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Almost Contra $g\omega\alpha$ -Continuous Functions in Topological Spaces

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Abstract: *In this paper, the notion of $g\omega\alpha$ -open sets in topological space is applied to present and study a new class of functions called almost contra $g\omega\alpha$ -continuous functions as a generalization of contra continuity and contra $g\omega\alpha$ -continuity, obtain their characterizations and properties. Also, the relationship with some other related functions are discussed.*
Keywords: *$g\omega\alpha$ -Closed sets, $g\omega\alpha$ -Continuous functions, **A**lmost contra $g\omega\alpha$ -continuous functions, Contra $g\omega\alpha$ -functions.*

I. Introduction

Many topologists studied the various types of generalizations of continuity [1], [2], [3], [4], [5]. In 1996, Dontchev [6] introduced the notion of contra continuity and strong S-closedness in topological spaces. A new weaker form of this class of functions called contra semi continuous function was introduced and investigated by Dontchev and Noiri [7]. Caldas and Jafari [8] introduced and studied the contra β -continuous functions and contra almost β -continuity is introduced and investigated by Baker [9].

In this paper, the notion of $g\omega\alpha$ -open sets in topological spaces is applied to introduce and study a new class of functions called almost contra $g\omega\alpha$ -continuous functions as a generalization of contra continuity and obtain their characterizations and properties. Also discuss the relationship with some other existing functions.

II. Preliminaries

Throughout this paper (X, τ), (Y, μ) and (Z, σ) (or simply X, Y and Z) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X the closure and interior of A with respect to τ are denoted by cl(A) and int(A) respectively.

Definition 2.1 *A subset A of a space X is called a,*
(i) semiopen set [10] if $A \subset cl(int(A))$.
(ii) α -open set [11] if $A \subset int(cl(int(A)))$.
(iii) regular open set [12] if A = int(cl(A)).

The complements of the above mentioned sets are called their respective closed sets. The α -closure of a subset A of a space X is the intersection of all α -closed sets that contain A and is denoted by α cl(A). The α -interior of a subset A of space X is the union of all α -open sets contained in A and is denoted by α int(A).

Definition 2.2 *[13] A subset A of X is $g\omega\alpha$ -closed if α cl(A) $\subset U$ whenever $A \subset U$ and U is $\omega\alpha$ -open in X. The family of all $g\omega\alpha$ -closed subsets of the space X is denoted by $G\omega\alpha$ C(X).*

Definition 2.3 *[14] A function $f : X \to Y$ is called $g\omega\alpha$ -continuous if the inverse image of every closed set in Y is $g\omega\alpha$ -closed in X.*

Definition 2.4 *[15] A function $f : X \to Y$ is said to be almost continuous if $f^{-1}(V)$ is open in X for each regular open set V of Y.*

Definition 2.5 *[16] A function $f : X \to Y$ is said to be (θ, s)-continuous if $f^{-1}(V)$ is closed in X for each regular open set V of Y.*

Definition 2.6 *[17] A space X is called locally $g\omega\alpha$ -indiscrete if every $g\omega\alpha$ -open set is closed in X.*

Definition 2.7 *[17] A function $f : X \to Y$ is said to be contra $g\omega\alpha$ -continuous if $f^{-1}(V)$ is $g\omega\alpha$ -closed in X for each open set V in Y.*

Definition 2.8 *[18] A function $f : X \to Y$ is said to be strongly $g\omega\alpha$ -open (resp. strongly $g\omega\alpha$ -closed) if image of every $g\omega\alpha$ -open (resp. $g\omega\alpha$ -closed) set of X is $g\omega\alpha$ -open (resp. $g\omega\alpha$ -closed) set in Y .*

Definition 2.9 *[18] A topological space X is said to be $g\omega\alpha$ -T_1 space if for any pair of distinct points x and y , there exist a $g\omega\alpha$ -open sets G and H such that $x \in G$, $y \notin G$ and $x \notin H$, $y \in H$.*

Definition 2.10 *[18] A topological space X is said to be $g\omega\alpha$ -T_2 space if for any pair of distinct points x and y there exist disjoint $g\omega\alpha$ -open sets G and H such that $x \in G$ and $y \in H$.*

Definition 2.11 *[18] A topological space X is said to be $g\omega\alpha$ -normal if each pair of disjoint closed sets can be separated by disjoint $g\omega\alpha$ -open sets.*

Definition 2.12 *[17] A space X is called $g\omega\alpha$ -connected provided that X is not the union of two disjoint nonempty $g\omega\alpha$ -open sets.*

Definition 2.13 *[17] A function $f : X \to Y$ is called weakly $g\omega\alpha$ -continuous if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists $U \in G\omega\alpha O(X, x)$ such that $f(U) \subset cl(V)$.*

III. Almost Contra $g\omega\alpha$ -Continuous Function

In this section, a new type of continuity called an almost contra $g\omega\alpha$ -continuity, which is weaker than contra $g\omega\alpha$ -continuity is introduced and studied some of their properties and characterizations.

Definition 3.1 *A function $f : X \to Y$ is said to be almost contra $g\omega\alpha$ -continuous if $f^{-1}(V)$ is $g\omega\alpha$ -closed in X for each regular open set V in Y .*

Theorem 3.2 *If X is $T_{g\omega\alpha}$-space and $f : X \to Y$ is almost contra $g\omega\alpha$ continuous, then f is (θ, s)-continuous.*

Proof. Let U be a regular open set in Y . Since f is almost contra $g\omega\alpha$ -continuous, $f^{-1}(U)$ is $g\omega\alpha$ -closed set in X and X is $T_{g\omega\alpha}$-space, which implies $f^{-1}(U)$ is closed set in X . Therefore f is (θ, s)-continuous.

Theorem 3.3 *If a function $f : X \to Y$ is almost contra $g\omega\alpha$ -continuous and X is locally $g\omega\alpha$ -indiscrete space then f is almost continuous.*

Proof. Let U be a regular open set in Y . Since f is almost contra $g\omega\alpha$ -continuous $f^{-1}(U)$ is $g\omega\alpha$ -closed set in X and X is locally $g\omega\alpha$ -indiscrete space, which implies $f^{-1}(U)$ is an open set in X . Therefore f is almost continuous.

Theorem 3.4 *The following are equivalent for a function $f : X \to Y$:*

(i) f is almost contra $g\omega\alpha$ -continuous.

(ii) $f^{-1}(int(cl(G)))$ is $g\omega\alpha$ -closed set in X for every open subset G of Y .

(iii) $f^{-1}(cl(int(F)))$ is $g\omega\alpha$ -open set in X for every closed subset F of Y .

Proof. (i)$\Rightarrow$(ii) Let G be an open set in Y . Then $int(cl(G))$ is regular open set in Y . By (i), $f^{-1}(int(cl(G)) \in G\omega\alpha C(X)$.

(ii)$\Rightarrow$(i) Proof is obvious.

(i)$\Rightarrow$(iii) Let F be a closed set in Y . Then $cl(int(G))$ is regular closed set in Y . By (i), $f^{-1}(cl(int(G)) \in G\omega\alpha O(X)$.

(iii)$\Rightarrow$(i) Proof is obvious.

Theorem 3.5 *The following are equivalent for a function $f : X \to Y$:*

(i) f is almost contra $g\omega\alpha$ -continuous.

(ii) For every regular closed set F of Y , $f^{-1}(F)$ is $g\omega\alpha$ -open set of X .

(iii) For each $x \in X$ and each regular closed set F of Y containing $f(x)$ there exists $g\omega\alpha$ -open set U containing x such that $f(U) \subset F$.

(iv) For each $x \in X$ and each regular open set V of Y not containing $f(x)$ there exists $g\omega\alpha$ -closed set K not containing x such that $f^{-1}(V) \subset K$.

Proof. (i)$\Rightarrow$(ii) Let F be a regular closed set in Y then $Y - F$ is a regular open set in Y. By (i), $f^{-1}(Y-F) = X - f^{-1}(F)$ is $g\omega\alpha$ -closed set in X. This implies $f^{-1}(F)$ is $g\omega\alpha$ -open set in X. Therefore, (ii) holds.

(ii)$\Rightarrow$(i) Let G be a regular open set of Y. Then $Y - G$ is a regular closed set in Y. By (ii), $f^{-1}(Y-G)$ is $g\omega\alpha$ -open set in X. This implies $X - f^{-1}(G)$ is $g\omega\alpha$ -open set in X, which implies $f^{-1}(G)$ is $g\omega\alpha$ -closed set in X. Therefore, (i) hold.

(ii)$\Rightarrow$(iii) Let F be a regular closed set in Y containing $f(x)$ which implies $x \in f^{-1}(F)$. By (ii), $f^{-1}(F)$ is $g\omega\alpha$ -open in X containing x. Set $U = f^{-1}(F)$, which implies U is $g\omega\alpha$ -open in X containing x and $f(U) = f(f^{-1}(F)) \subset F$. Therefore (iii) holds.

(iii)$\Rightarrow$(ii) Let F be a regular closed set in Y containing $f(x)$ which implies $x \in f^{-1}(F)$. From (iii), there exists $g\omega\alpha$ -open U_x in X containing x such that $f(U_x) \subset F$. That is $U_x \subset f^{-1}(F)$. Thus $f^{-1}(F) = \cup\{U_x : x \in f^{-1}(F)\}$, which is union of $g\omega\alpha$ -open sets. Therefore, $f^{-1}(F)$ is $g\omega\alpha$ -open set of X.

(iii)$\Rightarrow$(iv) Let V be a regular open set in Y not containing $f(x)$. Then $Y - V$ is a regular closed set in Y containing $f(x)$. From (iii), there exists a $g\omega\alpha$ -open set U in X containing x such that $f(U) \subset Y - V$. This implies $U \subset f^{-1}(Y-V) = X - f^{-1}(V)$. Hence, $f^{-1}(V) \subset X - U$. Set $K = X - U$ then K is $g\omega\alpha$ -closed set not containing x in X such that $f^{-1}(V) \subset K$.

(iv)$\Rightarrow$(iii) Let F be a regular closed set in Y containing $f(x)$. Then $Y - F$ is a regular open set in Y not containing $f(x)$. From (iv), there exists $g\omega\alpha$ -closed set K in X not containing x such that $f^{-1}(Y-F) \subset K$. This implies $X - f^{-1}(F) \subset K$. Hence, $X - K \subset f^{-1}(F)$, that is $f(X-K) \subset F$. Set $U = X - K$, then U is $g\omega\alpha$ -open set containing x in X such that $f(U) \subset F$.

Definition 3.6 *[19] A space X is said to be weakly Hausdorff if each element of X is an intersection of regular closed sets.*

Theorem 3.7 *If $f : X \to Y$ is an almost contra $g\omega\alpha$ -continuous injection and Y is weakly Hausdorff then X is $g\omega\alpha$ -T_1.*

Proof. Suppose Y is weakly Hausdorff. For any distinct points x and y in X, there exist V and W regular closed sets in Y such that $f(x) \in V$, $f(y) \notin V$, $f(y) \in W$ and $f(x) \notin W$. Since f is almost contra $g\omega\alpha$ -continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are $g\omega\alpha$ -open subsets of X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$, $y \in f^{-1}(W)$ and $x \notin f^{-1}(W)$. This shows that X is $g\omega\alpha$ -T_1.

Corollary 3.8. *If $f : X \to Y$ is a contra $g\omega\alpha$ -continuous injection and Y is weakly Hausdorff then X is $g\omega\alpha$ -T_1.*

Definition 3.9 *[20] A topological space X is called Ultra Hausdroff space, if for every pair of distinct points x and y in X, there exist disjoint clopen sets U and V in X containing x and y respectively.*

Theorem 3.10 *If $f : X \to Y$ is an almost contra $g\omega\alpha$ -continuous injective function from space X into a Ultra Hausdroff space Y then X is $g\omega\alpha$ -T_2.*

Proof. Let x and y be any two distinct points in X. Since f is an injective $f(x) \neq f(y)$ and Y is Ultra Hausdroff space, there exist disjoint clopen sets U and V of Y containing $f(x)$ and $f(y)$ respectively. Then $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $g\omega\alpha$ -open sets in X. Therefore X is $g\omega\alpha$ -T_2.

Definition 3.11 *[20] A topological space X is called Ultra normal space if each pair of disjoint closed sets can be separated by disjoint clopen sets.*

Theorem 3.12 *If $f: X \to Y$ is an almost contra $g\omega\alpha$ -continuous closed injection and Y is ultra normal then X is $g\omega\alpha$ -normal.*

Proof. Let E and F be disjoint closed subsets of X. Since f is closed and injective $f(E)$ and $f(F)$ are disjoint closed sets in Y. Since Y is ultra normal there exists disjoint clopen sets U and V in Y such that $f(E) \subset U$ and $f(F) \subset V$. This implies $E \subset f^{-1}(U)$ and $F \subset f^{-1}(V)$. Since f is an almost contra $g\omega\alpha$ -continuous injection, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $g\omega\alpha$ -open sets in X. This shows X is $g\omega\alpha$ -normal.

Definition 3.13 *Let A be a subset of X. Then $(g\omega\alpha\text{-}cl(A) - g\omega\alpha\text{-}int(A))$ is called $g\omega\alpha$ -frontier of A and is denoted by $g\omega\alpha\text{-}Fr(A)$*

Theorem 3.14 *The set of all points x of X at which $f: X \to Y$ is not almost contra $g\omega\alpha$ -continuous is identical with the union of $g\omega\alpha$ -frontier of the inverse images of closed sets of Y containing $f(x)$.*

Proof. Assume that f is not almost contra $g\omega\alpha$-continuous at $x \in X$. Then, there exists $F \in RC(Y, f(x))$ such that $f(U) \cap (Y - F) \neq \phi$ for every $U \in G\omega\alpha O(X, x)$. This implies $U \cap f^{-1}(Y - F) \neq \phi$ for every $U \in G\omega\alpha O(X, x)$. Therefore, $x \in g\omega\alpha\text{-}cl(f^{-1}(Y - F))$ = $g\omega\alpha\text{-}cl(X - f^{-1}(F))$ and also $x \in f^{-1}(F) \subset g\omega\alpha\text{-}cl(f^{-1}(F))$. Thus, $x \in g\omega\alpha\text{-}cl(f^{-1}(F)) \cap g\omega\alpha\text{-}cl(X - f^{-1}(F))$. This implies, $x \in g\omega\alpha\text{-}cl(f^{-1}(F)) - g\omega\alpha\text{-}int(f^{-1}(F))$. Therefore, $x \in g\omega\alpha\text{-}Fr(f^{-1}(F))$.

Conversely, suppose $x \in g\omega\alpha\text{-}Fr(f^{-1}(F))$ for some $F \in RC(Y, f(x))$ and f is almost contra $g\omega\alpha$ -continuous at $x \in X$, then there exists $U \in G\omega\alpha O(X, x)$ such that $f(U) \subset F$. Therefore, $x \in U \subset f^{-1}(F)$ and hence $x \in g\omega\alpha\text{-}int(f^{-1}(F)) \subset X - g\omega\alpha\text{-}Fr(f^{-1}(F))$. This contradicts that $x \in g\omega\alpha\text{-}Fr(f^{-1}(F))$. Therefore f is not almost contra $g\omega\alpha$ -continuous.

Theorem 3.15 *If $f: X \to Y$ is an almost contra $g\omega\alpha$ -continuous surjection and X is $g\omega\alpha$ -connected space then Y is connected.*

Proof. Let $f: X \to Y$ be an almost contra $g\omega\alpha$ -continuous surjection and X is $g\omega\alpha$ -connected space. Suppose Y is not connected, then there exist disjoint open sets U and V such that $Y = U \cup V$. Therefore U and V are clopen in Y. Since f is almost contra $g\omega\alpha$ -continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $g\omega\alpha$ -open sets in X. Moreover $f^{-1}(U)$ and $f^{-1}(V)$ are non empty disjoint and $X = f^{-1}(U) \cup f^{-1}(V)$. This is contradiction to the fact that X is $g\omega\alpha$ -connected space. Therefore, Y is connected.

Definition 3.16 *[21] A function $f: X \to Y$ is said to be R-map if $f^{-1}(V)$ is regular open in X for each regular open set V of Y.*

Definition 3.17 *[22] A function $f: X \to Y$ is said to be perfectly continuous if $f^{-1}(V)$ is clopen in X for each open set V of Y.*

Theorem 3.18 *For two functions $f: X \to Y$ and $g: Y \to Z$, let $g \circ f: X \to Z$ is a composition function.Then, the following properties holds:*

(i) If f is almost contra $g\omega\alpha$ -continuous and g is an R-map then $g \circ f$ is almost contra $g\omega\alpha$ -continuous.

(ii) If f is almost contra $g\omega\alpha$ -continuous and g is perfectly continuous then $g \circ f$ is $g\omega\alpha$ -continuous and contra $g\omega\alpha$ -continuous.

(iii) If f is contra $g\omega\alpha$ -continuous and g is almost continuous then $g \circ f$ is almost contra $g\omega\alpha$ -continuous.

Proof. (i) Let V be any regular open set in Z. Since g is an R-map, $g^{-1}(V)$ is regular open in Y. Since f is an almost contra $g\omega\alpha$ -continuous $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $g\omega\alpha$ -closed set in X. Therefore, $g \circ f$ is almost contra $g\omega\alpha$ -continuous.

(ii) Let V be any open set in Z. Since g is perfectly continuous, $g^{-1}(V)$ is clopen in Y. Since f is an almost contra $g\omega\alpha$ -continuous $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $g\omega\alpha$ -open and $g\omega\alpha$ -closed set in X. Therefore, $g \circ f$ is $g\omega\alpha$ -continuous and contra $g\omega\alpha$ -continuous.

(iii) Let V be any regular open set in Z. Since g is almost continuous, $g^{-1}(V)$ is open in Y. Since f is contra $g\omega\alpha$ -continuous $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $g\omega\alpha$ -closed set in X. Therefore, $g \circ f$ is almost contra $g\omega\alpha$ -continuous.

Theorem 3.19 *Let $f : X \to Y$ be a contra $g\omega\alpha$ -continuous and $g : Y \to Z$ be $g\omega\alpha$ -continuous. If Y is $T_{g\omega\alpha}$-space then $g \circ f : X \to Z$ is an almost contra $g\omega\alpha$ -continuous.*

Proof. Let V be any regular open and hence open set in Z. Since g is $g\omega\alpha$ -continuous $g^{-1}(V)$ is $g\omega\alpha$ -open in Y and Y is $T_{g\omega\alpha}$-space implies $g^{-1}(V)$ is open in Y. Since f is contra $g\omega\alpha$ -continuous $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $g\omega\alpha$ -closed set in X. Therefore, $g \circ f$ is an almost contra $g\omega\alpha$ -continuous.

Theorem 3.20 *If $f : X \to Y$ is surjective strongly $g\omega\alpha$ -open (or strongly $g\omega\alpha$ -closed) and $g : Y \to Z$ is a function such that $g \circ f : X \to Z$ is an almost contra $g\omega\alpha$ -continuous then g is an almost contra $g\omega\alpha$ -continuous.*

Proof. Let V be any regular closed (resp. regular open) set in Z. Since $g \circ f$ is an almost contra $g\omega\alpha$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g\omega\alpha$ -open (resp. $g\omega\alpha$ -closed) in X. Since f is surjective and strongly $g\omega\alpha$ -open (or strongly $g\omega\alpha$ -closed), $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is $g\omega\alpha$ -open(or $g\omega\alpha$ -closed). Therefore g is an almost contra $g\omega\alpha$ -continuous.

Definition 3.21 *A topological space X is said to be $g\omega\alpha$ -ultra-connected if every two nonempty $g\omega\alpha$ -closed subsets of X intersect.*

Definition 3.22 *[23] A topological space X is said to be hyper connected if every open set is dense.*

Theorem 3.23 *If X is $g\omega\alpha$ -ultra-connected and $f : X \to Y$ is an almost contra $g\omega\alpha$ -continuous surjection, then Y is hyperconnected.*

Proof. Let X be a $g\omega\alpha$ -ultra-connected and $f : X \to Y$ be an almost contra $g\omega\alpha$ -continuous surjection. Suppose Y is not hyperconnected. Then there exists an open set V such that V is not dense in Y. Therefore, there exist nonempty regular open subsets $B_1 = int(cl(V))$ and $B_2 = Y - cl(V)$ in Y. Since f is an almost contra $g\omega\alpha$ -continuous surjection, $f^{-1}(B_1)$ and $f^{-1}(B_2)$ are disjoint $g\omega\alpha$ -closed sets in X. This is contrary to the fact that X is $g\omega\alpha$ -ultra-connected. Therefore, Y is hyperconnected.

Definition 3.24 *A space X is said to be a*

(i) $g\omega\alpha$ -compact if every $g\omega\alpha$ -open cover of X has a finite subcover.

(ii) $G\omega\alpha$ -closed compact [17] if every $g\omega\alpha$ -closed cover of X has a finite subcover.

(iii) Nearly compact [24] if every regular open cover of X has a finite subcover.

(iv) Countably $g\omega\alpha$ -compact if every countable cover of X by $g\omega\alpha$ -open sets has a finite subcover.

(v) Countably $G\omega\alpha$ -closed compact [17] if every countable cover of X by $g\omega\alpha$ -closed sets has a finite subcover.

(vi) Nearly countably compact [24] if every countable cover of X by regular open sets has a finite subcover.

(vii) $g\omega\alpha$ -Lindelof if every $g\omega\alpha$ -open cover of X has a countable subcover.

(viii) $G\omega\alpha$ -Lindelof [17] if every $g\omega\alpha$ -closed cover of X has a countable subcover.

(ix) Nearly Lindelof [24] if every regular open cover of X has a countable subcover.

(x) Mildly $g\omega\alpha$ -compact if every $g\omega\alpha$ -clopen cover of X has a finite subcover.

(xi) Mildly countably $g\omega\alpha$ -compact if every countable cover of X by $g\omega\alpha$ -clopen sets has a finite subcover.

(xii) Mildly $g\omega\alpha$ -Lindelof if every $g\omega\alpha$ -clopen cover of X has a countable subcover.

Theorem 3.25 *Let $f : X \to Y$ be an almost contra $g\omega\alpha$ -continuous surjection. Then, the following properties hold.*

(i) If X is $G\omega\alpha$ -closed compact then Y is nearly compact.

(ii) If X is countably $G\omega\alpha$ -closed compact then Y is nearly countabaly compact.

(iii) If X is $G\omega\alpha$ -Lindelof then Y is nearly Lindelof.

Proof.(i) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra $g\omega\alpha$ -continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is $g\omega\alpha$ -closed cover of X. Since X is $G\omega\alpha$ -closed compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$, which is finite subcover for Y. Therefore, Y is nearly compact.

(ii) Let $\{V_\alpha : \alpha \in I\}$ be any countable regular open cover of Y. Since f is almost contra $g\omega\alpha$ -continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable $g\omega\alpha$ -closed cover of X. Since X is countably $G\omega\alpha$ -closed compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite subcover for Y. Therefore, Y is nearly countably compact.

(iii) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra $g\omega\alpha$ -continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is $g\omega\alpha$ -closed cover of X. Since X is $G\omega\alpha$ -Lindelof, there exists a countable subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite subcover for Y. Therefore, Y is nearly Lindelof.

Theorem 3.26 *Let $f : X \to Y$ be an almost contra $g\omega\alpha$ -continuous surjection. Then, the following properties hold.*

(i) If X is $g\omega\alpha$ -compact then Y is S -closed.

(ii) If X is countably $g\omega\alpha$ -closed, then Y is countably S -closed.

(iii) If X is $g\omega\alpha$ -Lindelof then Y is S -Lindelof.

Proof.(i) Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra $g\omega\alpha$ -continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is $g\omega\alpha$ -open cover of X. Since X is $g\omega\alpha$ -compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite subcover for Y. Therefore, Y is S-closed.

(ii) Let $\{V_\alpha : \alpha \in I\}$ be any countable regular closed cover of Y then as f is almost contra $g\omega\alpha$ -continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable $g\omega\alpha$ -open cover of X. Since X is countably $g\omega\alpha$ -compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite subcover for Y. Therefore, Y is countably S-closed.

(iii) Let $\{V_\alpha : \alpha \in I\}$ be any regular closed cover of Y. Since f is almost contra $g\omega\alpha$ -continuous, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is $g\omega\alpha$ -open cover of X. Since X is $g\omega\alpha$ -Lindelof, there exists a countable subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite subcover for Y. Therefore, Y is S-Lindelof.

Definition 3.27 *A function $f : X \to Y$ is said to be almost $g\omega\alpha$ -continuous if $f^{-}(V)$ is $g\omega\alpha$ -open in X for each regular open set V of Y.*

Theorem 3.28 *Let $f : X \to Y$ be an almost contra $g\omega\alpha$ -continuous and almost $g\omega\alpha$ -continuous surjection. Then, the following properties hold.*

(i) If X is mildly $g\omega\alpha$ -closed then Y is nearly compact.
(ii) If X is mildly countably $G\omega\alpha$ -closed then Y is nearly countabaly compact.
(iii) If X is mildly $g\omega\alpha$ -Lindelof then Y is nearly Lindelof.

Proof.(i) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra $g\omega\alpha$ -continuous and almost $g\omega\alpha$ surjection, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is $g\omega\alpha$ -clopen cover of X. Since X is mildly $g\omega\alpha$ -compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha) : \alpha \in I_0\}$, which is finite subcover for Y. Therefore, Y is nearly compact.

(ii) Let $\{V_\alpha : \alpha \in I\}$ be any countable regular open cover of Y. Since f is almost contra $g\omega\alpha$ -continuous and almost $g\omega\alpha$ surjection, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is countable $g\omega\alpha$ -closed cover of X. Since X is mildly countably $g\omega\alpha$ -compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ is finite subcover for Y. Therefore, Y is nearly countably compact.

(iii) Let $\{V_\alpha : \alpha \in I\}$ be any regular open cover of Y. Since f is almost contra $g\omega\alpha$ -continuous and almost $g\omega\alpha$ surjection,, $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is $g\omega\alpha$ -closed cover of X. Since X is mildly $g\omega\alpha$ -Lindelof, there exists a countable subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Since f is surjective, $Y = \cup\{V_\alpha) : \alpha \in I_0\}$ is finite subcover for Y. Therefore, Y is nearly Lindelof.

IV. contra closed graphs

In this section, $g\omega\alpha$ -regular graphs and contra $g\omega\alpha$ -closed graphs are defined and investigated the relationships between the graphs and contra functions.
Recall that for a function $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$

Definition 4.1 *The graph $G(f)$ of a function $f : X \to Y$ is said to be contra $g\omega\alpha$ -closed if for each $(x, y) \in (X, Y) - G(f)$, there exist $U \in G\omega\alpha O(X, x)$ and $V \in C(Y, y)$ such that $(U \times V) \cap G(f) = \phi$.*

Theorem 4.2 *Let $f : X \to Y$ be a function and let $g : X \to X \times Y$ be the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is almost contra $g\omega\alpha$ -continuous function, then f is an almost contra $g\omega\alpha$ -continuous.*

Proof. Let $V \in RC(Y)$, then $X \times V = X \times cl(int(V)) = cl(int(X)) \times cl(int(V)) = cl(int(X \times V))$. Therefore, $X \times V \in RC(X \times Y)$. Since g is almost contra $g\omega\alpha$ -continuous, $f^{-1}(V) = g^{-1}(X \times V) \in G\omega\alpha O(X)$. Thus, f is an almost contra $g\omega\alpha$ -continuous.

Lemma 4.3 *[25] Let $G(f)$ be the graph of f, for any subset $A \subset X$ and $B \subset Y$, we have $f(A) \cap B = \phi$ if and only if $(A \times B) \cap G(f) = \phi$.*

Lemma 4.4 *The graph $G(f)$ of $f : X \to Y$ is contra $g\omega\alpha$ -closed in $X \times Y$ if and only if for each $(x, y) \in (X, Y) - G(f)$, there exist $U \in G\omega\alpha O(X, x)$ and $V \in C(Y, y)$ such that $f(U) \cap V = \phi$.*

Proof. This is a direct consequences of definition 4.1 and lemma 4.3.

Theorem 4.5 *If $f : X \to Y$ is contra $g\omega\alpha$ -continuous and Y is Urysohn, then $G(f)$ is contra $g\omega\alpha$ -closed in $X \times Y$.*

Proof. Let $(x, y) \in (X, Y) - G(f)$. Then $y \neq f(x)$. Since Y is Urysohn, there exist open sets V and W such that $f(x) \in V$, $y \in W$ and $cl(V) \cap cl(W) = \phi$. Since f is contra $g\omega\alpha$ -continuous, there exists $U \in G\omega\alpha O(X, x)$ such that $f(U) \subset cl(V)$. Therefore, $(x, y) \in U \times cl(W) \subset X \times Y - G(f)$. This shows that $G(f)$ is contra $g\omega\alpha$ -closed in $X \times Y$.

Theorem 4.6 *If* $f: X \to Y$ *is* $g\omega\alpha$ *-continuous and* Y *is* T_1*, then* $G(f)$ *is contra* $g\omega\alpha$ *-closed in* $X \times Y$.

Proof. Let $(x, y) \in (X, Y) - G(f)$. Then $y \neq f(x)$ and there exists open set V of Y such that $f(x) \in V$, $y \notin V$. Since f is $g\omega\alpha$ -continuous there exists $U \in G\omega\alpha O(X, x)$ such that $f(U) \subset V$. Therefore, $f(U) \cap (Y - V) = \phi$. Thus, for each $(x, y) \in (X, Y) - G(f)$, there exist $U \in G\omega\alpha O(X, x)$ and $Y - V \in C(Y, y)$ such that $f(U) \cap Y - V = \phi$. Therefore, $G(f)$ is contra $g\omega\alpha$ -closed in $X \times Y$.

Definition 4.7 *The graph* $G(f)$ *of a function* $f: X \to Y$ *is said to be* $g\omega\alpha$ *-regular (resp. strongly contra* $g\omega\alpha$ *-closed) if for each* $(x, y) \in (X, Y) - G(f)$*, there exist* $g\omega\alpha$ *-closed (resp.* $g\omega\alpha$ *-open) set* U *in* X *containing* x *and* $V \in RO(Y, y)$ *(resp.* $V \in RC(Y, y)$*) such that* $(U \times V) \cap G(f) = \phi$.

Lemma 4.8 *The graph* $G(f)$ *of* $f: X \to Y$ *is* $g\omega\alpha$ *-regular (resp. strongly contra* $g\omega\alpha$ *-closed) in* $X \times Y$ *if and only if for each* $(x, y) \in (X, Y) - G(f)$*, there exist* $g\omega\alpha$ *-closed (resp.* $g\omega\alpha$ *-open) set* U *in* X *containing* x *and* $V \in RO(Y, y)$ *(resp.* $V \in RC(Y, y)$*)such that* $f(U) \cap V = \phi$.

Proof. Proof is obvious from Lemma 4.8.

Theorem 4.9 *Let* $f: X \to Y$ *have a* $g\omega\alpha$ *-regular graph* $G(f)$*. If* f *is surjective, then* Y *is weakly Hausdorff.*

Proof. Let y_1 and y_2 be any two distinct points of Y. Since f is surjective, $f(x) = y_1$ for some $x \in X$ and $(x, y_2) \in (X, Y) - G(f)$. Since $G(f)$ is $g\omega\alpha$ -regular, there exist $g\omega\alpha$ -closed set U in X containing x and $F \in RO(Y, y_2)$ such that $f(U) \cap F = \phi$ by Lemma 4.8 and hence $y_1 \notin F$. Then $y_1 \in Y - F$ and $y_2 \notin Y - F$ and $Y - F$ is regular closed set in Y. This implies Y is weakly Hausdorff.

Theorem 4.10 *If* $f: X \to Y$ *is almost* $g\omega\alpha$ *-continuous and* Y *is* T_2*, then* $G(f)$ *is* $g\omega\alpha$ *-regular in* $X \times Y$.

Proof. Let $(x, y) \in (X, Y) - G(f)$. Then $y \neq f(x)$. Since Y is T_2, there exist regular open sets V and W in Y, such that $f(x) \in V$, $y \in W$ and $V \cap W = \phi$. Since f is almost $g\omega\alpha$ -continuous $f^{-1}(V)$ is $g\omega\alpha$ -closed set in X containing x. Set $U = f^{-1}(V)$, then $f(U) \subset V$. Therefore, $f(U) \cap W = \phi$ and $G(f)$ is $g\omega\alpha$ -regular in $X \times Y$.

Theorem 4.11 *Let* $f: X \to Y$ *have a strongly contra* $g\omega\alpha$ *-closed graph G(f). If* f *is an almost contra* $g\omega\alpha$ *-continuous injection, then* X *is* $g\omega\alpha$ *-*T_2.

Proof. Let x and y be any two distinct points of X. Since X is injective, $f(x) \neq f(y)$. Then, $(x, f(y)) \in (X, Y) - G(f)$. Since $G(f)$ is strongly contra $g\omega\alpha$ -closed, by Lemma 4.8, there exist $g\omega\alpha$ -open set U in X containing x and $V \in RC(Y, y)$ such that $f(U) \cap V = \phi$ and hence $U \cap f^{-1}(V) = \phi$. Since f is an almost contra $g\omega\alpha$ -continuous, $f^{-1}(V)$ is $g\omega\alpha$ -open in X containing y. This shows that X is $g\omega\alpha$ -T_2.

Theorem 4.12 *Let* $f: X \to Y$ *have a* $g\omega\alpha$ *-regular* $G(f)$*. If* f *is injective, then* X *is* $g\omega\alpha$ -T_0.

Proof. Let x and y be any two distinct points of X. Then, $(x, f(y)) \in (X, Y) - G(f)$. Since $G(f)$ is $g\omega\alpha$ -regular, there exists $g\omega\alpha$ -closed set U in X containing x and $V \in RO(Y, f(y))$ such that $f(U) \cap V = \phi$ by lemma 4.8, and hence $U \cap f^{-1}(V) = \phi$. Therefore, $y \notin U$. Thus, $y \in X - U$ and $x \notin X - U$ and $X - U$ is $g\omega\alpha$ -open set in X. This implies X is $g\omega\alpha$ -T_0.

Definition 4.13 *A function* $f: X \to Y$ *is called almost weakly* $g\omega\alpha$ *-continuous if for each* $x \in X$ *and each open set* V *of* Y *containing* $f(x)$*, there exists* $U \in G\omega\alpha O(X, x)$ *such that* $f(U) \subset cl(V)$.

Theorem 4.14 *If* $f: X \to Y$ *is almost contra* $g\omega\alpha$ *-continuous, then* f *is almost weakly* $g\omega\alpha$ *-continuous.*

Proof. Let $x \in X$ and V be any open set of Y containing $f(x)$. Then $cl(V)$ is a regular closed set of Y containing $f(x)$. Since f is almost contra $g\omega\alpha$ -continuous by theorem 3.5 there exists $g\omega\alpha$ -open set in X containing x such that $f(U) \subset cl(V)$. By definition 4.13 f is almost weakly $g\omega\alpha$ -continuous.

Corollary 4.15. *If $f : X \to Y$ is almost contra $g\omega\alpha$ -continuous and Y is Urysohn, then $G(f)$ strongly contra $g\omega\alpha$ -closed in $X \times Y$.*

We recall that a topological space X is said to be extremely disconnected $[E.D]$ if the closure of every open set of X is open in X.

Theorem 4.16 *Let Y be E.D. Then a function $f : X \to Y$ is almost contra $g\omega\alpha$ -continuous if and only if it is almost $g\omega\alpha$ -continuous*

Proof. Let $x \in X$ and V be any regular open set of Y containing $f(x)$. Since Y is E.D then V is clopen and hence V is regular closed set of Y containing $f(x)$. Since f is almost contra $g\omega\alpha$ -continuous then there exists $g\omega\alpha$ -open set in X containing x such that $f(U) \subset V$. Then f is almost $g\omega\alpha$ -continuous.

Conversely, let F be any regular closed set of Y. Since Y is E.D, F is also regular open and $f^{-1}(F)$ is $g\omega\alpha$ -open in X. This shows that f is almost contra $g\omega\alpha$ -continuous

Theorem 4.17 *If $f : X \to Y$ is almost weakly $g\omega\alpha$ -continuous and Y is Urysohn, then $G(f)$ strongly contra $g\omega\alpha$ -closed in $X \times Y$.*

Proof. Let $(x, y) \in (X,Y) - G(f)$ implies, $y \neq f(x)$. Since Y is Urysohn there exist open sets V and W in Y such that $y \in V$, $f(x) \in W$ and $cl(V) \cap cl(W) = \phi$. Since f is almost weakly $g\omega\alpha$ -continuous, then there exists $U \in G\omega\alpha O(X, x)$ such that $f(U) \subset cl(W)$. This shows that $f(U) \cap cl(V)$ = $f(U) \cap cl(int(V)) = \phi$, where $cl(int(V)) \in RC(Y)$ and hence by lemma 4.8, we have $G(f)$ strongly contra $g\omega\alpha$ -closed in $X \times Y$.

V. Conclusion

In this paper, the study of contra $g\omega\alpha$ -continuous functions is continued. Further almost contra $g\omega\alpha$ -continuous functions and $g\omega\alpha$ -closed graphs in topological spaces are introduced and investigated. The notions contra $g\omega\alpha$ -continuous functions and almost contra $g\omega\alpha$ -continuous functions can be used to study some more stronger forms of $g\omega\alpha$ -continuous functions.

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