration per unit time.
7. δ : The backlogging rate ($0 < \delta < 1$).
8. A : Ordering cost per order during the cycle period.
9. C_d : Deterioration cost per unit per unit time.
10. C_h : Inventory holding cost per unit per unit time.
11. Cs : Shortage cost per unit.
12. *l* : Opportunity cost due to lost sales per unit.
13. IM : The maximum inventory level during [0, T].
14. IB : The maximum inventory level during shortage period.
15. Q : Order quantity in one cycle.
16. p_c : Purchase cost per unit.
17. μ : The time at which the inventory level reaches zero in RW ($0 < \mu < t_2$).
18. t_2 : The time at which the inventory level reaches zero in OW ($\mu < t_2 < T$).
19. T : The length of cycle time.
20. TC : Total cost per unit time.

III. Assumptions

1. The annual demand rate is a quadratic function of time and it is $R(t) = a+bt+ct^2$ (a, b, c > 0)
2. The deterioration rate

$$\theta(t) = \theta \quad ; 0 < t < \mu$$
$$= \alpha\beta(t-\mu)^{\beta-1} \quad ; \mu \le t \le T \qquad (0 < \alpha << 1, \beta > 0, \mu > 0)$$

3. Holding cost is a linear function of time in the OW and RW and it is $C_h = h+rt$ (h, r > 0)
4. The OW has fixed capacity 'w' and RW has unlimited capacity.
5. First the units kept in RW are used and then of OW.
6. Replenishment rate is infinite and instantaneous.
7. Shortages occur and they are partially backlogged.
8. Repair or replacement of the deteriorated items does not take place during a given cycle.

IV. Mathematical Model And Analysis

At time t = 0 the inventory level is S units. From these 'w' units are kept in owned warehouse (OW) and rest in the rented warehouse (RW). Firstly the units kept in rented warehouse (RW) are consumed and then of owned warehouse (OW). Due to the market demand and deterioration of the items, the inventory level decreases during the period [0, μ] and the inventory in RW reaches to zero. Again with the same effects, the inventory level decreases during the period [μ, t_2] and the inventory in OW will become zero. Thereafter, shortages are allowed to occur during the time interval [t_2, T], and all of the demand during the period [t_2, T] is partially backlogged.

The pictorial presentation is shown in the Figure – 1.

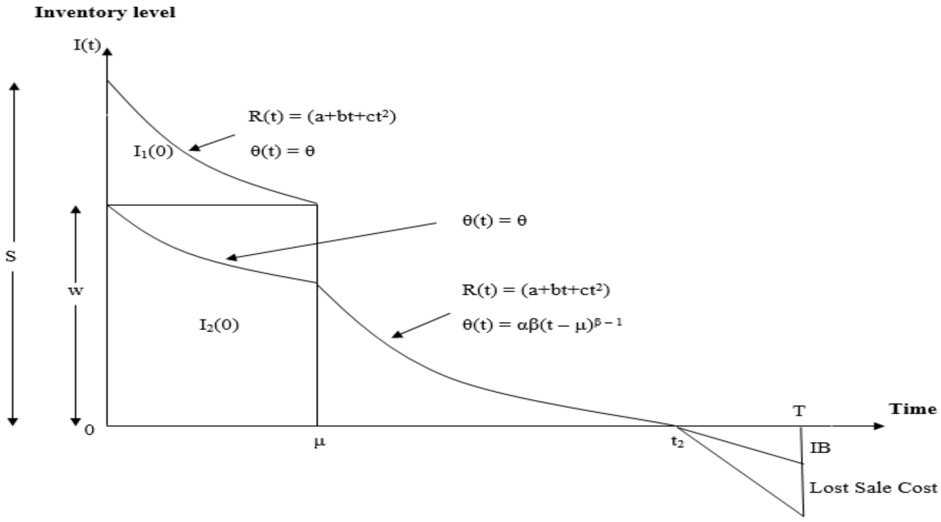

Figure – 1: Graphical presentation of the inventory system

The differential equations which describe the instantaneous state of I(t) over the period (0, T) are given by

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -(a + bt + ct^2) \qquad (0 \le t \le \mu) \qquad (1)$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = 0 \qquad (0 \le t \le \mu) \qquad (2)$$

$$\frac{dI_2(t)}{dt} + \alpha\beta(t-\mu)^{\beta-1} I_2(t) = -(a + bt + ct^2) \qquad (\mu \le t \le t_2) \qquad (3)$$

$$\frac{dI_3(t)}{dt} = -(a + bt + ct^2)e^{-\delta(T-t)} \qquad (t_2 \le t \le T) \qquad (4)$$

Under the boundary conditions $I_1(\mu) = 0$, $I_2(0) = w$, $I_2(t_2) = 0$, & $I_3(t_2) = 0$, solutions of equations (1) to (4) are given by

$$I_1(t) = c_1(1-\theta t) - \left\{at + \frac{1}{2}(b - a\theta)t^2 + \frac{1}{3}(2c - b\theta)t^3 - \frac{1}{12}c\theta t^4\right\} \qquad (0 \le t \le \mu) \qquad (5)$$

where, $c_1 = a\mu + \frac{1}{2}(b + a\theta)\mu^2 + \frac{1}{3}(c + b\theta)\mu^3 + \frac{1}{4}c\theta\mu^4$

$$I_2(t) = we^{-\theta t} \qquad (0 \le t \le \mu) \qquad (6)$$

$$I_2(t) = c_3\left(1 - \alpha(t-\mu)^\beta\right) - \left[k + (a + b\mu + c\mu^2)(t-\mu) + \left(\frac{b}{2} + c\mu\right)(t-\mu)^2 + \frac{c}{3}(t-\mu)^3 - \alpha k(t-\mu)^\beta + \frac{\beta}{\beta+1}(a + b\mu + c\mu^2)(t-\mu)^{\beta+1} + \frac{\beta(b + 2c\mu)}{2(\beta+2)}(t-\mu)^{\beta+2} + \frac{\beta c}{3(\beta+3)}(t-\mu)^{\beta+3}\right]$$

$$(\mu \le t \le t_2) \qquad (7)$$

where, $c_3 = \left(at_2 + b\frac{t_2^2}{2} + c\frac{t_2^3}{3}\right) + \alpha\left[(a + b\mu + c\mu^2)\frac{(t_2-\mu)^{\beta+1}}{\beta+1} + (b + 2c\mu)\frac{(t_2-\mu)^{\beta+2}}{\beta+2} + c\frac{(t_2-\mu)^{\beta+3}}{\beta+3}\right]$

$$I_3(t) = \frac{1}{\delta^2}(b\delta + c\delta T + c)(t - t_2) + \frac{c}{2\delta}(t^2 - t_2^2) + \varepsilon \ln\left[\frac{1+\delta(T-t)}{1+\delta(T-t_2)}\right] \qquad (t_2 \le t \le T) \qquad (8)$$

where $\varepsilon = \frac{1}{\delta^2}\left(a + bT + cT^2\right) + \frac{1}{\delta^2}(b + 2cT) + \frac{1}{\delta^3}c$

Now, Maximum inventory level

$$IM = S = I_1(0) + I_2(0)$$

$$\Rightarrow IM = S = a\mu + \frac{1}{2}(b + a\theta)\mu^2 + \frac{1}{3}(c + b\theta)\mu^3 + \frac{1}{4}c\theta\mu^4 + w \qquad (9)$$

$$IB = -I_3(T)$$

$$\Rightarrow IB = -\left[\frac{1}{\delta^2}(b\delta + c\delta T + c)(T - t_2) + \frac{c}{2\delta}(T^2 - t_2^2) - \varepsilon\ln\{1 + \delta(T - t_2)\}\right] \qquad (10)$$

Thus, the order size during total interval [0, T] is given by

$$Q = IM + IB$$

$$\Rightarrow Q = a\mu + \frac{1}{2}(b + a\theta)\mu^2 + \frac{1}{3}(c + b\theta)\mu^3 + \frac{1}{4}c\theta\mu^4 + w - \left[\frac{1}{\delta^2}(b\delta + c\delta T + c)(T - t_2) + \frac{c}{2\delta}(T^2 - t_2^2) - \varepsilon \ln\{1 + \delta(T - t_2)\}\right] \quad (11)$$

The total cost comprises of the following costs

1) Ordering Cost

The operating cost (OC) over the period [0, T] is

$$OC = A \quad (12)$$

2) Deterioration Cost

The deterioration cost (DC) over the period $[0, t_2]$ is

$$DC = C_d\left\{\int_0^{\mu}\theta I_1(t)\,dt + \int_0^{\mu}\theta I_2(t)\,dt + \int_{\mu}^{t_2}\alpha\beta(t-\mu)^{\beta-1}I_2(t)\,dt\right\}$$

$$\Rightarrow \quad DC = C_d\Bigg[\theta\left\{(w + c_1)\mu - \left(\frac{a}{2}\mu^2 + \frac{b}{6}\mu^3 + \frac{c}{12}\mu^4\right)\right\} + \alpha\beta\Bigg[(c_3 - k)\frac{(t_2-\mu)^{\beta}}{\beta} - \left\{(a + b\mu + c\mu^2)\frac{(t_2-\mu)^{\beta+1}}{\beta+1} + \left(\frac{b}{2} + c\mu\right)\frac{(t_2-\mu)^{\beta+2}}{(\beta+2)} + (c)\frac{(t_2-\mu)^{\beta+3}}{3(\beta+3)}\right\}\Bigg]\Bigg] \quad (13)$$

3) Inventory Holding Cost

The inventory holding cost (IHC) over the period $[0, t_2]$ is

$$IHC = \int_0^{\mu}(h + rt)I_1(t)\,dt + \int_0^{\mu}(h + rt)I_2(t)\,dt + \int_{\mu}^{t_2}(h + rt)I_2(t)\,dt$$

$$\begin{aligned}\Rightarrow IHC = &\Bigg[h\left\{c_1\left(\mu - \frac{1}{2}\theta\mu^2\right) - \left(\frac{1}{2}a\mu^2 + \frac{1}{6}(b - a\theta)\mu^3 + \frac{1}{24}(2c - b\theta)\mu^4 - \frac{1}{60}c\theta\mu^5\right)\right\}\\
&+ r\left\{c_1\left(\frac{1}{2}\mu^2 - \frac{1}{3}\theta\mu^3\right) - \left(\frac{1}{3}a\mu^3 + \frac{1}{8}(b - a\theta)\mu^4 + \frac{1}{30}(2c - b\theta)\mu^5 - \frac{1}{72}c\theta\mu^6\right)\right\}\Bigg]\\
&+ \left[w\left\{h\mu + \frac{1}{2}(r - \theta h)\mu^2 - \frac{1}{3}r\theta\mu^3\right\}\right]\\
&+ \Bigg[(h + r\mu)\Bigg[\left\{c_3(t_2 - \mu) - \frac{\alpha}{\beta+1}(t_2-\mu)^{\beta+1}\right\}\\
&- \left\{k(t_2-\mu) + \frac{1}{2}(a + b\mu + c\mu^2)(t_2-\mu)^2 + \left(\frac{b}{6} + \frac{c\mu}{3}\right)(t_2-\mu)^3 + \frac{c}{12}(t_2-\mu)^4\right\}\\
&- \alpha\Bigg\{\frac{k}{\beta+1}(t_2-\mu)^{\beta+1} + \frac{\beta(a + b\mu + c\mu^2)}{(\beta+1)(\beta+2)}(t_2-\mu)^{\beta+2} + \frac{\beta(b + 2c\mu)}{2(\beta+2)(\beta+3)}(t_2-\mu)^{\beta+3}\\
&+ \frac{\beta c}{3(\beta+3)(\beta+4)}(t_2-\mu)^{\beta+4}\Bigg\}\Bigg]\\
&+ r\Bigg[c_3\left\{\frac{(t_2-\mu)^2}{2} - \frac{\alpha}{\beta+2}(t_2-\mu)^{\beta+2}\right\}\\
&- \Bigg[\left\{\frac{k}{2}(t_2-\mu)^2 + \frac{(a + b\mu + c\mu^2)}{3}(t_2-\mu)^3 + \left(\frac{b}{8} + \frac{c\mu}{4}\right)(t_2-\mu)^4 + \frac{c}{15}(t_2-\mu)^5\right\}\\
&- \alpha\Bigg\{\frac{k}{\beta+2}(t_2-\mu)^{\beta+2} + \frac{\beta(a + b\mu + c\mu^2)}{(\beta+2)(\beta+3)}(t_2-\mu)^{\beta+3} + \frac{\beta(b + 2c\mu)}{2(\beta+2)(\beta+4)}(t_2-\mu)^{\beta+4}\\
&+ \frac{\beta c}{3(\beta+3)(\beta+5)}(t_2-\mu)^{\beta+5}\Bigg\}\Bigg]\Bigg]\Bigg] \end{aligned} \quad (14)$$

4) Shortage Cost

The shortage cost (SC) over the period $[t_2, T]$ is

$$SC = -C_S \int_{t_2}^{T} I_3(t)\,dt$$

$$\Rightarrow SC = C_S\left[-\frac{1}{6\delta^2}(3b\delta + 4c\delta T + 2c\delta t_2 + 3c)(t - t_2)^2 + \varepsilon(T - t_2) - \frac{1}{\delta}\varepsilon\ln\{1 + \delta(T - t_2)\}\right] \tag{15}$$

5) Lost Sale Cost

Opportunity cost due to lost sale (LSC) over the period $[t_2, T]$ is

$$LSC = l\int_{t_2}^{T}\left(1 - e^{-\delta(T-t)}\right)\left(a + bt + ct^2\right)dt$$

$$\Rightarrow LSC = l\left[\frac{1}{\delta^2}(a\delta^2 + b\delta + c\delta T + c)(T - t_2) + \left(\frac{b\delta + c}{2\delta}\right)(T^2 - t_2^2) + \frac{c}{3}(T^3 - t_2^3) - \varepsilon\ln\{1 + \delta(T - t_2)\}\right] \tag{16}$$

6) Purchase cost

The purchase cost (PC) during the period $[0, T]$ is

$$PC = p_c \cdot Q$$

$$\Rightarrow PC = p_C\left[c_1 + w - \left[\frac{1}{\delta^2}(b\delta + c\delta T + c)(T - t_2) + \frac{c}{2\delta}(T^2 - t_2^2) - \varepsilon\ln\{1 + \delta(T - t_2)\}\right]\right] \tag{17}$$

Hence the total cost per unit time is given by

$$TC = \frac{1}{T}[OC + DC + IHC + SC + LSC + PC] \tag{18}$$

Substituting the values of OC, DC, IHC, SC, LSC and PC from equations (12) to (17), we get

$$\begin{aligned}
\Rightarrow TC = \frac{1}{T}\Bigg[& A + C_d\Bigg[\theta\left\{(w + c_1)\mu - \left(\frac{a}{2}\mu^2 + \frac{b}{6}\mu^3 + \frac{c}{12}\mu^4\right)\right\} + \alpha(t_2 - \mu)^\beta(c_3 - k) \\
& - \alpha\beta\left\{(a + b\mu + c\mu^2)\frac{(t_2 - \mu)^{\beta+1}1}{\beta + 1} + \frac{1}{(\beta + 2)}\left(\frac{b}{2} + c\mu\right)(t_2 - \mu)^{\beta+2} \right.\\
& \left. + \frac{c}{3(\beta + 3)}(t_2 - \mu)^{\beta+3}\right\}\Bigg] \\
& + h\left\{c_1\left(\mu - \theta\frac{\mu^2}{2}\right) - \left(a\frac{\mu^2}{2} + \frac{(b - a\theta)}{6}\mu^3 + \frac{2c - b\theta}{24}\mu^4 - \frac{c\theta}{60}\mu^5\right)\right\} \\
& + r\left\{c_1\left(\frac{\mu^2}{2} - \frac{\theta}{3}\mu^3\right) - \left(\frac{a}{3}\mu^3 + \frac{(b - a\theta)}{8}\mu^4 + \frac{(2c - b\theta)}{30}\mu^5 - \frac{c\theta}{72}\mu^6\right)\right\} \\
& + w\left\{h\mu + \frac{(r - \theta h)}{2}\mu^2 - \frac{r\theta}{3}\mu^3\right\} \\
& + \Bigg[(h + r\mu)\Bigg[\left\{c_3(t_2 - \mu) - \frac{\alpha}{\beta + 1}(t_2 - \mu)^{\beta+1}\right\} - k(t_2 - \mu) - \frac{(a + b\mu + c\mu^2)}{2}(t_2 - \mu)^2 \\
& - \frac{\left(\frac{b}{2} + c\mu\right)}{3}(t_2 - \mu)^3 - \frac{c}{12}(t_2 - \mu)^4 - \frac{\alpha k}{\beta + 1}(t_2 - \mu)^{\beta+1} - \frac{\alpha\beta(a + b\mu + c\mu^2)}{(\beta + 1)(\beta + 2)}(t_2 - \mu)^{\beta+2} \\
& - \frac{\alpha\beta(b + 2c\mu)}{2(\beta + 2)(\beta + 3)}(t_2 - \mu)^{\beta+3} - \frac{\alpha\beta c}{3(\beta + 3)(\beta + 4)}(t_2 - \mu)^{\beta+4}\Bigg] \\
& + r\Bigg[c_3\left\{\frac{(t_2 - \mu)^2}{2} - \frac{\alpha}{\beta + 2}(t_2 - \mu)^{\beta+2}\right\} \\
& - \Bigg[\left\{\frac{k}{2}(t_2 - \mu)^2 + \frac{(a + b\mu + c\mu^2)}{3}(t_2 - \mu)^3 + \frac{\left(\frac{b}{2} + c\mu\right)}{4}(t_2 - \mu)^4 + \frac{c}{15}(t_2 - \mu)^5\right\} \\
& - \alpha\left\{\frac{k}{\beta + 2}(t_2 - \mu)^{\beta+2} + \frac{\beta(a + b\mu + c\mu^2)}{(\beta + 2)(\beta + 3)}(t_2 - \mu)^{\beta+3} + \frac{\beta(b + 2c\mu)}{2(\beta + 2)(\beta + 4)}(t_2 - \mu)^{\beta+4} \right.\\
& \left. + \frac{\beta c}{3(\beta + 3)(\beta + 5)}(t_2 - \mu)^{\beta+5}\right\}\Bigg]\Bigg]\Bigg] \\
& + C_s\left[-\frac{(3b\delta + 4c\delta T + 2c\delta t_2 + 3c)}{6\delta^2}(t - t_2)^2 + \varepsilon(T - t_2) - \frac{\varepsilon}{\delta}\ln\{1 + \delta(T - t_2)\}\right] \\
& + l\Bigg[\frac{(a\delta^2 + b\delta + c\delta T + c)}{\delta^2}(T - t_2) + \left(\frac{b\delta + c}{2\delta}\right)(T^2 - t_2^2) + \frac{c}{3}(T^3 - t_2^3) \\
& - \varepsilon\ln\{1 + \delta(T - t_2)\}\Bigg] \\
& + p_C\left\{c_1 + w - \left[\frac{(b\delta + c\delta T + c)}{\delta^2}(T - t_2) + \frac{c}{2\delta}(T^2 - t_2^2) - \varepsilon\ln\{1 + \delta(T - t_2)\}\right]\right\}\Bigg]
\end{aligned} \tag{19}$$

Our objective is to determine optimum values μ*, t_2* and T* of μ, t_2 and T respectively so that cost function TC is minimum. Note that μ*, t_2* and T* are the solutions of the equations $\frac{\partial TC}{\partial \mu} = 0, \frac{\partial TC}{\partial t_2} = 0, \frac{\partial TC}{\partial T} = 0$

which can satisfy the following sufficient conditions:

$$\left.\begin{array}{l} \left|\begin{matrix} \frac{\partial^2 TC}{\partial \mu^2} & \frac{\partial^2 TC}{\partial \mu \partial t_2} & \frac{\partial^2 TC}{\partial \mu \partial T} \\ \frac{\partial^2 TC}{\partial t_2 \partial \mu} & \frac{\partial^2 TC}{\partial t_2^2} & \frac{\partial^2 TC}{\partial t_2 \partial T} \\ \frac{\partial^2 TC}{\partial T \partial \mu} & \frac{\partial^2 TC}{\partial T \partial t_2} & \frac{\partial^2 TC}{\partial T^2} \end{matrix}\right|_{\mu=\mu^*,\ t_2=t_2^*,\ T=T^*} > 0 \\ \left|\begin{matrix} \frac{\partial^2 TC}{\partial \mu^2} & \frac{\partial^2 TC}{\partial \mu \partial t_2} \\ \frac{\partial^2 TC}{\partial t_2 \partial \mu} & \frac{\partial^2 TC}{\partial t_2^2} \end{matrix}\right|_{\mu=\mu^*,\ t_2=t_2^*,\ T=T^*} > 0 \\ \text{and } \left.\frac{\partial^2 TC}{\partial \mu^2}\right|_{\mu=\mu^*,\ t_2=t_2^*,\ T=T^*} > 0 \end{array}\right\} \qquad (20)$$

The optimal values μ*, t_2* and T* can be obtained by using Maple software.
The above developed model is illustrated by means of the following numerical example.

V. Numerical Example

To illustrate the proposed model, an inventory system with the following hypothetical values is considered. By taking a = 4, b = 3, c = 2, α = 0.001, β = 8, δ = 0.02, h = 1, r = 0.5, θ = 0.01, ℓ = 9, w = 400, C_p = 20, C_d = 10, Cs = 7 and *A* = 500 (with appropriate units).

The optimal values of μ, t_2 and T are μ* = 2.184643676, t_2* = 4.482121695, T* = 5.686851905 units and the optimal total cost per unit time TC = 2283.594995 units.

VI. Sensitivity Analysis

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes in its input parameter values. Here, we study the sensitivity for the total cost per time unit TC with respect to the changes in the values of the parameters a, b, c, α, β, δ, h, r, θ, ℓ, w, C_p, C_d, Cs and A.

The sensitivity analysis is performed by considering variation in each one of the above parameters keeping all other remaining parameters as fixed.

Table – 1: Partial Sensitivity Analysis

Parameter	**%**	**□**	**t_2**	**T**	**TC**	**% TC**
a	– 10	2.187309599	4.484921594	5.692517868	2277.446457	– 0.27
	– 05	2.185979995	4.483524799	5.689685571	2280.522910	– 0.13
	+ 05	2.183300724	4.480712357	5.684016880	2286.674839	+ 0.13
	+ 10	2.181951220	4.479296863	5.681180504	2289.736943	+ 0.27
b	– 10	2.159912359	4.460348820	5.710928437	2270.686964	– 0.57
	– 05	2.170226862	4.469457991	5.701329672	2275.863867	– 0.34
	+ 05	2.195767482	4.491831394	5.674727224	2289.958884	+ 0.28
	+ 10	2.206148282	4.500837621	5.662559354	2296.284485	+ 0.56
c	– 10	2.071884682	4.382632222	5.801983443	2242.948988	– 1.78
	– 05	2.136738914	4.440056700	5.737604838	2265.677230	– 0.78
	+ 05	2.227617710	4.519551682	5.638439501	2300.644598	+ 0.75
	+ 10	2.266209198	4.552873307	5.592143278	2316.895715	+ 1.46
α	– 10	2.188259508	4.507573712	5.691553592	2279.106396	– 0.20
	– 05	2.186918566	4.497807187	5.689747927	2280.836967	– 0.12
	+ 05	2.182329006	4.467116603	5.684086089	2286.190815	+ 0.11
	+ 10	2.180925237	4.458414646	5.682484159	2287.699467	+ 0.18
β	– 10	2.174047834	4.674251825	5.721539905	2246.687270	– 1.62
	– 05	2.183323317	4.575293092	5.703609148	2266.277940	– 0.76
	+ 05	2.179877133	4.394265645	5.671173760	2298.965287	+ 0.67
	+ 10	2.170416791	4.311258274	5.656478084	2312.655851	+ 1.27
δ	– 10	2.190523348	4.487933261	5.686249212	2283.863705	+ 0.01
	– 05	2.187592327	4.485036162	5.686550123	2283.731671	+ 0.01
	+ 05	2.181678922	4.479191370	5.687154536	2283.463338	– 0.01
	+ 10	2.178695143	4.476242301	5.687458093	2283.325245	– 0.01
Parameter	**%**	**□**	**t_2**	**T**	**TC**	**% TC**
	– 10	2.340449950	4.632075738	5.722847459	2264.913603	– 0.82

h	– 05	2.264483970	4.558950831	5.705304547	2274.405244	– 0.40
	+ 05	2.100260347	4.400949211	4.400949211	2292.460295	+ 0.39
	+ 10	2.010466056	4.314603742	5.646618617	2300.995397	+ 0.76
r	– 10	2.399913482	4.690899420	5.749624123	2270.554763	– 0.57
	– 05	2.291396333	4.585604650	5.717544837	2277.340782	– 0.27
	+ 05	2.079875034	4.380651514	5.657565865	2289.322712	+ 0.25
	+ 10	1.977339978	4.281423698	5.629709639	2294.567556	+ 0.48
θ	– 10	2.196729197	4.493523738	5.689490283	2282.166167	– 0.06
	– 05	2.190701024	4.487836754	5.688174778	2282.889279	– 0.03
	+ 05	2.178556704	4.476378132	5.685521560	2284.292082	+ 0.03
	+ 10	2.172439651	4.470605625	5.684183635	2285.011153	+ 0.06
ℓ	– 10	2.181856988	4.479367367	5.687165668	2283.440873	– 0.007
	– 05	2.183252277	4.480746447	5.687008653	2283.518235	– 0.004
	+ 05	2.186031201	4.483493127	5.686695424	2283.665305	+ 0.003
	+ 10	2.187414871	4.484860762	5.686539210	2283.740394	+ 0.006
w	– 10	2.296151804	4.578963060	5.562816659	2115.224366	– 7.37
	– 05	2.243079922	4.533268078	5.626663265	2200.082538	– 3.66
	+ 05	2.120636282	4.425343907	5.743614443	2365.776659	+ 3.60
	+ 10	2.050768252	4.362672032	5.797146119	2446.690956	+ 7.14
p_c	– 10	1.891256116	1.891256116	5.571059252	2103.017912	– 7.91
	– 05	2.043940883	4.340081127	5.631824237	2193.799868	– 3.93
	+ 05	2.315224381	4.614455747	5.736855580	2372.464783	+ 3.89
	+ 10	2.437158814	4.738478524	5.782434518	2460.491007	+ 7.75
C_d	– 10	2.201590708	4.514352772	5.693409554	2278.548880	– 0.22
	– 05	2.193216841	4.498131222	5.690116199	2281.106768	– 0.11
	+ 05	2.175886109	4.466299059	5.683613201	2285.988190	+ 0.10
	+ 10	2.166956217	4.450641063	5.680396943	2288.313591	+ 0.21
Cs	– 10	2.063344547	4.362279373	5.699884433	2277.192870	– 0.28
	– 05	2.127503180	4.425655147	5.693149730	2280.528397	– 0.13
	+ 05	2.235906797	4.532798806	5.680957652	2286.394746	+ 0.12
	+ 10	2.282189393	4.578567416	5.675434850	2288.973650	+ 0.24
A	– 10	2.173283922	4.470894181	5.674750896	2274.795294	– 0.39
	– 05	2.178972664	4.476516592	5.680808817	2279.202120	– 0.19
	+ 05	2.190297073	4.487709603	5.692880246	2287.982416	+ 0.19
	+ 10	2.195932972	4.493280429	5.698893920	2292.370961	+ 0.38

VII. Graphical Presentation

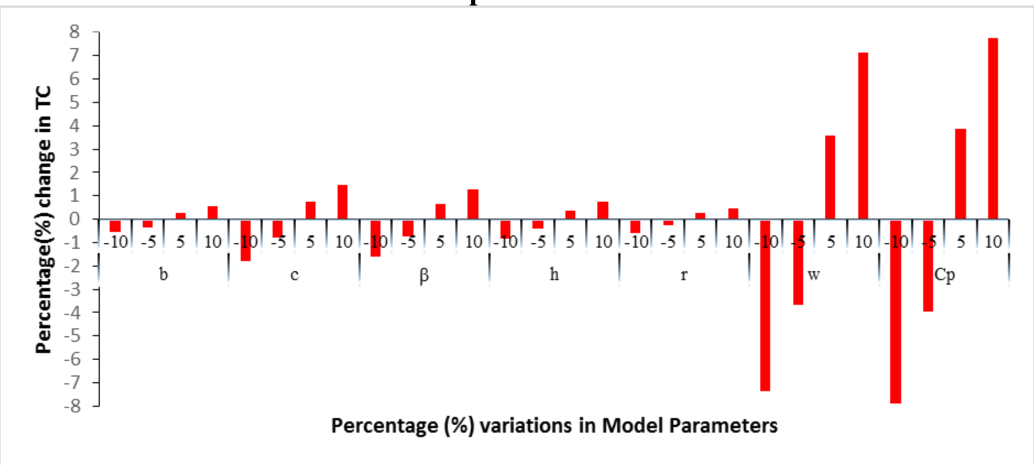

Figure – 2

VIII. Conclusions

- From the **Table – 1**, we observe that as the values of the parameters a, b, c, α, β, h, r, θ, ℓ, w, C_p, C_d, Cs, and A increase the average total cost increases and if the values of the parameter δ increases the average total cost decreases.
- **Figure – 2** shows that the total cost per time unit is highly sensitive to changes in the values of w and C_p**.**
- We observe from **Figure – 2** that the total cost per time unit is moderately sensitive to changes in the values of b, c, β, h, and r.
- Also, From the **Figure - 2** the total cost per time unit is less sensitive to changes in the values of a, α, δ, ℓ, θ, C_d, Cs and A.

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