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Multi-Item Inventory Model of Deteriorating Items with Partial Backordering and Linear Demand

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Abstract: *We develop a model for deteriorating item with linear demand and partial backorder. Often customers want to buy multiple items in one order, especially when delivery cost is high and there are alternative sellers available to turn to. In this situation, the rate of purchase for one inventory item is influenced by stock outs of another inventory items that were requested together. In this inventory model for a set of inter-dependent inventory items whose purchases are partially interdependent, we find an optimal solution for ordering terms and fill rate that minimize the costs of lost sales, backordering, ordering, and holding inventories. We find that purchase dependency influences the behavior of cost function, making it very different from the typical cost functions reported in previous research.*

I. Introduction

Ghare, P. M. and Scharder, G. F. (1963) gives the concept of deteriorating items develop a model for exponentially decaying.Yadav,R.K., etl.(2014), developed an inventory model for deteriorating and repairable items with linear demand. Yadav, R.K.,etl.(2015),also worked on repairable items for production inventory with random deterioration .Yadav,R.K.,(2015) etl,developed an inventory model for variable demand, constant holding cost and without shortages

There has been much work regarding inventory modeling for lost sales and partial backordering (Montgomery et al, 1973; Park 1982; Rosenberg 1979; Pentico and Drake 2009). Most of the studies determine optimal inventory policies for one inventory item, assuming that the inventory policy for one item does not influence the cost of inventory for other inventory items (Bijvank and Vis, 2011).Multi-item inventory systems were first discussed by Federgruen, Groenevelt, and Tijms (1984) who found out that coordinated replenishments for multiple items can significantly reduce total inventory costs because placing orders for multiple items in one replenishment order would reduce set-up costs. Their algorithm, however, did not consider the possibility of the effect of an item's inventory position on other items' sales. Recently, researchers have started to realize the complicated natures of the multi-item inventory systems. The demands for multiple inventory items might be correlated, affecting the optimal can-order policies for a two-item inventory system (Liu and Yuan, 2000). When one product is out of stock, the demand might be satisfied with other available products, requiring an inventory model for substitutable products (Yadavalli, Van Schoor, and Udayabaskaran, 2006). A lost sale for a major item might reduce the sales of minor items (Zhang, 2012).The existing literature, however, have not unveiled the inter-related nature of inventory policies across multiple items when purchase rate of one inventory item is influenced by other items' stockouts. This purchase pattern has been reported in empirical studies by Lee and Seo (2007) who found that in spare parts markets, many customers want to receive deliveries of all needed spare parts in one shipment because delivery cost is high and they have alternative sellers for the spare parts. We develop an inventory model for a set of interdependent inventory items whose purchases are partially inter-dependent.

Assuming partial backordering, deterioration and linear demands, we find an optimal solution for reorder terms and fill rate that minimize the costs of lost sales, backordering, ordering, and holding inventories.

II. Partial backorder

We develop our model based on the logic and notation of Pentico and Drake (2009)'s deterministic EOQ model for one inventory item. Pentico and Drake advanced the basic EOQ model by permitting partial backordering when stockout occurred. When the basic EOQ assumption of no stockouts is relaxed, two possible scenarios of customer response are backorders and lost sales. Because it is not realistic to assume that all customers are willing to wait for re-stocking, or that all customers are to walk away, researchers have developed inventory models for partial backorders (Montgomery et al 1973; Vijayan and Kumaran, 2008; Padmanabhan and Vrat, 1990). After reviewing the existing models for partial backordering, we find the Pentico and Drake (2009)'s model most appropriate for extending to our multi-item scenario.

III. Decision Variables and Given Information

Inventory policy is about determining when to order and how much to order for each inventory item. With the assumption of linear demand where a part of demand is deterministic and other part of demand varies with constant rate, however, the two decisions can be made with respect to the length of order cycle and the fraction of time that the item is out of stock (Refer to the Figure 1). Therefore, the decision variables for our model are as follows:

T_i order cycle length for item i
S_i stockout rate for i^{th} item i. e. , the fraction of time that item i is out of stock

In order to determine the order cycle length and the stockout rate for multiple items, we need information on demand, cost structure, and purchase pattern for each item. The information that should be given is defined below.

θ_i deterioration rate of i^{th} items θ_i<<1
d_i a_i+b_it, demand rate for item i in units pre unit time $a_i \geq 0, b_i$<<1
C_{oi} ordering cost of i^{th} items for placing and receiving an order
C_{pi} the opportunity cost per unit of lost sales of i^{th} items , including lost profit and any good will loss,
C_{hi} the unit holding cost of i^{th} items for one unit of time,
C_{bi} the cost to keep one unit backordered of i^{th} items for one unit time,
r_{li} the lost-sales rate of i^{th} items i. e. , the fraction of demands that are lost when the item is not in stock
α_{ij} the fraction of the demands for item i that are wanted together with j
P_i purchase rate for item i while the item i is in stock, in units per unit time

$$P_i = d_i(1-\sum_j r_{lj}\alpha_{ij}S_j),\ i \neq j \quad \ldots(1)$$

IV. Formulation of Model

The invenytory of i^{th} item is governed by the differential equation of

$$\frac{dI_i(t)}{dt} + \theta I_i(t) = -(a_i + b_i t), 0 \leq t \leq T_i \quad \ldots(2)$$

$$I_i(t) = -e^{-\theta_i t}\int (a_i + b_i t)e^{\theta_i t}dt$$

$$= -\{\frac{a_i}{\theta_i} + \frac{b_i}{\theta_i}(t - \frac{1}{\theta_i})\} + \mathrm{C}\,e^{-\theta_i t}$$

$$I_i(0) = -(\frac{a_i}{\theta_i} - \frac{b_i}{\theta_i^2}) + C$$

$$= Q_i$$

$$C = Q_i + (\frac{a_i}{\theta_i} - \frac{b_i}{\theta_i^2})$$

Now

$$I_i(t) = -\{\frac{a_i}{\theta_i} + \frac{b_i}{\theta_i}(t - \frac{1}{\theta_i})\} + (Q_i + \frac{a_i}{\theta_i} - \frac{b_i}{\theta_i^2})e^{-\theta_i t}$$

$$= -\{\frac{a_i}{\theta_i} + \frac{b_i}{\theta_i}(t - \frac{1}{\theta_i})\} + (Q_i + \frac{a_i}{\theta_i} - \frac{b_i}{\theta_i^2})(1 - \theta_i t)$$

where

$$= Q_i(1 - \theta_i t) - a_i + \frac{2b_i t}{\theta_i}$$

$$I_i(0) = \mathrm{Q}_i \quad \ldots(3)$$

Then

The demand for certain item refers to the quantity customers are willing to purchase. However, not all of the demand quantity is realized as sales. Customers make inquiries on stock availability, and then make decision about purchase based on stock availability of the needed items. If one of the items required in one order is not available the customer may not actually place the order.

Therefore, we distinguish the purchase rate from the demand rate. Purchase rate is influenced by stockouts of the other items that are demanded in the same order. Equation (1) explains how the purchase rate i affected by stockouts of other items. Item i's demand rate a_i+b_it is reduced by the portion of the demand that is ordered together with other items α_{ij} , multiplied by the probability that the other co-demanded item j is out of stock r_{li}, then multiplied by the fraction of times that customers walk away because the other item j is out of stock S_j. In Figure 1 that shows the inventory position of item i, the purchase rate is indicated as the reduction rate, or the slope of the inventory position P_j because the actual inventory level is determined by the purchase rate, not by the demand rate.

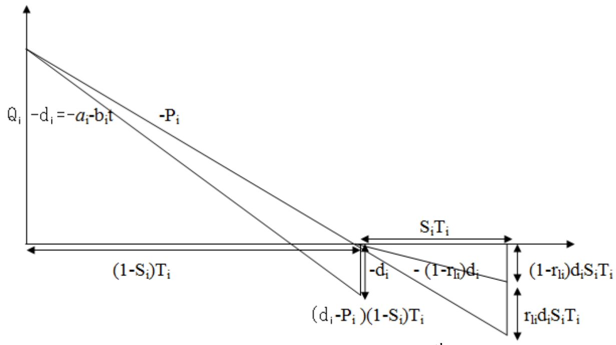


Figure1 Inventory Graph of i^{th} item

V. Objective Function

Next, we define the objective function for our deterministic inventory model with partial backordering and purchase-dependency. The objective is to minimize the total inventory-related cost. The total inventory-related cost TC(T, S) is defined as the average cost per unit time. The cost elements are; the ordering cost, inventory holding cost, backordering cost, the cost of lost sales due to the stockout of the item, and the cost of lost sales due to the stock out of other items even though the item is in stock.

Inventory holding cost of ith items

$$= C_{hi}\int_0^{T_i(1-S_i)} P_iI_i(t)dt$$

$$= C_{hi}\int_0^{T_i(1-S_i)} (1-\sum_j C_{li}\alpha_{ij}S_i)\{Q_i(1-\theta_i t)a_i - a_i^2 + \frac{2a_ib_it}{\theta_i} + Q_i(1-\theta_i t)\,\mathrm{b}_i\,t - a_i\,\mathrm{b}_i\,t + \frac{2b_i^2t^2}{\theta_i}\}dt$$

$$= C_{hi}(1-\sum_j C_{li}\alpha_{ij}S_i)\{Q_i(\mathrm{t}-\frac{\theta_i t^2}{2})a_i - a_i^2t + \frac{a_ib_it^2}{\theta_i} + Q_i(\frac{\mathrm{t}^2}{2}-\frac{\theta_i t^3}{3})\,\mathrm{b}_i - a_i\,\mathrm{b}_i\,\frac{t^2}{2} + \frac{2b_i^2t^3}{3\theta_i}\}_0^{T_i(1-S_i)}$$

$$= C_{hi}(1-\sum_j C_{li}\alpha_{ij}S_i)\{Q_i(T_i(1-S_i)-\frac{\theta_iT_i^2(1-S_i)^2}{2})a_i - a_i^2T_i(1-S_i) + \frac{a_ib_iT_i^2(1-S_i)^2}{\theta_i} +$$

$$Q_i(\frac{T_i^2(1-S_i)^2}{2}-\frac{\theta_iT_i^3(1-S_i)^3}{3})\,\mathrm{b}_i - a_i\,\mathrm{b}_i\,\frac{T_i^2(1-S_i)^2}{2} + \frac{2b_i^2T_i^3(1-S_i)^3}{3\theta_i}\}$$

…(4)

back ordering cost

$$= C_{bi}\int_0^{T_iS_i}(1-C_{li})(a_i+b_it)\,\mathrm{t}\,dt$$

$$= C_{bi}(1-C_{li})(a_i\frac{T_i^2S_i^2}{2}+\frac{b_iT_i^3S_i^3}{3})$$

…(5)

Cost of lost sales due to stock out of the item

$$= C_{pi}\int_0^{T_iS_i} C_{li}(a_i+b_it)dt$$

$$= C_{pi}C_{li}(a_iT_iS_i+\frac{b_iT_i^2S_i^2}{2})$$

…(6)

Cost of lost sales due to stock out of the other item

$$= C_{pi} \int_{0}^{T_i(1-S_i)} \{(a_i + b_i t) - \mathrm{P}_i\} dt$$

$$= C_{pi} \int_{0}^{T_i(1-S_i)} (a_i + b_i t) \sum_j r_{lj} \alpha_{ij} S_j dt$$

$$= C_{pi} a_i \sum_j r_{lj} \alpha_{ij} S_j T_i (1-S_i) + \frac{b_i \sum_j r_{lj} \alpha_{ij} S_j T_i^2 (1-S_i)^2}{2}$$

…(7)

Total inventory-related cost TC(T, S)

$$TC(T,\mathrm{S}) = \sum_{i=1}^{n} [\frac{C_{oi}}{T_i} + \frac{C_{hi}}{T_i} \int_{0}^{T_i(1-S_i)} P_i I_i(t) dt + \frac{C_{bi}}{T_i} \int_{0}^{T_i S_i} (1 - r_{li})(a_i + b_i t)\, \mathrm{t}\, dt +$$

$$\frac{C_{pi}}{T_i} \int_{0}^{T_i S_i} C_{li}(a_i + b_i t) dt + \frac{C_{pi}}{T_i} \int_{0}^{T_i(1-S_i)} \{(a_i + b_i t) - \mathrm{P}_i\} dt]$$

$$TC(T,\mathrm{S}) = \sum_{i=1}^{n} [\frac{C_{oi}}{T_i} + \frac{C_{hi}}{T_i} (1 - \sum_j r_{lj} \alpha_{ij} S_j) \{Q_i (T_i (1-S_i) - \frac{\theta_i T_i^2 (1-S_i)^2}{2}) a_i - a_i^2 T_i (1-S_i) + \frac{a_i b_i T_i^2 (1-S_i)^2}{\theta_i} +$$

$$Q_i (\frac{T_i^2 (1-S_i)^2}{2} - \frac{\theta_i T_i^3 (1-S_i)^3}{3}) \mathrm{b}_i - a_i \,\mathrm{b}_i \frac{T_i^2 (1-S_i)^2}{2} + \frac{2 b_i^2 T_i^3 (1-S_i)^3}{3\theta_i}\} + \frac{C_{bi}}{T_i} (1 - C_{li})(a_i \frac{T_i^2 S_i^2}{2} +$$

$$\frac{b_i T_i^3 S_i^3}{3}) + \frac{C_{pi}}{T_i} C_{pi} C_{li} (a_i T_i S_i + \frac{b_i T_i^2 S_i^2}{2}) + \frac{C_{pi}}{T_i} a_i \sum_j r_{lj} \alpha_{ij} S_j T_i (1-S_i) + \frac{C_{pi} b_i \sum_j r_{lj} \alpha_{ij} S_j T_i^2 (1-S_i)^2}{2}]$$

$$= \sum_{i=1}^{n} [\frac{C_{oi}}{T_i} + C_{hi} (1 - \sum_j r_{lj} \alpha_{ij} S_j) \{Q_i ((1-S_i) - \frac{\theta_i T_i (1-S_i)^2}{2}) a_i - a_i^2 (1-S_i) + \frac{a_i b_i T_i (1-S_i)^2}{\theta_i} +$$

$$Q_i (\frac{T_i (1-S_i)^2}{2} - \frac{\theta_i T_i^2 (1-S_i)^3}{3}) \mathrm{b}_i - a_i \,\mathrm{b}_i \frac{T_i (1-S_i)^2}{2} + \frac{2 b_i^2 T_i^2 (1-S_i)^3}{3\theta_i}\} + C_{bi} (1 - r_{li})(a_i \frac{T_i S_i^2}{2} +$$

$$\frac{b_i T_i^2 S_i^3}{3}) + C_{pi} r_{li} (a_i S_i + \frac{b_i T_i S_i^2}{2}) + C_{pi} a_i \sum_j r_{lj} \alpha_{ij} S_j (1-S_i) + \frac{C_{pi} b_i \sum_j r_{lj} \alpha_{ij} S_j T_i (1-S_i)^2}{2}]$$

…(8)

VI. Solving For Optimal T And F

Taking the first partial derivatives of TC(T_i, F_i)with respect to T_i and F_i respectively, and setting them equal to 0,

$$\frac{\partial TC(T_i, \mathrm{S}_i)}{\partial T_i} = -\frac{C_{oi}}{T_i^2} + C_{hi} (1 - \sum_j C_{lj} \alpha_{ij} S_j) \{Q_i (-\frac{\theta_i a_i (1-S_i)^2}{2}) a_i + \frac{a_i b_i (1-S_i)^2}{\theta_i} +$$

$$Q_i (\frac{(1-S_i)^2}{2} - \frac{2\theta_i T_i \ (1-S_i)^3}{3}) \mathrm{b}_i - a_i \,\mathrm{b}_i \frac{(1-S_i)^2}{2} + \frac{4 b_i^2 T_i \ (1-S_i)^3}{3\theta_i}\} +$$

$$C_{bi} (1 - C_{li})(a_i \frac{S_i^2}{2} + \frac{2 b_i T_i S_i^3}{3}) + C_{pi} C_{li} (a_i S_i + \frac{b_i S_i^2}{2}) + \frac{C_{pi} b_i \sum_j C_{lj} \alpha_{ij} S_j (1-S_i)^2}{2}$$

$$= 0.$$

Considering b_i<<1 and S_i<<1 then neglecting higher power of these variables
We have

$$-C_{oi}+C_{hi}(1-\sum_j r_{lj}\alpha_{ij}S_j)T_i^2\{Q_i(-\frac{\theta_i a_i(1-S_i)^2}{2})a_i+\frac{a_i b_i(1-S_i)^2}{\theta_i}+Q_i\frac{(1-S_i)^2}{2}\mathrm{b}_i-a_i\,\mathrm{b}_i\frac{(1-S_i)^2}{2}\}+$$
$$C_{bi}(1-r_{li})\frac{a_iS_i^{\,2}}{2}T_i^2+C_{pi}r_{li}(a_iS_i+\frac{b_iS_i^{\,2}}{2})T_i^2+\frac{b_iC_{pi}\sum_j r_{lj}\alpha_{ij}S_j(1-S_i)^2}{2}T_i^2=0.$$

and

$$T_i^*=\sqrt{\frac{C_{oi}}{C_{hi}(1-\sum_j r_{lj}\alpha_{ij}S_j)\{-\frac{Q_i\theta_i a_i^2(1-S_i)^2}{2}+\frac{a_ib_i(1-S_i)^2}{\theta_i}+\frac{Q_i(1-S_i)^2\mathrm{b}_i}{2}-a_i\,\mathrm{b}_i\frac{(1-S_i)^2}{2}\}+C_{bi}(1-r_{li})\frac{a_iS_i^{\,2}}{2}+C_{pi}r_{li}(a_iS_i+\frac{b_iS_i^{\,2}}{2})+\frac{b_iC_{pi}\sum_j r_{lj}\alpha_{ij}S_j(1-S_i)^2}{2}}}$$

…(9)

$$\frac{\partial TC(T_i,\mathrm{S}_i)}{\partial S_i}=C_{hi}(1-\sum_j r_{lj}\alpha_{ij}S_j)\{-Q_i+2a_i\theta_iT_i(1-S_i)+a_i^2-\frac{2a_ib_iT_i(1-S_i)}{\theta_i}+$$
$$-Q_iT_i(1-S_i)\mathrm{b}_i+a_i\,\mathrm{b}_i\,T_i(1-S_i)\}+C_{bi}(1-r_{li})(a_iT_iS_i+b_iT_i^2S_i^{\,2})+$$
$$C_{pi}r_{li}(a_i+\frac{2b_iT_iS_i}{2})-C_{pi}a_i\sum_j r_{lj}\alpha_{ij}S_j+\left(C_{pi}b_i\sum_j r_{lj}\alpha_{ij}S_jT_i\{S_i-1\}\right)$$

Or

$$\frac{\partial TC(T_i,\mathrm{S}_i)}{\partial S_i}=C_{bi}(1-r_{li})b_iT_i^2S_i^2+\{C_{hi}(1-\sum_j r_{lj}\alpha_{ij}S_j)(-2a_i\theta_iT_i+\frac{2a_ib_iT_i}{\theta_i}+Q_iT_i\,\mathrm{b}_i-a_i\,\mathrm{b}_i\,T_i)+$$
$$C_{bi}(1-r_{li})a_iT_i+C_{pi}r_{li}b_iT_i)+C_{pi}b_i\sum_j r_{lj}\alpha_{ij}S_jT_i\}S_i+C_{hi}(1-\sum_j r_{lj}\alpha_{ij}S_j)(-Q_i+2a_i\theta_iT_i$$
$$+a_i^2-\frac{2a_ib_iT_i}{\theta_i}-Q_iT_i\,\mathrm{b}_i+a_i\,\mathrm{b}_i\,T_i)+C_{pi}r_{li}a_i-C_{pi}a_i\sum_j r_{lj}\alpha_{ij}S_j-\left(C_{pi}b_i\sum_j r_{lj}\alpha_{ij}S_jT_i\right)$$
$$=0$$

$$\frac{\partial TC(T_i,\mathrm{S}_i)}{\partial S_i}=S_i^{\,2}\{a_iC_{hi}C_{li}\alpha_{ij}(\frac{\theta_iT_i}{2}-\frac{b_iT_i}{\theta_i}-\frac{Q_iT_i\,\mathrm{b}_i}{2a_i}+\frac{\mathrm{b}_i\,T_i}{2})-C_{hi}C_{li}\alpha_{ij}(-2a_i\theta_iT_i+\frac{2a_ib_iT_i}{\theta_i}+$$
$$+Q_iT_i\,\mathrm{b}_i-a_i\,\mathrm{b}_i\,T_i)+C_{bi}(1-C_{li})b_iT_i^2+\frac{3C_{pi}b_iC_{li}\alpha_{ij}T_i}{2}\}+S_i\{C_{hi}C_{li}\alpha_{ij}Q_i-a_i\theta_iT-a_i^2+\frac{2a_ib_iT_i}{\theta_i}+$$
$$Q_iT_i\,\mathrm{b}_i-T_ia_i\,\mathrm{b}_i+C_{bi}(1-C_{li})a_iT_i+b_iT_iC_{pi}C_{li}-2C_{pi}a_iC_{li}\alpha_{ij}-2C_{pi}b_iC_{li}\alpha_{ij}T_i\}$$
$$-C_{hi}\{C_{li}\alpha_{ij}(Q_i-\frac{2a_i\theta_iT_i}{2}-a_i^2+\frac{a_ib_iT_i}{\theta_i}+\frac{Q_iT_i\,\mathrm{b}_i}{2}-\frac{a_i\,\mathrm{b}_i\,T_i}{2})+Q_i-2a_i\theta_iT+a_i^2+\frac{2a_ib_iT_i}{\theta_i}+$$
$$+Q_iT_i-a_i\,\mathrm{b}_i\,T_i\}+C_{pi}C_{li}a_i+C_{pi}a_iC_{li}\alpha_{ij}+\frac{\left(C_{pi}b_iC_{li}\alpha_{ij}T_i\right)}{2}$$
$$=0$$

We have

$$S_i^* = \frac{\begin{array}{l} -\{C_{hi}(1-\sum_j r_{lj}\alpha_{ij}S_j)(-2a_i\theta_i T_i^* + \frac{2a_ib_iT_i^*}{\theta_i} + Q_iT_i^*\,\mathrm{b}_i - a_i\,\mathrm{b}_i\,T_i^*) + C_{bi}(1-r_{li})a_iT_i^* + \\ C_{pi}r_{li}b_iT_i^*) + C_{pi}b_i\sum_j r_{lj}\alpha_{ij}S_jT_i^*\} \\ \pm\sqrt{\begin{array}{l}\{C_{hi}(1-\sum_j r_{lj}\alpha_{ij}S_j)(-2a_i\theta_i T_i^* + \frac{2a_ib_iT_i^*}{\theta_i} + Q_iT_i^*\,\mathrm{b}_i - a_i\,\mathrm{b}_i\,T_i^*) + C_{bi}(1-r_{li})a_iT_i^* + C_{pi}r_{li}b_iT_i^*) \\ +C_{pi}b_i\sum_j r_{lj}\alpha_{ij}S_jT_i^*\}^2 - 4C_{bi}(1-r_{li})b_iT_i^{*2}C_{hi}(1-\sum_j r_{lj}\alpha_{ij}S_j)(-Q_i + 2a_i\theta_i T^*{}_i \\ +a_i{}^2 - \frac{2a_ib_iT_i^*}{\theta_i} - Q_iT_i^*\,\mathrm{b}_i + a_i\,\mathrm{b}_i\,T_i^*) + C_{pi}r_{li}a_i - C_{pi}a_i\sum_j r_{lj}\alpha_{ij}S_j - \left(C_{pi}b_i\sum_j r_{lj}\alpha_{ij}S_jT_i^*\right)\end{array}}\end{array}}{C_{bi}(1-r_{li})b_iT_i^{*2}}$$

…(10)

The optimal solution indicates that the optimal reorder term is a function of other items' stockout rates. The optimal stockout rate is a function of the other items' re-order term. The optimal solution shown in (9),(10) makes sense only when the following condition is met.

$$C_{hi}(1-\sum_j r_{lj}\alpha_{ij}S_j)\{\frac{a_ib_i(1-S_i)^2}{\theta_i} + \frac{Q_i(1-S_i)^2\,\mathrm{b}_i}{2}\} + C_{bi}(1-r_{li})\frac{a_iS_i{}^2}{2} + C_{pi}r_{li}(a_iS_i + \frac{b_iS_i{}^2}{2}) +$$
$$\frac{b_iC_{pi}\sum_j r_{lj}\alpha_{ij}S_j(1-S_i)^2}{2} \geq_{hi} (1-\sum_j r_{lj}\alpha_{ij}S_j)\{\frac{Q_i\theta_ia_i{}^2(1-S_i)^2}{2} + a_i\,\mathrm{b}_i\,\frac{(1-S_i)^2}{2}\}$$

…(11)

In addition, S_i values should be a fractional value.

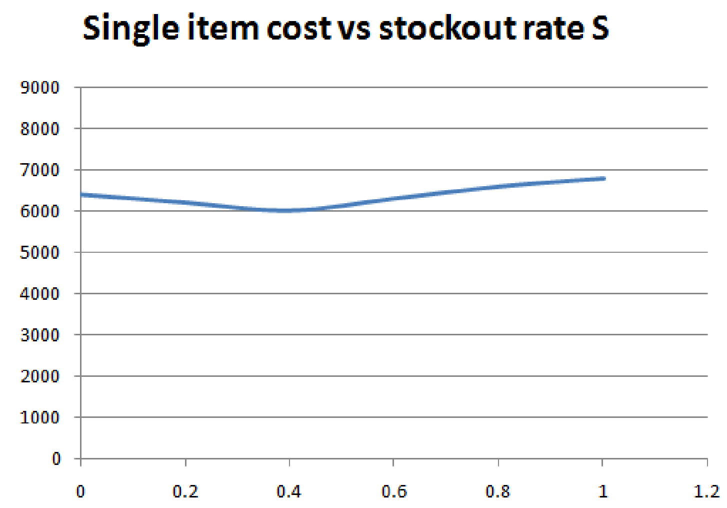
$$0 \leq \frac{\begin{array}{l} -\{C_{hi}(1-\sum_j r_{lj}\alpha_{ij}S_j)(-2a_i\theta_i T_i^* + \frac{2a_ib_iT_i^*}{\theta_i} + Q_iT_i^*\,\mathrm{b}_i - a_i\,\mathrm{b}_i\,T_i^*) + C_{bi}(1-r_{li})a_iT_i^* + \\ C_{pi}r_{li}b_iT_i^*) + C_{pi}b_i\sum_j r_{lj}\alpha_{ij}S_jT_i^*\} \\ \pm\sqrt{\begin{array}{l}\{C_{hi}(1-\sum_j r_{lj}\alpha_{ij}S_j)(-2a_i\theta_i T_i^* + \frac{2a_ib_iT_i^*}{\theta_i} + Q_iT_i^*\,\mathrm{b}_i - a_i\,\mathrm{b}_i\,T_i^*) + C_{bi}(1-r_{li})a_iT_i^* + C_{pi}r_{li}b_iT_i^*) \\ +C_{pi}b_i\sum_j r_{lj}\alpha_{ij}S_jT_i^*\}^2 - 4C_{bi}(1-r_{li})b_iT_i^{*2}C_{hi}(1-\sum_j r_{lj}\alpha_{ij}S_j)(-Q_i + 2a_i\theta_i T^*{}_i \\ +a_i{}^2 - \frac{2a_ib_iT_i^*}{\theta_i} - Q_iT_i^*\,\mathrm{b}_i + a_i\,\mathrm{b}_i\,T_i^*) + C_{pi}r_{li}a_i - C_{pi}a_i\sum_j r_{lj}\alpha_{ij}S_j - \left(C_{pi}b_i\sum_j r_{lj}\alpha_{ij}S_jT_i^*\right)\end{array}}\end{array}}{C_{bi}(1-r_{li})b_iT_i^{*2}} \leq 1$$

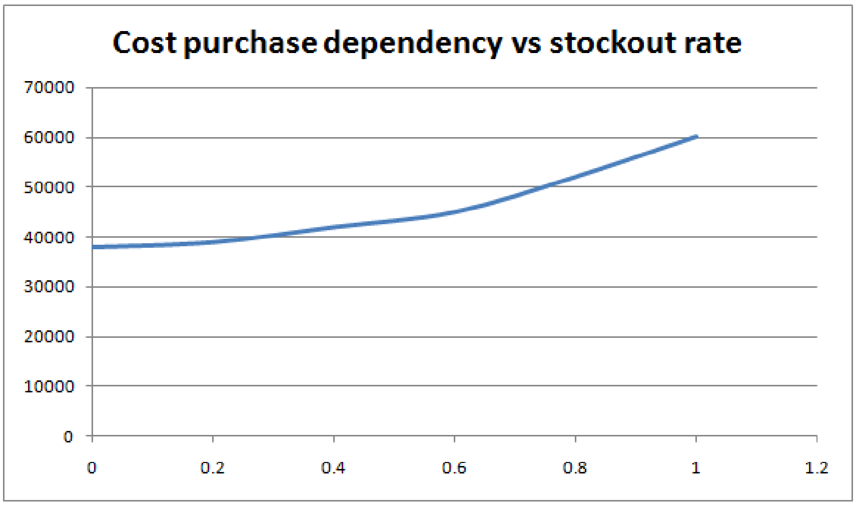
…(12)

If the above condition is not met, the optimal condition is not determined by (9) and (10). Past study shown partial backordering models found that there is certain conditions on the backordering rate that makes a partial backordering solution makes sense. If all customers will wait (r_{li}=0), it is optimal to allow some stockouts. If no customers will wait(r_{li}=1) it is optimal to either allow no stockouts or lose all sales.or lose all sales. However, the conditions for our optimal solution are convoluted with the decision variables for other inventory items.

VII. Relation Between Purchase Dependency And Inventory Cost

In order to understand the effect of purchase dependency on inventory cost, we compared the cost function under the single-item assumption as calculated by Pentico and Drake (2009) and the cost function with the purchase dependency, under the same conditions. The following two example sets of comparison demonstrate that considering the purchase dependency changes the costs function significantly, resulting in a very different optimal S_i values.





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