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Presic Type Common Fixed Point Theorem for Four Maps in Complex Valued b-Metric Spaces

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Abstract: *In this paper, we obtain a Presic type fixed point theorem for two pairs of jointly $2k$-weakly compatible maps in complex valued b-metric spaces.We also give an example to illustrate our main theorem.*
Keywords: *b-metric spaces, Jointly $2k$-weakly compatible pairs,Presic type theorem.*

I. Introduction and Preliminaries

There are several generalizations of the Banach contraction principle in literature on fixed point theory. Presic [1] generalized the Banach contraction principle as follows.Throughout this paper $\mathbb{N}$ and $\mathbb{C}$ denote the set of all positive integers and complex numbers respectively.

Theorem 1.1.([1]) Let (X,d) be a complete metric space, k be a positive integer and $T: X^k \to X$ be a mapping satisfying

(1.1.1) $d(T(x_1,x_2,\cdots,x_k),T(x_2,x_3,\cdots,x_{k+1})) \le \sum_{i=1}^{k} q_i d(x_i,x_{i+1})$

for all $x_1,x_2,\cdots,x_k,x_{k+1} \in X$, where $q_i \ge 0$ and $\sum_{i=1}^{k} q_i < 1$. Then there exists a unique point $x \in X$ such that $T(x,x,....,x) = x$. Moreover, if $x_1,x_2,\cdots,x_{k+1}$ are arbitrary points in X and for $n \in \mathbb{N}$, $x_{n+k} = T(x_n,x_{n+1},\cdots,x_{n+k-1})$, then the sequence $\{x_n\}$ is convergent and $\lim_{n\to\infty} x_n = T(lim x_n, lim x_n, ...lim x_n)$.

Inspired by the Theorem 1.1, Ciric and Presic [2] proved following theorem.

Theorem 1.2. ([2]) Let (X,d) be a complete metric space, k a positive integer and $T: X^k \to X$ be a mapping satisfying

$$d(T(x_1,x_2,\cdots,x_k),T(x_2,x_3,\cdots,x_{k+1})) \le \lambda\, max\{d(x_i,x_{i+1}): 1 \le i \le k\}$$

for all $x_1,x_2,\cdots,x_k,x_{k+1}$ in X, where $\lambda \in [0,1)$. Then there exists a point $x \in X$ such that $x = T(x,x,....,x)$. Moreover, if $x_1,x_2,\cdots,x_{k+1}$ are arbitrary points in X and for $n \in \mathbb{N}$, $x_{n+k} = T(x_n,x_{n+1},\cdots,x_{n+k-1})$,then the sequence $\{x_n\}$ is convergent and $\lim_{n\to\infty} x_n = T(lim x_n, lim x_n, ...lim x_n)$. If in addition, we suppose that on diagonal $\Delta \subset X^k$, $d(T(u,u,...,u),T(v,v,...,v)) < d(u,v)$ holds for $u,v \in X$ with $u \ne v$, then x is the unique fixed point satisfying $x = T(x,x,...,x)$.

Recently Rao et al.[3,4] obtained some Presic type theorems for two and three maps in metric spaces.Now we give the following definition of [3,4] .

Definition 1.3. Let X be a non empty set and $T: X^{2k} \to X$ and $f: X \to X$ .The pair (f,T) is said to be $2k$-weakly compatible if $f(T(x,x,...,x)) = T(fx,fx,...,fx)$ whenever $x \in X$ such that $fx = T(x,x,...,x)$.

Using this definition, Rao et al. [3] proved the following

Theorem 1.4 .([3]) Let (X,d) be a metric space, k a positive integer and $S,T : X^{2k} \to X$, $f : X \to X$ be mappings satisfying

(1.4.1) $d(S(x_1, x_2, \cdots, x_{2k}), T(x_2, x_3, \cdots, x_{2k+1})) \le \lambda \max\{d(fx_i, fx_{i+1}) : 1 \le i \le 2k\}$

for all $x_1, x_2, \cdots, x_{2k}, x_{2k+1}$ in X ,

(1.4.2) $d(T(y_1, y_2, \cdots, y_{2k}), S(y_2, y_3, \cdots, y_{2k+1})) \le \lambda \max\{d(fy_i, fy_{i+1}) : 1 \le i \le 2k\}$

for all $y_1, y_2, \cdots, y_{2k}, y_{2k+1}$ in X , where $0 < \lambda < 1$,

(1.4.3) $d(S(u, \cdots, u), T(v, \cdots, v)) < d(fu, fv)$, for all $u, v \in X$ with $u \ne v$.

(1.4.4) Suppose that $f(X)$ is complete and either (f,S) or (f,T) is a $2k-$ weakly compatible pair.

Then there exists a unique point $p \in X$ such that $fp = p = S(p, \cdots, p) = T(p, \cdots, p)$.

Azam et al.[5] introduced the concept of complex valued metric spaces and obtained sufficient conditions for the existence of common fixed points of a pair of mappings satisfying a contractive condition. Later several authors for example,refer [6-14] proved fixed and common fixed point theorems in the setting of complex valued metric spaces.

In this paper, we obtain a common fixed point theorem of Presic type for four mappings in complex valued b -metric spaces.We present one example to illustrate our main theorem. We also obtain some corollaries. To begin with, we recall some basic definitions, notations and results.

Let $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ follows:

$z_1 \precsim z_2$ if and only if $Re(z_1) \le Re(z_2),\ Im(z_1) \le Im(z_2)$.

Thus $z_1 \precsim z_2$ if one of the following holds:

(1) $Re(z_1) = Re(z_2)$ and $Im(z_1) = Im(z_2)$,

(2) $Re(z_1) < Re(z_2)$ and $Im(z_1) = Im(z_2)$,

(3) $Re(z_1) = Re(z_2)$ and $Im(z_1) < Im(z_2)$,

(4) $Re(z_1) < Re(z_2)$ and $Im(z_1) < Im(z_2)$.

Clearly $z_1 \precsim z_2 \Rightarrow |z_1| \le |z_2|$.

We will write $z_1 \precnsim z_2$ if $z_1 \ne z_2$ and one of (2), (3) and (4) is satisfied. Also we will write $z_1 \prec z_2$ if only (4) is satisfied.

Definition 1.5. ([5]) Let X be a non empty set. A function $d : X \times X \to \mathbb{C}$ is called a complex valued metric on X if for all $x, y, z \in X$ the following conditions are satisfied:

(i) $0 \precsim d(x, y)$ and $d(x, y) = 0$ if and only if $x = y$;

(ii) $d(x, y) = d(y, x)$;

(iii) $d(x, y) \precsim d(x, z) + d(z, y)$.

The pair (X,d) is called a complex valued metric space.

Now, we briefly recall the definitions and lemmas about complex valued b -metric spaces introduced by Rao et al.[15].

Definition 1.6.([15]) Let X be a non empty set and $s \ge 1$. A function $d : X \times X \to \mathbb{C}$ is called a complex valued b - metric on X if for all $x, y, z \in X$ the following conditions are satisfied*:*

(i) $0 \precsim d(x, y)$ and $d(x, y) = 0$ if and only if $x = y$;

(ii) $d(x, y) = d(y, x)$;

(iii) $d(x, y) \precsim s[d(x, z) + d(z, y)]$.

The pair (X,d) is called a complex valued b - metric space.

Note. If $z_1 = a + ib$ and $z_2 = \alpha + i\beta$ then we define $\max\{z_1, z_2\} = \max\{a, \alpha\} + i\max\{b, \beta\}$.

Definition 1.7.([15]) Let (X,d) be a complex valued b -metric space.

1. A point $x \in X$ is called interior point of a set $A \subseteq X$ whenever there exists $0 \prec r \in \mathbb{C}$ such that $B(x,r) = \{y \in X : d(x,y) \prec r\} \subseteq A$.
2. A point $x \in X$ is called a limit point of a set $A \subseteq X$ whenever there exists $0 \prec r \in \mathbb{C}$ such that $B(x,r) \cap (X - A) \neq \phi.$
3. A subset $B \subseteq X$ is called open whenever each point of B is an interior point of B.
4. A subset $B \subseteq X$ is called closed whenever each limit point of B is in B.
5. The family $F = \{B(x,r) : x \in X \text{ and } 0 \prec r\}$ is a sub basis for a topology on X. We denote this complex topology by τ_c. Indeed, the topology τ_c is Hausdorff.

Let $\{x_n\}$ be a sequence in X and $x \in X$ .If for every $c \in \mathbb{C}$ with $0 \precsim c$ there is $n_0 \in \mathbb{N}$ such that for all $n > n_0$, $d(x_n, x) \prec c$,then $\{x_n\}$ is said to be convergent to x and x is the limit point of $\{x_n\}$.We denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$. If for every $c \in \mathbb{C}$ with $0 \prec c$ there is $n_0 \in \mathbb{N}$ such that for all $n > n_0, d(x_n, x_{n+m}) \prec c$, where $m \in \mathbb{N}$,then $\{x_n\}$ is called a Cauchy sequence in (X,d). If every Cauchy sequence is convergent in (X,d) then (X,d) is called a complete complex valued b -metric space. We require the follwing lemmas.

Lemma 1.8.([15]) Let (X,d) be a complex valued b -metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.

Lemma 1.9.([15]) Let (X,d) be a complex valued b -metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n, m \to \infty$.

One can easily prove the following lemma

Lemma 1.10. Let (X,d) be a complex valued b -metric space and let $\{x_n\}$ and $\{y_n\}$ be sequences in X converging to x and y respectively. Then

(i) $\frac{1}{s}|d(x,z)| \le \lim_{n\to\infty} |d(x_n, z)| \le s\,|d(x,z)|$ for all $z \in X$,

(ii) $\frac{1}{s^2}|d(x,y)| \le \lim_{n\to\infty} |d(x_n, y_n)| \le s^2 |d(x,y)|$.

Before proving our main theorem we give the following new definition.

Definition 1.11. Let X be a nonempty set, k a positive integer and $S,T : X^{2k} \to X$ and $f,g : X \to X$. The pairs (f,S) and (g,T) are said to be jointly $2k$-weakly compatible if $f(S(x,x,...,x)) = S(fx, fx,..., fx)$ and $g(T(x,x,...,x)) = T(gx, gx,..., gx)$ whenever there exists $x \in X$ such that $fx = S(x,x,...,x)$ and $gx = T(x,x,...,x)$.

Now we give our main theorem.

II. Main Result

Theorem 2.1. Let (X,d) be a complete complex valued b -metric space with $s \ge 1$ and k be any positive integer. Let $S,T : X^{2k} \to X$ and $f,g : X \to X$ be mappings satisfying

(2.1.1) $S(X^{2k}) \subseteq g(X), T(X^{2k}) \subseteq f(X)$,

(2.1.2) $$d(S(x_1,x_2,...,x_{2k}), T(y_1,y_2,...,y_{2k})) \precsim \lambda \max\left\{\begin{matrix} d(gx_1, fy_1), d(fx_2, gy_2), \\ d(gx_3, fy_3), d(fx_4, gy_4), \\ \dots\dots\dots\dots \\ d(gx_{2k-1}, fy_{2k-1}), d(fx_{2k}, gy_{2k}) \end{matrix}\right\}$$

$\forall x_1, x_2, ..., x_{2k}, y_1, y_2, .., y_{2k} \in X, where\, \lambda \in (0, \frac{1}{s^{2k}})$.

(2.1.3) (f, S) and (g, T) are jointly $2k$ -weakly compatible pairs,

(2.1.4) Suppose $z = fu = gu$ for some $u \in X$ whenever there exists a sequence $\{y_{2k+n}\}_{n=1}^{\infty}$ in X such that $\lim_{n\to\infty} y_{2k+n} = z \in X$.

Then z is the unique point in X such that $z = fz = gz = S(z, z, .., z, z) = T(z, z, ..., z, z)$.

Proof. Suppose $x_1, x_2, ..., x_{2k}$ are arbitrary points in X , From (2.1.1), define

$$y_{2k+2n-1} = S(x_{2n-1}, x_{2n}, ..., x_{2k+2n-2}) = gx_{2k+2n-1},$$

$$y_{2k+2n} = T(x_{2n}, x_{2n+1}, ..., x_{2k+2n-1}) = fx_{2k+2n}, \text{ for } n = 1, 2, ...$$

Let $\alpha_{2n} = d(fx_{2n}, gx_{2n+1})$ and $\alpha_{2n-1} = d(gx_{2n-1}, fx_{2n})$, for $n = 1, 2, ..$

Write $\theta = \lambda^{\frac{1}{2k}}$ and $\mu = \max\{\frac{|\alpha_1|}{\theta}, \frac{|\alpha_2|}{(\theta)^2}, ..., \frac{|\alpha_{2k}|}{(\theta)^{2k}}\}$.

Then $0 < \theta < 1$ and by the selection of μ , we have

$$|\alpha_n| \le \mu\, (\theta)^n \text{ for } n = 1, 2, ..., 2k \qquad (1)$$

Consider

$$\alpha_{2k+1} = d(gx_{2k+1}, fx_{2k+2})$$
$$= d(S(x_1, x_2, ..., x_{2k-1}, x_{2k}), T(x_2, x_3, ..., x_{2k}, x_{2k+1}))$$
$$\precsim \lambda \max \begin{Bmatrix} d(gx_1, fx_2), d(fx_2, gx_3), \\ d(gx_3, fx_4), d(fx_4, gx_5), \\ .... \\ d(gx_{2k-1}, fx_{2k}), d(fx_{2k}, gx_{2k+1}) \end{Bmatrix}$$
$$= \lambda \max\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, ..., \alpha_{2k-1}, \alpha_{2k}\}.$$

$$|\alpha_{2k+1}| \le \lambda \max\{\mu\theta, \mu(\theta)^2, ..., \mu(\theta)^{2k}\},\ from(1)$$
$$= \lambda\mu\theta = \mu\theta(\theta)^{2k} = \mu(\theta)^{2k+1} \qquad (2)$$

Also

$$\alpha_{2k+2} = d(fx_{2k+2}, gx_{2k+3})$$
$$= d(T(x_2, x_3, ..., x_{2k}, x_{2k+1}), S(x_3, x_4, ..., x_{2k+1}, x_{2k+2}))$$
$$= d(S(x_3, x_4, ..., x_{2k+1}, x_{2k+2}), T(x_2, x_3, ..., x_{2k}, x_{2k+1}))$$
$$\precsim \lambda \max \begin{Bmatrix} d(gx_3, fx_2), d(fx_4, gx_3), \\ d(gx_5, fx_4), d(fx_6, gx_5), \\ .... \\ d(gx_{2k+1}, fx_{2k}), d(fx_{2k+2}, gx_{2k+1}) \end{Bmatrix}$$
$$= \lambda \max\{\alpha_2, \alpha_3, \alpha_4, \alpha_5 ...., \alpha_{2k}, \alpha_{2k+1}\}.$$

$$|\alpha_{2k+2}| \le \lambda \max\{\mu(\theta)^2, \mu(\theta)^3, ..., \mu(\theta)^{2k+1}\},\ from(1)$$
$$= \lambda\mu(\theta)^2 = \mu(\theta)^2(\theta)^{2k} = \mu(\theta)^{2k+2} \qquad (3)$$

Continuing in this way, we get $|\alpha_n| \le \mu(\theta)^n$, for $n = 1,2,....$ (4)

Consider

$$d(y_{2k+2n-1}, y_{2k+2n})$$
$$= d(S(x_{2n-1}, x_{2n},...,, x_{2k+2n-2}), T(x_{2n}, x_{2n+1},..., x_{2k+2n-1}))$$
$$\precsim \lambda \max\left\{\begin{matrix} d(gx_{2n-1}, fx_{2n}), d(fx_{2n}, gx_{2n+1}), \\ d(gx_{2n+1}, fx_{2n+2}), d(fx_{2n+2}, gx_{2n+3}), \\ \dots\dots\dots\dots \\ d(gx_{2k+2n-3}, fx_{2k+2n-2}), d(fx_{2k+2n-2}, gx_{2k+2n-1}) \end{matrix}\right\}$$
$$= \lambda \max\{\alpha_{2n-1}, \alpha_{2n},....,\alpha_{2k+2n-3}, \alpha_{2k+2n-2}\}.$$

$$|d(y_{2k+2n-1}, y_{2k+2n})|$$
$$\le \lambda \max\{\mu(\theta)^{2n-1}, \mu(\theta)^{2n},..., \mu(\theta)^{2k+2n-3}, \mu(\theta)^{2k+2n-2}\}$$
$$= \lambda\mu(\theta)^{2n-1} = \mu(\theta)^{2k}(\theta)^{2n-1} = \mu(\theta)^{2k+2n-1} \qquad (5)$$

Also

$$d(y_{2k+2n}, y_{2k+2n+1})$$
$$= d(T(x_{2n}, x_{2n+1},..., x_{2k+2n-1}), S(x_{2n+1}, x_{2n+2},..., x_{2k+2n}))$$
$$= d(S(x_{2n+1}, x_{2n+2},..., x_{2k+2n}), T(x_{2n}, x_{2n+1},..., x_{2k+2n-1}))$$
$$\precsim \lambda \max\left\{\begin{matrix} d(gx_{2n+1}, fx_{2n}), d(fx_{2n+2}, gx_{2n+1}), \\ d(gx_{2n+3}, fx_{2n+2}), d(fx_{2n+4}, gx_{2n+3}), \\ \dots\dots\dots\dots \\ d(gx_{2k+2n-1}, fx_{2k+2n-2}), d(fx_{2k+2n}, gx_{2k+2n-1}) \end{matrix}\right\}$$

$$= \lambda \max\{\alpha_{2n}, \alpha_{2n+1}, \alpha_{2n+2}, \alpha_{2n+3}....,\alpha_{2k+2n-1}\}.$$

$$|d(y_{2k+2n}, y_{2k+2n})|$$
$$\le \lambda \max\{\mu(\theta)^{2n}, \mu(\theta)^{2n+1},..., \mu(\theta)^{2k+2n-2}, \mu(\theta)^{2k+2n-1}\}$$
$$= \lambda\mu(\theta)^{2n} = \mu(\theta)^{2k}(\theta)^{2n} = \mu(\theta)^{2k+2n} \qquad (6)$$

From (5),(6), we have $|d(y_{2k+n}, y_{2k+n+1})| \le \mu(\theta)^{2k+n}$,for $n = 1,2,3,...$ .....(7)

Now, using(7),for $m > n$ consider

$$|d(y_{2k+n}, y_{2k+m})| \le \begin{pmatrix} s|d(y_{2k+n}, y_{2k+n+1})| + s^2 |d(y_{2k+n+1}, y_{2k+n+2})| \\ + s^3 |d(y_{2k+n+2}, y_{2k+n+3})| + ... + \\ s^{m-n-1} |d(y_{2k+m-1}, y_{2k+m})| \end{pmatrix}$$

$$\le \begin{pmatrix} s\,\mu(\theta)^{2k+n} + s^2\,\mu(\theta)^{2k+n+1} + s^3\,\mu(\theta)^{2k+n+2} \\ + ... + s^{m-n-1}\mu(\theta)^{2k+m-1} \end{pmatrix}$$

$$\le \mu\begin{bmatrix} (s\theta)^{2k+n} + (s\theta)^{2k+n+1} + (s\theta)^{2k+n+2} \\ + ... + (s\theta)^{2k+m-1} \end{bmatrix}, since\ s \ge 1$$

$$\leq \mu(s\theta)^{2k}\left[\frac{(s\theta)^{n}}{1-s\theta}\right] since\ \ s\theta = s\lambda^{\frac{1}{2k}} < s.\frac{1}{s} = 1$$

$$\to 0\ as\ n \to \infty, m \to \infty.$$

Hence $\{y_{2k+n}\}$ is a Cauchy sequence in (X,d).

Since X is complete,there exists $z \in X$ such that $y_{2k+n} \to z$ as $n \to \infty$.

From(2.1.4),there exists $u \in X$ such that $z = fu = gu.$ (8)

Now consider

$$\left|d(S(u,u,...,u), y_{2k+2n})\right|.$$

$$= \left|d(S(u,u,...,u), T(x_{2n}, x_{2n+1},..., x_{2n+2k-1}))\right|$$

$$\leq \lambda \max\begin{Bmatrix}\left|d(gu, fx_{2n})\right|, \left|d(fu, gx_{2n+1})\right|, \\ ...., \\ \left|d(gu, fx_{2k+2n-2})\right|, \left|d(fu, gx_{2k+2n-1})\right|\end{Bmatrix}$$

Letting $n \to \infty$ and using (8),Lemma 1.10(i), we get

$\frac{1}{s}\left|d(S(u,u,...,u), fu)\right| \leq 0$ so that $S(u,u,...,u) = fu$ (9)

Similarly we have $T(u,u,...,u) = gu$ (10)

Since (f,S) and (g,T) are jointly $2k$ -weakly compatible pairs and from (9),(10), we have

$$fz = f(fu) = f(S(u,u,...,u)) = S(fu, fu,..., fu) = S(z,z,...,z) \quad ....(11)$$

and $$gz = T(z,z,...,z,z) \quad ....(12)$$

Now using (10), (11), we get

$$d(fz,z) = d(fz,gu)$$

$$= d(S(z,z,...,z,z), T(u,u,...,u,u))$$

$$\precsim \lambda \max\begin{Bmatrix} d(gz, fu), d(fz,gu), \\ d(gz, fu), d(fz,gu), \\ .... \\ d(gz, fu), d(fz,gu) \end{Bmatrix}$$

$$= \lambda \max\{d(gz,z), d(fz,z)\}.$$

Thus $d(fz,z) \precsim \lambda \max\{d(gz,z), d(fz,z)\}$ (13)

Similarly,we have $d(gz,z) \precsim \lambda \max\{d(gz,z), d(fz,z)\}$ (14)

From (13)and (14), we have

$$\max\{\left|d(gz,z)\right|, \left|d(fz,z)\right|\} \leq \lambda \max\{\left|d(gz,z)\right|, \left|d(fz,z)\right|\}$$

which in turn yields that $fz = z = gz$ (15)

From (11),(12)and (15),we have $fz = z = gz = S(z,z,...,z,z) = T(z,z,...,z,z)$ (16)

Suppose that there exists $z' \in X$ such that

$$z' = fz' = gz' = S(z',z',...,z',z') = T(z',z',...,z',z').$$

Then from (2.1.2), we have

$$\left|d(z,z')\right| = \left|d(S(z,z,...,z,z), T(z',z',...,z',z'))\right|$$

$$\leq \lambda \max\begin{Bmatrix} |d(gz, fz')|, |d(fz, gz')|, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ |d(gz, fz')|, |d(fz, gz')| \end{Bmatrix}$$
$$= \lambda |d(z, z')|.$$

This implies that $z' = z$.
Thus z is the unique point in X satisfying (16).

Now we give an example to illustrate our main Theorem 2.1.

Example 2.2. Let $X = [0,1]$ and $d(x, y) = i\,|x - y|^2$ and $k = 1$.

Define $S(x, y) = \frac{3x^2 + 2y}{\sqrt{4608}}$, $T(x, y) = \frac{2x + 3y^2}{\sqrt{4608}}$, $fx = \frac{x}{6}$ and $gx = \frac{x^2}{4}$

for all $x, y \in X$. Then clearly $s = 2$. Then for all $x_1, x_2, y_1, y_2 \in X$,we have

$$d(S(x_1, x_2), T(y_1, y_2)) = i\,|\frac{3x_1^2 + 2x_2}{\sqrt{4608}} - \frac{2y_1 + 3y_2^2}{\sqrt{4608}}|^2$$
$$= i\,\frac{1}{4608}|(3x_1^2 - 2y_1) + (2x_2 - 3y_2^2)|^2$$
$$\precsim i\,\frac{1}{4608}(|3x_1^2 - 2y_1| + |2x_2 - 3y_2^2|)^2$$
$$\precsim i\,\frac{1}{2304}(|3x_1^2 - 2y_1|^2 + |2x_2 - 3y_2^2|^2)$$
$$= i\,\frac{1}{16}(|\frac{x_1^2}{4} - \frac{y_1}{6}|^2 + |\frac{x_2}{6} - \frac{y_2^2}{4}|^2)$$
$$\precsim i\,\frac{1}{8}\max\{|\frac{x_1^2}{4} - \frac{y_1}{6}|^2, |\frac{x_2}{6} - \frac{y_2^2}{4}|^2\}$$
$$= \frac{1}{8}\max\{d(gx_1, fy_1), d(fx_2, gy_2)\}.$$

Here $\lambda = \frac{1}{8} \in (0, \frac{1}{4}) = (0, \frac{1}{2^2}) = (0, \frac{1}{s^{2k}})$.

Thus $(2.1.2)$ is satisfied.

One can easily verify the remaining conditions of Theorem 2.1.

Clearly 0 is the unique point in X such that f 0 = 0 = g0 = S(0, 0,...,0, 0) = T(0, 0,...,0, 0).

Corollary 2.3. Let (X, d) be a complex valued b -metric space with $s \geq 1$ and k be any positive integer. Let $S, T : X^{2k} \to X$ and $f : X \to X$ be mappings satisfying

(2.3.1) $S(X^{2k}) \subseteq f(X), T(X^{2k}) \subseteq f(X)$,

(2.3.2) $d(S(x_1, x_2, ..., x_{2k}), T(y_1, y_2, ..., y_{2k})) \leq \lambda \max\{d(fx_i, fy_i) : 1 \leq i \leq 2k\}$

$\forall x_1, x_2, ..., x_{2k}, y_1, y_2, .., y_{2k} \in X, where\, \lambda \in (0, \frac{1}{s^{2k}})$,

(2.3.3) $f(X)$ is a complete sub space of X,

(2.3.4) (f, S) or (f, T) is a $2k$ -weakly compatible pair.

Then there exists a unique point $u \in X$ such that $u = fu = S(u, u, .., u, u) = T(u, u, .., u, u)$.

Corollary 2.4. Let (X, d) be a complex valued b -metric space with $s \geq 1$ and k be any positive integer. Let $S : X^k \to X$ and $f : X \to X$ be mappings satisfying

(2.4.1) $S(X^k) \subseteq f(X)$,

(2.4.2) $d(S(x_1, x_2,..., x_k), S(y_1, y_2,..., y_k)) \leq \lambda \max\{d(fx_i, fy_i) : 1 \leq i \leq k\}$

$$\forall x_1, x_2,..., x_k, y_1, y_2,.., y_k \in X, where\, \lambda \in (0, \frac{1}{s^k}),$$

(2.4.3) $f(X)$ is a complete sub space of X ,

(2.4.4) (f, S) is a k -weakly compatible pair.

Then there exists a unique point $u \in X$ such that $u = fu = S(u, u,.., u, u)$.

Corollary 2.5. Let (X, d) be a complete complex valued b -metric space with $s \geq 1$ and k be any positive integer. Let $S, T : X^{2k} \to X$ be mappings satisfying

(2.5.1) $d(S(x_1, x_2,..., x_{2k}), T(y_1, y_2,..., y_{2k})) \leq \lambda \max\{d(x_i, y_i) : 1 \leq i \leq 2k\}$

$$\forall x_1, x_2,..., x_{2k}, y_1, y_2,.., y_{2k} \in X, where\, \lambda \in (0, \frac{1}{s^{2k}}).$$

Then there exists a unique point $u \in X$ such that $u = S(u, u,.., u, u) = T(u, u,.., u, u)$.

References

[1]. S.B.Presic, Sur une classe d' inequations aux differences finite et sur la convergence de certaines suites, Pub de l^1 inst. Math. Belgrade, 5(19), (1965), 75-78.

[2]. Lj.B.Ciric and S.B. Presic, On Presic type generalization of Banach contraction mapping principle, Acta. Math. Univ. Comenianae, LXXVI (2), (2007), 143-147.

[3]. K.P.R.Rao, G.N.V.Kishore and Md.Mustaq Ali, Generalization of Banach contraction principle of Presic type for three maps, Math.Sci. 3(3), (2009), 273 - 280.

[4]. K.P.R.Rao, Md.Mustaq Ali and B.Fisher, Some Presic type generalization of Banach contraction principle, Math.Moravica 15(2011), 41 - 47.

[5]. A.Azam, B.Fisher and M.Khan, Common fixed point theorems in complex valued metric spaces, Numer. Funct. Anal. Optim. 32(3),(2011),243-253.

[6]. C.Klin-eam and C.Suanoom, Some common fixed point theorems for generralized contractive type mappings on complex valued metric spaces, Abstr.Appl.Anal.,2013,6 pages, Article ID 604215, doi.10.1155/2013/604215.

[7]. F. Rouzkard and M. Imdad, Some common fixed point theorems on complex valued metric spaces, Comp. Math.Appls., 64,(2012), 1866-1874.

[8]. H. K. Nashine, M. Imdad and M. Hasan, Common fixed point theorems under rational contractions in complex valued metric spaces,J. Nonlinear Sci. Appl.,7,(2014), 42-50.

[9]. K.Sitthikul and S.Saejung, Some fixed points in complex valued metric spaces, Fixed point theory Appl.,2012,11 pages,Article ID 189 .

[10]. M.A.Kutbi, A.Azam, J.Ahmad, and C.Di Bari, Some common fixed point results for generalized contraction in complex valued metric spaces, J.Appl.Math.,2013,10 pages,ArticleID 35297.

[11]. M.Kumar,P. Kumar and S. Kumar,Common fixed point theorems in complex valued metric spaces,J. Ana. Num. Theor. 2(2),(2014), 103-109 .

[12]. N.Singh, D.Singh, A.Badal and V.Joshi,Fixed point theorems in complex valued metric spaces, Journal of the Egyptian Mathematical Society, (2015),1-8 .

[13]. R.K.Verma and H.K.Pathak,Common fixed point theorems for a pair of mappings in complex valued metric spaces,Journal of mathematics and computer Science, 6 (2013), 18-26.

[14]. W.Sintunavarat and P.Kumam, Generalized common fixed point theorems in complex valued metric spaces and applications, J.Inequal.appl.,2012, 11 pages, Article ID 84.

[15]. K.P.R.Rao, P.Ranga Swamy and J.Rajendra Prasad,A common fixed point theorem in complex valued b-metric spaces, Bulletin of Mathematics and Statistics Research,1(1),(2013),1-8.