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Srinivasa Ramanujan's Contributions in Mathematics

Dharminder Singh[1], Arun Kumar Chopra[2], Sukhdeep Singh Bal[3]
[1,2]*Asst. Professor Guru Nanak College, Budhlada, Dist. Mansa (Pb.), India*
[3]*Student of B.Sc. III (N.M.) Guru Nanak College, Budhlada, Dist. Mansa (Pb.), India*

Abstract: *The Indian govt. celebrated 125^{th} anniversary of the great Mathematician of Indian soil Srinivasa Ramanujan on 22 December in the year 2012. Without any formal education and extreme poverty conditions, he emerged as one of great mathematician of India.His mathematical ideas transformed and reshaped 20^{th} century mathematics and their ideas are inspiration for 21^{st} century mathematicians. His excellence can be realized from the fact that he discovered some results which are supposed to be true but have not been proved till date. His foresightedness was so scintillating that he gave those ideas in Mathematics that no one can imagine to invent them.*

Keywords: *G.H. Hardy, Highly composite numbers, Partition function, Ramanujan.*

I. Introduction

Ramanujan, one of the elegant Mathematician of India was born in Erode on 22nd December 1887.Erode is a small village (in that time), 400 Km away from Tamilnadu's present capital Chennai. His father was a clerk in Kumbkonam.At the age of five, SrinivasaRamanujan made his first appearance in school as a student. It was only a matter of time before it came to be known that he had extraordinary talent. He showed flashes of brilliance which were not to be seen in any ordinary kid at that age. He completed his primary education in a couple of years and then went to Town High School for further studies. He showed extraordinary liking for mathematics. When he was yet in school, he mathematically calculated the approximate length of earth's equator. He very clearly knew the values of the square root of two and the pie value. At the age of 16, he got scholarship. But his love only for mathematics cost him the scholarship as he neglected and failed in other subjects. His loss of scholarship was a great blow to him. He could not afford to study on his own. He had to find work and leave studies for good. He found a job of an accounts clerk in the office of the Madras Port Trust. Despite being rejected two times, his work was recognized by both G. H. Hardy and J. E. Littlewood and he went to England in 1914. In 1916,he was awarded with a degree of B.Sc. (later named Ph.D.) by Cambridge University for his work on highly composite number.In 1916, when he was at his best while working with his colleagues Hardy & Littlewood, he met with health problems. He was hospitalized in Cambridge and was diagnosed with T.B. and vitamin deficiency. After two years struggle, in 1919, he showed some recovery and he decided to return back to India. However, the improvement was temporary and after his arrival at Bombay, his health deteriorated again and finally he passed away on 26thApril, 1920.His main contributions in mathematics lie in the field of Analysis, Infinite Series, Number Theory & Game Theory.His geniusness was that he discovered his own theorems.Due to his great achievements in the field of Mathematics, Indian govt. decided to celebrated his birthday 22ndDecember as Mathematics Day. ISTE, New Delhi and NBHM, Mumbai have taken initiative to hold Mathematical competition for students as well as teachers of colleges on the name of Srinivasa Ramanujan from 2012 to till date so that students and teachers of India know about the legacy of such great mathematician of India.

II. Hardy-Ramanujan Number

Once Hardy visited to Putney where Ramanujan was hospitalized. He visited there in a taxi cab having number 1729. Hardy was very superstitious due to his such nature when he entered into Ramanujan's room, he quoted that he had just came in a taxi cab having number 1729 which seemed to him an unlucky number but at that time, he prayed that his perception may go wrong as hewanted that his friend would get well soon, but Ramanujan promptly replied that this was a very interesting number as it is the smallest number which can be expressed as the sum of cubes of two numbers in two different ways as given below:

$$1729 = 1^3 + 12^3 = 10^3 + 9^3$$

Later some theorems were established in theory of elliptic curves which involves this fascinating number.

Infinite Series Forπ:

Srinivasa Ramanujan also discovered some remarkable infinite series of π around 1910. The series

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k!)(1103 + 26390k)}{(k!)^4\, 396^{4k}}$$

computes a further eight decimal places of π with each term in the series.Later on, a number of efficient algorithms have been developed bynumber theorists using the infinite series of π given by Ramanujan.

III. Goldbach's Conjecture

Goldbach's conjecture is one of the important illustrations of Ramanujan contribution towards the proof of the conjecture. The statement is every even integer > 2 is the sum of two primes, that is, 6=3+3. Ramanujan and his associates had shown that every large integer could be written as the sum of at most four (Example: 43=2+5+17+19).

IV. Theory Of Equations

Ramanujan was shown how to solve cubic equations in 1902 and he went on to find his own method to solve the quadratic. He derived the formula to solve biquadratic equations.The following year, he tried to provide the formula for solving quintic but he couldn't as he was not aware of the fact that quintic could not be solved by radicals.

V. Ramanujan-Hardy Asymptotic Formula

Ramanujan's one of the major work was in the partition of numbers. By using partition function $p(n)$, he derived a number of formulae in order to calculate the partition of numbers. In a joint paper with Hardy, Ramanujan gave an asymptotic formulas for$p(n)$.In fact, a careful analysis of the generating function for $p(n)$ leads to the Hardy-Ramanujan asymptotic formulagiven by

$$p(n) \sim \frac{1}{4n\sqrt{3}} e^{\pi\sqrt{\frac{2n}{3}}},\ n \to \infty$$

In their proof, they discovered a new method called the 'circle method' which made fundamental use of the modular property of the Dedekind η-function. We see from the Hardy-Ramanujan formula that $p(n)$has exponential growth. It had the remarkable property that it appeared to give the correct value of $p(n)$and this was later proved by Rademacher using special functions andthen KenOno gave the algebraic formula to calculate partition function for any natural number n.

VI. Ramanujan's Congruences

Ramanujan's congruences are some remarkable congruences for the partition function. He discovered the congruences

$$p(5n + 4) \equiv 0(mod\ 5)$$
$$p(7n + 5) \equiv 0(mod\ 7)$$
$$p(11n + 6) \equiv 0\ (mod\ 11),\ \forall n \in N.$$

In his 1919 paper, he gave proof for the first two congruencesusing the following identities using Pochhammer symbol notation. After the death of Ramanujan, in 1920, the proof of all above congruences extracted from his unpublished work.

VII. Highly Composite Numbers

A natural number n is said to behighly composite number if it has more divisors than any smaller natural number. If we denote the number of divisors of n *by* $d(n)$, then we say $n \in$ N is called a highly composite if $d(m) < d(n)\ \forall m < n$ where$m \in$ N.For example, $n = 36$is highly composite because it has $d(36) = 9$ and smaller natural numbers have less number of divisors.If

$n = 2^{k_2} 3^{k_3} \ldots\ldots p^{k_p}$ (by Fundamental theorem of Arithmetic)

is the prime factorization of a highly composite number $n$then the primes$2, 3, \ldots, p$form a chain of consecutive primes where the sequence of exponents is decreasing; i.e.$k_2 \geq k_3 \geq \ldots\ldots \geq k_p$and the final exponent $k_p$is 1, except for $n = 4$ and $n = 36$.

VIII. Some Other Contributions

Apart from the contributions mentioned above, he worked in some other areas of mathematics such as hypogeometric series, Bernoulli numbers, Fermat's lasttheorem. He focused mainly on developing the relationship between partial sums and products of hyper-geometric series. He independently discovered Bernoulli numbers and using these numbers, he formulated the value of Euler's constant up to 15 decimal places. He nearly verified Fermat's last theorem which states that no three natural number x, y and z satisfy the equation$x^n + y^n = z^n$ for any integer $n > 2$.

IX. Srinivasa Ramanujan's Publications

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