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The Inventory Model for Deteriorating items with Triangular type Demand Rate

K. D. Rathod[1] And P. H. Bhathawala[2]
[1]*Assistant Professor, Department of Mathematics, Shri R. R. Lalan College, Bhuj, Gujarat State, India.*
[2]*Professor, Department of Mathematics, Auro University, Surat, Gujarat State, India.*

Abstract: *In this paper, we study the inventory model for deteriorating items with triangular type demand rate, that is, the demand rate is a piecewise linear function. The inventory system consists of several replenishments and all the ordering cycles are of fixed length. We have considered two cases in first case we have considered that* $0 < t_1 < \lambda$ *and in the other we have taken* $\lambda \leq t_1 < T$*; Where* t_1 *is the time when the inventory level reaches zero and* λ *is time point changing from the increasing linearly demand to decreasing linearly demand. In first case* $C(t_1)$ *obtains its minimum at* $t_1 = {t_1}^*$ *and the optimal order quantity denoted by* Q^* *is* $Q^* = S^* + \Delta_1$*, where* S^* *denotes the optimal value of S. In second case* $C(t_1)$ *obtains its minimum at* $t_1 = \lambda$ *and the optimal order quantity denoted by* Q^* *is* $Q^* = S^* + \Delta$*, where* S^* *denotes the optimal value of S.*

I. Introduction

There are many researchers who studied inventory models for deterioration items like Plastic items, kid's items, sports items, electronic components, food items, drugs and fashion goods. Deterioration is defined as decay, change or spoilage of items from being used for its original purpose. A few examples of items in which appreciable deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the model. Therefore, many authors have considered economic order quantity (EOQ) models for deteriorating items. The inventory model with ramp type demand rate was pro-posed the first time by Hill (1995). The ramp type demand is very commonly seen when some fresh fruit come to the market. In case of ramp type demand rate, the demand increases linearly at the beginning and then the market grows into a stable stage such that the demand becomes a constant until the end of the inventory cycle.

Hill (1995) first considered the inventory models for increasing demand followed by a constant demand. He derived the exact solution to compare with the Silver-Meal heuristic. Mandal and Pal (1998) extended the inventory model with ramp type demand for deterioration items and allowing shortage. Wu and Ouyang (2000) extended the inventory model to include two different replenishment policies: (a) models starting with no shortage and (b) models starting with shortage. Deng, Lin, and Chu (2007) point out some questionable results of Mandal and Pal (1998) and Wu and Ouyang (2000), and then resolved the similar problem by offering a rigorous and efficient method to derive the optimal solution. Wu (2001) further investigated the inventory model with ramp type demand rate such that the deterioration followed the Weibull distribution. Giri, Jalan, and Chaudhuri (2003) ex-tended the ramp type demand inventory model with a more generalized Weibull deterioration distribution. Various types of order-level inventory models for deteriorating items at a constant rate with time-dependent demand were discussed recently. Readers can consult the performance work by Wu, Lin, Tan, and Lee (1999), Manna and Chaudhuri (2006), Benkherouf (1995), Gosw-ami and Chaudhuri (1991), Hariga (1996), Panda, Senapati, and Basu (2008).

In following, Mingbao Cheng and Guoqing Wang's trapezoidal type demand rate model is extended to the Triangular type demand. The inventory system consists of several replenishments and all the ordering cycles are of fixed length, and consider only one ordering cycle in this paper. Such type of demand pattern is generally seen in the case of any fad or seasonal goods coming to market. The demand rate for such items increases with the time up to certain time and finally the demand rate approximately decrease to a constant or zero, and then begin the next replenishment cycle. We think that such type of demand rate is quite realistic and a useful inventory replenishment policy for such type of inventory model is also proposed. We think that our work will provide a solid foundation for the further study of this kind of important inventory models with Triangular type demand rate.

The rest of the paper is organized as follows. In Section 2, we de-scribe the assumptions and notation used throughout this paper. In Section 3, we establish the mathematical model with shortage in inventory and the necessary conditions to find an optimal solution. In Section 4, we use some numerical examples to illustrate the solution procedure. Finally, we make a summary and provide some suggestions for future research.

II. Notations and assumption

1) The replenishment rate is infinite, thus, replenishment is instantaneous.
2) The demand rate, R(t), which is positive and consecutive, is assumed to be a Triangular type function of time, that is,

$$R(t) = \begin{cases} a_1 + b_1 t\ ; t \leq \lambda \\ a_2 - b_2 t\ ; \lambda \leq t \leq T \end{cases}$$

Where λ is time point changing from the increasing linearly demand to decreasing linearly demand (see Fig. 1).

3) I(t) is the level of inventory at time t, $0 \leq t \leq T$
4) T is the fixed length of each ordering cycle.
5) β is the constant deteriorating rate, $0 < \beta < 1$.
6) t_1 is the time when the inventory level reaches zero.
7) A is the fixed ordering cost per order.
8) c is the cost of each deteriorated item.
9) H is the inventory holding cost per unit per unit
10) SC is the shortage cost per unit per time unit.
11) S is the inventory level for the ordering cycle, such that S= I(0).
12) Q is the ordering quantity per cycle

III. Mathematical Formulation

We consider the deteriorating inventory model with triangular type demand rate. Replenishment occurs at time t = 0 so at t = 0 inventory level reaches to maximum.

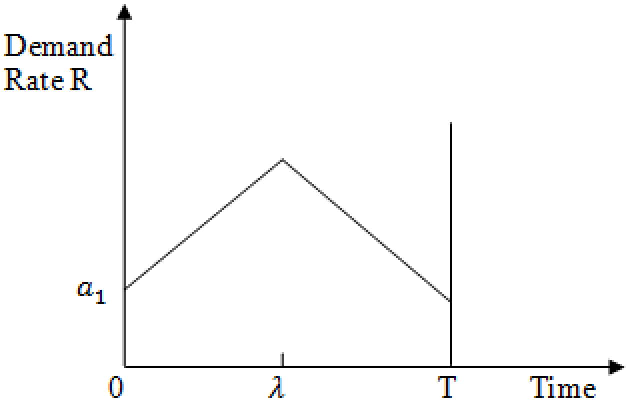

Fig. 1. A triangular type demand rate function.

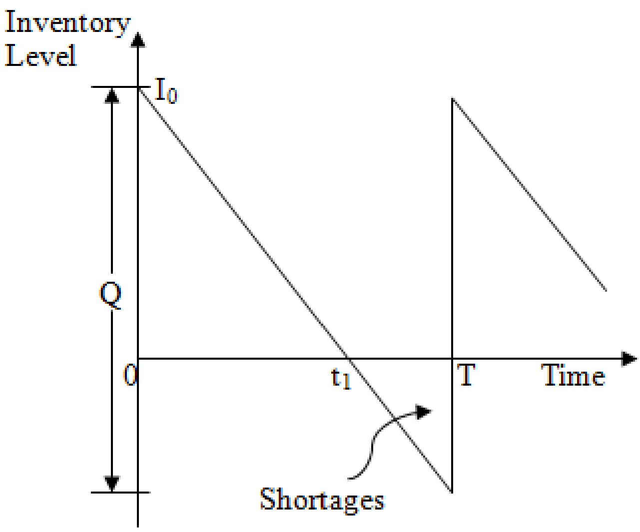

Fig. 2. Inventory level vs Time

From time t = 0 to t = t_1 the inventory level reduces due to demand and deterioration (See Fig. 2.). At time t = t_1, the inventory level achieves zero, and then shortages occurs during the time period (t_1, T). The demand during the shortage period is completely backlogged and satisfied by the next replenishment. The behavior of inventory system at any time can be described by the following differential equations:

$$\frac{dI(t)}{dt} = -\beta I(t) - R(t); 0 < t < t_1 \qquad 3.1$$

Here; $-\beta I(t)$ denotes the deterioration of inventory
$-R(t)$ denotes the decreasing demand rate

$$\frac{dI(t)}{dt} = -R(t); t_1 < t < T \qquad 3.2$$

This is the shortage period and such that all of the demand backlogged.
With the boundary condition $I(t_1) = 0$.
In follows we consider two possible cases based on the values of t_1 and λ.

Case 1
Let us consider $0 < t_1 < \lambda$.

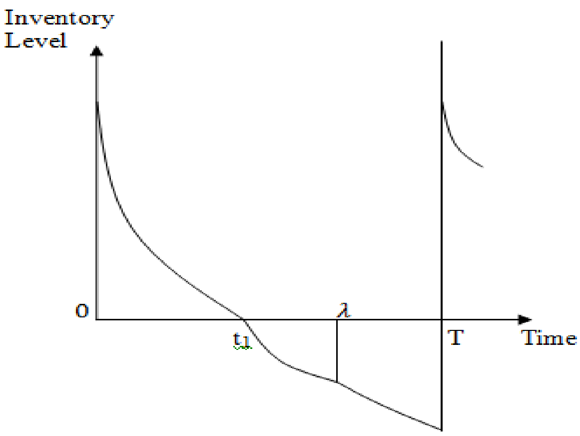

Fig. 3.

Due to triangular type market demand and deteriorating items the inventory level gradually diminishes during the time period$[0, t_1]$and ultimately falls to zero at timet_1. Then from equation 3.1, we have

$$\frac{dI(t)}{dt} = -\beta I(t) - (a_1 + b_1 t);\ 0 < t < t_1 \qquad 3.3$$

$$\frac{dI(t)}{dt} = -(a_1 + b_1 t);\ \ t_1 < t < \lambda \qquad 3.4$$

$$\frac{dI(t)}{dt} = -(a_2 - b_2 t);\ \lambda < t < T \qquad 3.5$$

Solving the equations 3.3 to 3.5 with $I(t_1) = 0$, we have

$$I(t) = \left(\frac{a_1 + b_1 t_1}{\beta} - \frac{b_1}{\beta^2}\right) e^{\beta(t_1 - t)} - \left(\frac{a_1 + b_1 t}{\beta}\right) + \frac{b_1}{\beta^2}; 0 \le t \le t_1 \qquad 3.6$$

$$I(t) = a_1(t_1 - t) + \frac{b_1}{2}({t_1}^2 - t^2); t_1 \le t \le \lambda \quad 3.7$$

$$I(t) = a_1(t_1 - \lambda) + \frac{b_1}{2}\left({t_1}^2 - \lambda^2\right) - a_2(t - \lambda) - \frac{b_2}{2}\left(t^2 - \lambda^2\right);\ \lambda \le t \le T \qquad 3.8$$

The initial inventory level can be computed as follows

$$I(0) = S = \left(e^{\beta t_1} - 1\right)\left(\frac{a_1}{\beta} - \frac{b_1}{\beta^2}\right) - \frac{b_1 t_1}{\beta} e^{\beta t_1} \quad 3.9$$

- The total number of items that perish in the interval $[0, t_1]$, say R_T is

$$R_T = S - \int_0^{t_1} R(t)dt$$

$$= S - \int_0^{t_1} (a_1 + b_1 t)dt$$

$$= \left(e^{\beta t_1} - 1\right)\left(\frac{a_1}{\beta} - \frac{b_1}{\beta^2}\right) - \frac{b_1 t_1}{\beta} e^{\beta t_1} - a_1 t - \frac{b_1 t^2}{2} \quad 3.10$$

- The total number of inventory carried during the interval $[0, t_1]$, say H_T is

$$H_T = \int_0^{t_1} I(t)dt$$

$$= \int_0^{t_1} \left[\left(\frac{a_1 + b_1 t_1}{\beta} - \frac{b_1}{\beta^2}\right) e^{\beta(t_1 - t)} - \left(\frac{a_1 + b_1 t}{\beta}\right) + \frac{b_1}{\beta^2}\right] dt$$

$$= \left(\frac{a_1 + b_1 t_1}{\beta^2} - \frac{b_1}{\beta^3}\right)\left(e^{\beta t_1} - 1\right) - \left(\frac{a_1 \beta - b_1}{\beta^2}\right) t_1 - \frac{b_1}{2\beta} {t_1}^2 \quad 3.11$$

- The total quantities short during the interval $[t_1, T]$, say SQ, is

$$SQ = -\int_{t_1}^{T} I(t)dt$$

$$= -\int_{t_1}^{\lambda} \left[a_1(t_1 - t) + \frac{b_1}{2}({t_1}^2 - t^2)\right] dt - \int_{\lambda}^{T} \begin{bmatrix} a_1(t_1 - \lambda) + \frac{b_1}{2}\left({t_1}^2 - \lambda^2\right) \\ -a_2(t - \lambda) - \frac{b_2}{2}\left(t^2 - \lambda^2\right) \end{bmatrix} dt$$

$$= \frac{a_1}{2}(t_1 - \lambda)(t_1 + \lambda - 2T) + \frac{b_1}{6}\left(2{t_1}^3 - 2\lambda^3 + 2\lambda^2 T - 2{t_1}^2 T\right) + \frac{a_2}{2}(T - \lambda)T + \frac{b_2}{6}(T - \lambda)\left(T^2 + 2\lambda - 2\lambda^2\right) \quad 3.11$$

The average total cost per unit time under the condition $0 < t_1 < \lambda$. Can be given as follows:

$$C(t_1) = \frac{1}{T}[A + cR_T + hH + SC(SQ)] \quad 3.12$$

For finding the minimum cost let us differentiate equation 3.12 with respect to t_1 we get

$$\frac{dC(t_1)}{dt_1} = \frac{1}{T}\left[0 + c\left[\left(e^{\beta t_1}\beta\right)\left(\frac{a_1}{\beta} - \frac{b_1}{\beta^2}\right) - \frac{b_1}{\beta} e^{\beta t_1} - \frac{b_1 t_1}{\beta} e^{\beta t_1}\right] + h\left[\left(\frac{a_1 + b_1 t_1}{\beta^2} - \frac{b_1}{\beta^3}\right)\left(e^{\beta t_1}\beta\right) + \right.\right.$$

b1β3eβt1−1−a1β−b1β2t1−b1βt1+ SCa12t1−λ+a12t1+λ−2T+
b166t12−6t1T

$$= \frac{1}{T}\left[\left(c + \frac{h}{\beta}\right)\left(e^{\beta t_1} - 1\right) + SC(t_1 - T)\right](a_1 + b_1 t_1) \quad 3.13$$

Now the necessary condition for a function being extremum is $\frac{dC(t_1)}{dt_1} = 0$.

So, we get

$$\frac{1}{T}\left[\left(c+\frac{h}{\beta}\right)\left(e^{\beta t_1}-1\right)+SC(t_1-T)\right](a_1+b_1t_1)=0 \qquad 3.14$$

From this equation we can say that

$$\left[\left(c+\frac{h}{\beta}\right)\left(e^{\beta t_1}-1\right)+SC(t_1-T)\right]=0$$

$$OR$$

$$(a_1+b_1t_1)=0$$

Let

$$f(t_1)=\left[\left(c+\frac{h}{\beta}\right)\left(e^{\beta t_1}-1\right)+SC(t_1-T)\right]$$

$$f'(t_1)=\left[\left(c+\frac{h}{\beta}\right)\left(e^{\beta t_1}\beta\right)+SC\right]>0$$

So we can say that $f(t_1)$ is strictly increasing function and since $f(0)=SC(-T)<0$ *and*

$$f(T)=\left(c+\frac{h}{\beta}\right)\left(e^{\beta T}-1\right)>0$$

So $f(t_1)$ has unique solution say t_1^* within $(0,T)$.

Conclusion 1

The deteriorating inventory model under the condition $0<t_1<\lambda$, $C(t_1)$ obtains its minimum at $t_1=t_1^*$.

From the above conclusion we can also say that the total back-order amount at the end of the cycle is $\Delta=\frac{a_1}{2}(t_1^*-\lambda)(t_1^*+\lambda-2T)+\frac{b_1}{6}\left(2t_1^{*3}-2\lambda^3+2\lambda^2T-2t_1^{*2}T\right)+\frac{a_2}{2}(T-\lambda)T+\frac{b_2}{6}(T-\lambda)\left(T^2+2\lambda-2\lambda^2\right)$.

Therefore the optimal order quantity denoted by Q^* is $Q^*=S^*+\Delta$, where S^* denotes the optimal value of S.

Case 2.

Let us consider $\lambda\le t_1<T$.

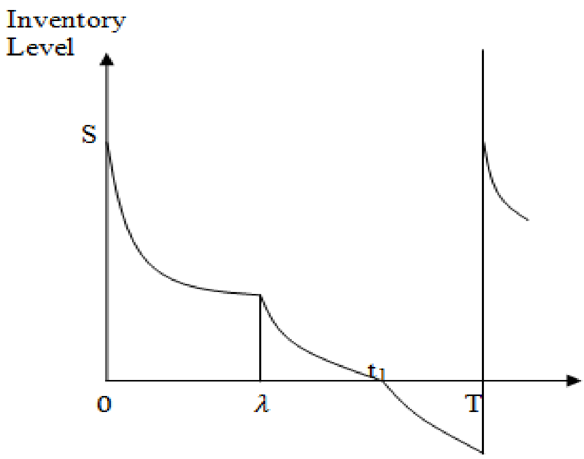

Fig. 4.

$$\frac{dI(t)}{dt}=-\beta I(t)-(a_1+b_1t);\ 0<t<\lambda \qquad 3.15$$

$$\frac{dI(t)}{dt}=-\beta I(t)-(a_2+b_2t);\ \lambda<t<t_1 \qquad 3.16$$

$$\frac{dI(t)}{dt}=-\beta I(t)-(a_2+b_2t);\ t_1<t<T \qquad 3.17$$

Solving the above equations with $I(t_1)=0$, we have

$$I(t) = \left(\frac{b_1}{\beta^2} + \frac{a_1 + b_1\lambda}{\beta}\right)e^{\beta(\lambda - t)} - \frac{a_1 + b_1 t}{\beta} - \frac{b_1}{\beta^2};\ 0 \le t \le \lambda \qquad 3.18$$

$$I(t) = \left(\frac{-b_2}{\beta^2} + \frac{a_2 - b_2\lambda}{\beta}\right)e^{\beta(\lambda - t)} - \frac{a_2 - b_2 t}{\beta} + \frac{b_2}{\beta^2};\ \lambda \le t \le t_1 \qquad 3.19$$

$$I(t) = a_2(t_1 - t) + \frac{b_2}{2}(t^2 - {t_1}^2); t_1 \le t \le T \quad 3.20$$

$$S = I(0) = \left(\frac{b_1}{\beta^2} + \frac{a_1 + b_1\lambda}{\beta}\right)e^{\beta\lambda} - \frac{a_1}{\beta} - \frac{b_1}{\beta^2} \quad 3.21$$

- The total number of items that perish in the interval $[0, t_1]$, say R_T is

$$R_T = S - \int_0^{t_1} R(t)dt$$

$$= S - \left[\int_0^{\lambda} (a_1 + b_1 t)dt + \int_{\lambda}^{t_1} (a_2 - b_2 t)dt\right]$$

$$= \left(\frac{b_1}{\beta^2} + \frac{a_1 + b_1\lambda}{\beta}\right)e^{\beta\lambda} - \frac{a_1}{\beta} - \frac{b_1}{\beta^2} - (a_1 - a_2)\lambda - (b_1 - b_2)\frac{\lambda^2}{2} + a_2 t_1 - \frac{b_2 {t_1}^2}{2} \qquad 3.22$$

- The total number of inventory carried during the interval $[0, t_1]$, say H_T is

$$H_T = \int_0^{t_1} I(t)dt$$

$$= \int_0^{\lambda} \left(\frac{b_1}{\beta^2} + \frac{a_1 + b_1\lambda}{\beta}\right)e^{\beta(\lambda - t)} - \frac{a_1 + b_1 t}{\beta} - \frac{b_1}{\beta^2}dt + \int_{\lambda}^{t_1} \left(\frac{-b_2}{\beta^2} + \frac{a_2 - b_2\lambda}{\beta}\right)e^{\beta(\lambda - t)} - \frac{a_2 - b_2 t}{\beta} + \frac{b_2}{\beta^2}dt$$

$$= \left(\frac{b_2 - b_1}{\beta^3}\right) - \left(\frac{(a_1 + a_2) + (b_1 - b_2)\lambda}{\beta^2}\right) - \left(\frac{\left((a_1 - a_2) + (b_1 + b_2)\right)\lambda}{\beta}\right) - \frac{(b_1 + b_2)\lambda^2}{2\beta} + \frac{e^{\beta\lambda}}{\beta}\left(\frac{b_1}{\beta^2} + \frac{a_1 + b_1\lambda}{\beta} + \frac{b_2}{\beta^2} - \frac{a_2 - b_2\lambda}{\beta}\right) - \left(\frac{-b_2}{\beta^3} + \frac{a_2 - b_2\lambda}{\beta}\right)e^{-\beta t_1} - \left(\frac{2a_2 - b_2 t_1}{\beta}\right)t_1 + \frac{b_2 t_1}{\beta^2} \qquad 3.23$$

- The total quantities short during the interval $[t_1, T]$, say SQ, is

$$SQ = -\int_{t_1}^{T} I(t)dt$$

$$= -\int_{t_1}^{T} a_2(t_1 - t) + \frac{b_2}{2}(t^2 - {t_1}^2)dt$$

$$= (t_1 - T)\left[a_2 t_1 - \frac{a_2(t_1 + T)}{2} + \frac{b_2(T^2 - t_1 T + {t_1}^2)}{2} - \frac{b_2 {t_1}^2}{2}\right] 3.24$$

The average total cost per unit time under the condition $\lambda \le t_1 \le T$. Can be given as follows

$$C(t_1) = \frac{1}{T}[A + cR_T + hH + SC(SQ)] \qquad 3.25$$

For finding the minimum cost let us differentiate equation 3.12 with respect to t_1 we get

$$\frac{dC(t_1)}{dt_1} = \frac{1}{T}\left\{0 + c[a_2 - b_2 t_1] + h\left[\frac{a_2 - b_2 t_1}{\beta} + \frac{b_2}{\beta^2} + \beta\left(\frac{-b_2}{\beta^3} + \frac{a_2 - b_2\lambda}{\beta}\right)e^{-\beta t_1}\right] + SC\left((a_2 - b_2 t_1)(t_1 - T)\right)\right\} = 0 \quad 3.26$$

$$\therefore\ \frac{(a_2 - b_2 t_1)}{T}[c + h + SC(t_1 - T)] + h\left[\frac{b_2}{\beta} + \left(\frac{-b_2}{\beta^2} + a_2 - b_2\lambda\right)e^{-\beta t_1}\right] = 0$$

Conclusion 2

The deteriorating inventory model under the condition $\le t_1 < T$, $C(t_1)$ obtains its minimum at $t_1 = \lambda$.

From the above conclusion we can also say that the total back-order amount at the end of the cycle is $\Delta_1 = (t_1{}^* - T)\left[a_2 t_1{}^* - \frac{a_2(t_1{}^* + T)}{2} + \frac{b_2\left(T^2 - t_1{}^* T + t_1{}^{*2}\right)}{2} - \frac{b_2 t_1{}^{*2}}{2}\right]$. Therefore the optimal order quantity denoted by Q^* is $Q^* = S^* + \Delta_1$, where S^* denotes the optimal value of S.

3. Numerical examples

In this part we provide some numerical examples to illustrate the above theory.

Example 1. The parameter values is given as follows: T = 10 weeks, λ = 3 weeks, a_1= 100 unit, b_1= 5 unit, a_2 = 200 unit, b_2= 10 unit, β = 0.2, A = 200, c = 3 per unit, h = 10 per unit, SC = 5 per unit. Based on the solution procedure as above, we have $f(\lambda) = 8.57 > 0.$ then it yields that the optimal replenishment time $t_1{}^* = 2.235$ weeks, the optimal order quantity, Q*, for each ordering cycle, is 1468.39836 unit.

Example 2. The parameter values is given as follows: T = 10 weeks, λ = 1 weeks, a_1= 100 unit, b_1= 5 unit, a_2 = 200 unit, b_2= 10 unit, β = 0.2, A = 200, c = 3 per unit, h = 10 per unit, SC = 5 per unit. Based on the solution procedure as above, we have $f(\lambda) = -1.4277 < 0.$ then it yields that the optimal replenishment time $t_1{}^* =$ 2.235 weeks, the optimal order quantity, Q*, for each ordering cycle, is 1348.39836 unit.

IV. Conclusion

In this paper, we study the inventory model for deteriorating items with triangular type demand rate, that is, the demand rate is a piecewise linearly functions. We proposed an inventory replenishment policy for this type of inventory model. From the market information, we can find that the triangular type demand rate model is more applicable than trapezoidal and ramp type demand rate model in the stage of product life cycle. Of course, the paper provides an interesting topic for the further study of such kind of important inventory models, and at same time, the following three problems can be considered in our future research, either. (1) In Deng et al. (2007), they considered the ramp type demand, and in this paper, we extended to trapezoid demand. However, for the inventory models starting with stock, why these two inventory models have the same optimal solution? (2) There is no set up cost (ordering cost) in this inventory model. What will happen, if we add set up cost, for example, A = 200, into this inventory model? (3) How about the inventory model stating with shortages?

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