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An Optimal Model for Deteriorating Items with Selling Price Demand and Time-Varying Holding Cost

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Abstract: *Inventory models in which the demand rate depends on the inventory level are based on the common real-life observation that greater product availability tends to stimulate more sales. In this model we discussed a selling price dependent demand rate with a time varying holding cost and shortages are not allowed. The holding cost per unit of the item per unit time is assumed to be an increasing function of the time spent in storage. At the end of the paper, numerical examples are provided to illustrate the problem and sensitivity analyses have been carried out for showing the effect of variation in the parameters.*
Keywords: *Constant Deterioration, Selling price dependent demand, Time varying holding cost.*

I. Introduction

Many authors have considered economic order quantity models for deteriorating items. Ghare&Schrader(1963) were the first who studied inventory models of deteriorating items. They assumed the constant market demand. With the passage of time several other researchers developed the inventory model for deteriorating items with dependent demand rate and the consumption of the deteriorating items was closely relative to a negative exponential function of time. They established the classical EOQ model with constant deterioration rate without shortages. Shah Y. K. and Jaiswal M. C. (1977) studied an order-level inventory model for a system with constant rate of deterioration.Dave and Patel (1997) studied firstly the inventory model on deteriorating items with linear increasing demand when shortages were not allowed. S. K. Goyal and B. C. Giri (2001) derived trends in deteriorating inventory models. Li and et al (2010)discussed the review on deteriorating inventory model.

Muhlemann A. P. and Valtis-Spanopoulos N. P. (1980),reviewed the variable holding cost EOQ model. Weiss H. J. (1982) discussed EOQ model with non-linear holding cost. J. Chang and C. Y. Dye(1999) discussed an EOQ Model for Deteriorating Items with Time Varying Demand and Partial Backlogging.V. K. Mishra and et al (2013) presentedan Inventory Model for Deteriorating Items with Time-Dependent Demand and Time-Varying Holding Cost under Partial Backlogging.R. Amutha and E. Chandrasekaran (2013)analyzed an EOQ Model for Deteriorating Items with Quadratic Demand and Time Dependent Holding Cost. In the classical inventory model, the demand rate may be time dependent, price dependent and stock dependent.Burwell (1997) discussed that an Economic lot size model for price dependent demand under quantity and freight discounts. Mondal et al. (2003) presented an inventory system of ameliorating items for price dependent demand rate.You (2005) developed an inventory model with price and time dependent demand, Teng et al. (2005) developed an inventory model with price dependent demand rate. Ajantha Roy (2008) developed that an inventory model where demand rate is function of selling price.

In this paper, we developed an economic order quantityinventory model for deteriorating items and selling price demand. Shortages are not allowed. Holding cost is assumed to be linear. This model is illustrates with numerical example.

II. Assumptions and Notations

- The inventory system considers a single item only.
- The demand rate is the function of selling price $(a\text{-}p) > 0$ where p is the selling price per unit time and a is a parameter used in demand function which hold the conditiona $> p$.
- θbe the deterioration rate. It follows the constant deterioration $\theta(t) = \theta$ where $0<\theta<1$.
- The inventory system is considered over a finite time horizon.
- Lead time is zero, Shortages are not allowed.
- A0, Cd describes the fixed ordering cost per order and the cost of each deteriorated unit.
- I(t): the inventory at time t, $0 \le t \le T$.
- h(t): the time-varying holding cost per unit time where $h(t) = h + \alpha t$, $h > 0$ and $\alpha > 0$.

- T, T* , TC,TC* denotes the length of the cycle, the an optimal length of the cycle, total cost per unit time, the minimum total cost per unit time.
- I0, I0* describes the economic order quantity and the optimal economic order quantity.

III. Mathematical Formulation and Solution of the Model

We consider the inventorymodel of deteriorating items with selling price demand rate. Let I(t) be the inventory level at time t ($0 \le t \le T$). The differential equations to describe instantaneous state over [0,T] are given by

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -(a-p),\ 0 \le t \le T \quad (3.1)$$

With boundary condition I(t) = 0

Solution of equation (3.1) is given by

$$I(t) = (a-p)[(T-t) + \theta(\frac{T^2}{2} + \frac{t^2}{2} - tT) + \theta^2(\frac{T^3}{6} - \frac{t^3}{6} - \frac{tT^2}{2} + \frac{t^2T}{2})]\ 0 \le t \le T \quad (3.2)$$

Putting the boundary condition I(0) = I_0 in equation (2) therefore,

$$I_0 = I(0) = (a-p)[T + \frac{\theta T^2}{2} + \frac{\theta^2 T^3}{6}] \quad (3.3)$$

The ordering cost is (OC) = $A_{0.}$

The total demand during the cycle period [0,T] is

$$\int_0^T D(t)dt = \int_0^T (a-p)dt$$

$$= (a-p)T$$

The number of deteriorated units is

$$I_0 - \int_0^T D(t)dt = (a-p)[T + \frac{\theta T^2}{2} + \frac{\theta^2 T^3}{6}] - (a-p)T$$

$$= (a-p)[\frac{\theta T^2}{2} + \frac{\theta^2 T^3}{6}]$$

The deterioration cost (DC) for the cycle [0.T] = $C_d \times$ (the number of deteriorated units)

$$= C_d[(a-p)[\frac{\theta T^2}{2} + \frac{\theta^2 T^3}{6}]]$$

$$= C_d(a-p)[\frac{\theta T^2}{2} + \frac{\theta^2 T^3}{6}] \quad (3.4)$$

The total inventory holding cost (HC) for the cycle [0.T] is ,

$$= \int_0^T (h+\alpha t)I(t)dt$$

$$= \int_0^T (h+\alpha t)[(a-p)[(T-t) + \theta(\frac{T^2}{2} + \frac{t^2}{2} - tT) + \theta^2(\frac{T^3}{6} - \frac{t^3}{6} - \frac{tT^2}{2} + \frac{t^2T}{2})]]dt$$

$$= (a-p)[\frac{hT^2}{2} + \frac{\theta hT^3}{6} + \frac{\theta^2 hT^4}{24} + \frac{\alpha T^3}{6} + \frac{\theta\alpha T^4}{24} + \frac{\theta^2\alpha T^5}{120}] \quad (3.5)$$

Total variable cost=Ordering cost(OC) + Deterioration cost(DC)+Holding cost (HC)

Therefore, the total variable cost per unit time TC(T) is

$$\frac{A_0}{T} + \frac{C_d(a-p)}{T}[\frac{\theta T^2}{2} + \frac{\theta^2 T^3}{6}] + \frac{(a-p)}{T}[\frac{hT^2}{2} + \frac{\theta hT^3}{6} + \frac{\theta^2 hT^4}{24} + \frac{\alpha T^3}{6} + \frac{\theta\alpha T^4}{24} + \frac{\theta^2\alpha T^5}{120}] \quad (3.6)$$

Our aim is to find minimum the variable cost per unit time.

The necessary and sufficient conditions to minimize TC(T) for a given value T are respectively

$\frac{\partial TC(T)}{dT}=0$ and $\frac{\partial^2 TC(T)}{\partial T^2}>0$.

Now $\frac{\partial TC(T)}{dT}=0$ gives the following equation in T:

$$\frac{\partial TC(T)}{\partial T}=\frac{-A_0}{T^2}+C_d(a-p)[\frac{\theta}{2}+\frac{\theta^2 T}{3}]+(a-p)[\frac{h}{2}+\frac{\theta hT}{3}+\frac{\theta^2 hT^2}{8}+\frac{\alpha T}{3}+\frac{\theta\alpha T^2}{8}+\frac{\theta^2\alpha T^3}{30}] \quad (3.7)$$

Provided that it satisfies the following condition $\frac{\partial^2 TC(T)}{\partial T^2}>0$

Again,

$$\frac{\partial^2 TC(T)}{\partial T}=\frac{2A_0}{T^3}+C_d(a-p)[\frac{\theta^2}{3}]+(a-p)[\frac{\theta h}{3}+\frac{\theta^2 hT}{4}+\frac{\alpha}{3}+\frac{\theta\alpha T}{4}+\frac{\theta^2\alpha T^2}{10}] \quad (3.8)$$

IV. Numerical Example

Consider an inventory system with the following parameter in proper unit A_0=500, θ=0.8, h=0.5, α = 0.2, C_d = 1 a =100, p=60.Solving equation (3.7) with the above parameters, we get T^* = 2.44875 and then using this in equation (3.6) and (3.3), we obtain the minimum total cost per unit time TC^* = 330.7044 and the economic order quantity

I_0^* = 256.5422 respectively.

V. Sensitivity Analysis

To study the effects of changes in the system parameters, A_0, θ, h, α, C_d, a, and p on the optimal cost and the number of reorder.The sensitivity analysis is performed by changing each of the parameters by 50%, 25%, -25%, and -50% taking oneparameter at a time, keeping the remaining parameters at their original values.

Case (i)

Using the above said parameters and only varying the parameter A_0, we get,

TABLE-1

Parameter	% change in parameter	T^*	TC^*	I_0^*
A_0	+50	2.80791	425.6488	332.9240
	+25	2.64158	379.7900	295.9569
	-25	2.21685	277.1690	213.7882
	-50	1.92017	216.8447	166.0067

Result

Increasing the value of the parameter A_0, the value of T^* , TC^*,and I_0^* be increased. Here T^*, TC^* and I_0^* are moderately sensitive to change in A_0.

Case (ii)

Using the above said parameters and only varying the parameter h we get,

TABLE-2

Parameter	% change in parameter	T^*	TC^*	I_0^*
H	+50	2.30297	353.6469	229.0914
	+25	2.37159	342.4637	241.7671
	-25	2.53657	318.2727	274.0451
	-50	2.63801	305.0421	295.1941

Result

T^* and I_0^* decrease while TC^* increases with increase in value of the parameter h. Here T^*, TC^* and I_0^* are moderately sensitive to change in h.

Case (iii)

Using the above said parameters and only varying the parameter α, we get,

TABLE-3

Parameter	% change in parameter	T^*	TC^*	I_0^*
A	+50	2.38398	337.2011	244.1021
	+25	2.41522	334.0074	250.0531
	-25	2.48489	327.2809	263.6556
	-50	2.52408	323.7240	271.5106

VI. Result

T^* and I_0^*decrease while TC^* increases with increase in value of the parameter α. Here T^*, TC^* andI_0^* are lowly sensitive to change in α..

VII. Conclusion

The model developed in this paper assumes demand of a product to be selling priceand follows constant deterioration rate with time varying holding cost. Shortages are not allowed.This model's demand pattern is applicable for vegetable, medicine, flowers, and so on.This model is solved analytically by minimizing the total inventory cost. This model has a future scope as shortages being allowed and changing the various demand.

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