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Mixed Convection Flow of Viscous Dissipating Fluid through Channel Moving in Opposite Direction

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Abstract: *The present paper pursuit significantly analyses the free convection flow of viscous incompressible fluid bounded by two vertical plates filled with the porous matrix in the presence of transverse magnetic field with periodic temperature at one plate is discussed when both the plates are moving in opposite directions. The non-dimensional governing equations are analytically solved by applying the homotopy perturbation technique. The effect of various physical parameters is discussed in the flow field numerically, and presented by figures. The effect of Skin friction and rate of heat transfer in terms of Nusselt number is shown by tables. It is observed that the magnetic parameter (M) has a retarding effect on the main flow velocity u, whereas the permeability parameter* k_p *or convective heat force parameter* (G_r) *has an accelerating effect on it. Both Prandtl number (Pr) and suction parameter (Re) reduce the temperature at all the points, but magnetic parameter (M) or dissipation parameter (Ec) reverses the effect. Further, a growing suction parameter (Re) enhances the skin friction at the plate while magnetic parameter diminishes it. The effect of increasing magnetic parameter (M) or viscous dissipation parameter (Ec) is to reduce the rate of heat transfer at the plate while a growing Prandtl number (Pr) or suction parameter (Re) reverses the effect. The problem is very much significant in view of its several engineering, geophysical and industrial applications.*

Keywords: *Free convection, Porous matrix, Suction, Skin-friction, Nusselt number, Viscous dissipation, Suction*

I. Introduction

The problem of hydro magnetic flow with heat transfer in a region occupied by fluid saturated with porous medium and free convection caused by temperature variations leading to accelerating flow field at every point of the fluid. The problem with this process of heat transfer is encountered in both the geophysical and industrial environmental including thermal energy storage system, a solar collector, hot plates, freezers and ovens, etc. with a porous absorber, but problem of heat transfer turns complex due to variation in the plate temperature.

The importance of channel flow through porous media in the field of applied engineering and geophysical such as is for filtration, purification processes, the study of underground water resources, seepage of water in river beds, in the petroleum industry to study the movement of natural gas, oil and water through the oil channels and reservoirs.

The principle of controlling the natural convection on a heated surface and the temperature of a heated body by suction of fluid finds its applications, of which a high temperature heat exchanger is one such example.

In view of these applications, a series of investigations has been made by different scholars. Gersten and Gross [1] studied the flow and heat transfer along a plane wall with periodic suction. Gulab and Mishra [2] analyzed the unsteady MHD flow of conducting fluid through a porous medium. Kaviany [3] explained the laminar flow through a porous channel bounded by isothermal parallel plates. Attia & Kotb [4] presented the MHD flow between two parallel plates with heat transfer. The unsteady hydro magnetic natural convection in a fluid saturated porous channel was studied by Chamkha [5]. Ahmed and Sharma [6] discussed three dimensional free convective flow and heat transfer through porous media. Taylor [7] and Richardson [8] continued the investigation in which they modeled fluid flow by Darcy number. MHD unsteady free convective flow between two vertical walls contained porous medium analyzed by Sarangi and Jose [9]. Sharma and yadav [10] analyzed the heat transfer through three dimensional Couette flow between a stationary porous plate bounded by porous medium and a moving porous plate. Nield and Bejan [11] and Kaviany [12], Vafai and Tien [13] also analyzed the influence of solid boundary and the inertia forces on flow and heat transfer using porosity. Rudriarh and Nagraj [14] discussed the fully free connective flow through a porous medium bounded by heated plates. Backermann [15] proved into natural convection in vertical enclosures with porous layers Singh [16] considered steady magneto hydrodynamic fluid flow between two parallel plates. Al-Hadhrami et al. [17] considered the flow of fluids through horizontal channels of porous materials and obtained velocity expressions. Singh [18] perceived the effect of mass transfer on free convection in MHD flow of viscous fluid. Singh and Gupta [19] considered the effect of mass transfer on transient MHD free convection flow of an incompressible viscous fluid. Das et al. [20] analyzed the periodic suction/ injection effect in three dimensional Couette flow. Bareletta and Celli [21] studied the mixed convection MHD flow in a vertical channel: effects of Joule heating and viscous dissipation. Seth et al. [22] described MHD Couette flow with porous channels. Deka and Bhattacharya [23] considered unsteady free convection Couette flow of heat generating/ absorbing fluid saturated with porous. Gupta et al. [24] investigated free convection flow between two plates moving opposite direction partially filled with porosity. Sharma & Saxena [25] studied viscous incompressible fluid with Co-sinusoidal variable temperature in the Presence of a magnetic field. Ahmed and Kalita, [26] incorporated the model of magneto hydrodynamic transient flow through a porous medium bounded by a hot vertical plate in the presence of radiation. Jain et al. [27] presented the three dimensional heat and mass transfer periodic flow through a vertical porous channel with transpiration cooling and slip boundary condition when plates are moving in opposite directions. Sharma et al. [28] presented the effect of radiation and free convection with periodic temperature in porous media. Raju et al. [29] reported the MHD convective flow Problem through a porous medium in a horizontal channel with expressing the effect of magnetic parameter and permeability.

Viscous dissipation changes the temperature distribution by playing a role like an energy source which leads to affect heat transfer rates. Dissipative effects are significant in tribology, instrumentation, food processing, lubrication and many other spheres. Pertinent information about its worth is vital to design an optimal thermal system. Physically, viscous dissipation accounts for the creation of thermal energy due to the viscous stresses. Though the viscous dissipation effect is a feeble effect in comparison to other counterpart effects, but in many situations it has made a decision effects. The effect is more pronounced for high Prandtl number flows. It has been viewed that though the viscous dissipation effect is quantitatively insignificant, in some cases it amounts to noticeable qualitative effect (Gebhart and Mollendorf [30] , Nield [31], Rees et al. [32].) The merit of the effect of viscous dissipation depends on whether the plates are being cooled or heated. The effect of Joule heating is usually characterized by the magnetic parameter. Both of the above effects are important in nuclear engineering (Alim et al. [33]). Soundalgekar [34] explored the viscous dissipative heating effect on the two dimensional unsteady free convective flow past an infinite vertical porous plate. Duwairi [35] reported the effect of Joule heating and viscous dissipation on the forced convection flow in the presence of thermal radiation. Gnaneswara Reddy [36] studied the effects of thermophoresis, viscous dissipation and joule heating on steady MHD flow over isothermal permeable surface. Bhuiyan et al. [37] presented Joule heating effects on MHD natural convection flows in presence of pressure stress work and viscous dissipation from a horizontal circular Cylinder.

He [38, 39] and Biazar and Ghazvini [40] perceived the solution of non-linear coupled equations by applying homotopy perturbation technique.

The proposed study of MHD fluid flow of viscous dissipation is through two vertical plates moving in opposite directions with periodic temperature at one plate.

II. Mathematical Analysis

Consider a channel of an incompressible, viscous and electrically conducting fluid past between two vertical plates filled with a porous medium. Let the plates be $\bar{x}\,\bar{z}$ - plane and $\bar{y}$ axis normal to it. The applied magnetic field $\beta_{0,}$ along the $\bar{y}$ axis in the Presence of constant suction velocity- v_0 is at one heated porous plate. The electrical field owing to the polarization of charges and Hall current is taken negligible. Both the plates are moving in the opposite direction along - $\bar{x}$ and $\bar{x}$ axis and one porous plate is heated with span-wise co-sinusoidal temperature $\bar{\theta} = \emptyset_0\left(1+\in\ \cos\frac{\pi\bar{z}}{a}\right)$ and the other is cooled. Under Bousinesque approximation, the flow and porous medium are governed by the following equations:

(a) Equation of momentum

$$v_0\ \frac{\partial\bar{u}}{\partial\bar{y}} = \ v\left(\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+\frac{\partial^2\bar{u}}{\partial\bar{z}^2}\right)+g\beta\bar{\theta}-\frac{\sigma\beta_0^2\bar{u}}{\rho}-\frac{v}{K_p}\bar{u}\ \ \ldots\ldots\ldots (1)$$

(b) Equation of Energy

$$\rho C_\rho v_0\ \frac{\partial\bar{\theta}}{\partial\bar{y}} = k\left(\frac{\partial^2\bar{\theta}}{\partial\bar{y}^2}+\frac{\partial^2\bar{\theta}}{\partial\bar{z}^2}\right)+\mu\left[\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)^2+\left(\frac{\partial\bar{u}}{\partial\bar{z}}\right)^2\right]+\sigma{\beta_0}^2\bar{u}^2\qquad\ldots\ldots\ldots (2)$$

Corresponding boundary conditions are:

$$\bar{u} = -u_0,\ \ \bar{\theta} = \emptyset_0\left(1+\in cos\frac{\pi\bar{z}}{a}\right)\quad at\ \bar{y}=0$$

$$\bar{u} = u_{0,}\quad \bar{\theta} = 0\qquad at\quad \bar{y}=a$$

$v = v_0$ Suction constant velocity………… (3)

Introducing dimensionless quantities:

$u = \frac{\bar{u}}{u_0}$, $y = \frac{\bar{y}}{a}$, $z = \frac{\bar{z}}{a}$, $\theta = \frac{\bar{\theta}}{\emptyset_0}$,
$R_e = \frac{-v_0 a}{\upsilon}$, $P_r = \frac{u_0 C_p}{K}$, $G_r = \frac{g\beta\emptyset_0\upsilon}{u_0 {v_0}^2}$,
$E_c = \frac{{u_0}^2}{C_p\emptyset_0}$, $M^2 = \frac{\sigma{\beta_0}^2 a^2}{\mu}$, $k_p = \frac{\bar{K}_p}{a^2}$, .. (3a)

Equation (1) and (2) become, using (3a)

$$\frac{\partial u}{\partial y} = -\frac{1}{R_e}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - G_r R_e\theta + \frac{M^2}{R_e}u + \frac{u}{k_p R_e} \text{............ (4)}$$

$$\frac{\partial \theta}{\partial y} = -\frac{1}{P_r R_e}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) - \frac{1}{R_e}E_c\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right] - \frac{E_c M^2}{R_e}u^2 \text{............... (5)}$$

The boundary and matching conditions in dimensionless form are:

$y = 0$, $u = -1$, $\theta = 1 + \in cos\pi z$
$y = 1$, $u = 1$, $\theta = 0$………………. (6)

Where g is the acceleration due to gravity, $\emptyset_0$ is the mean temperature, u_0 is the velocity of plate, β is the coefficient of volumetric expansion, σ is the coefficient of electrical conductivity, C_p is specific heat, β_0 is electromagnetic induction, ν is kinematic viscosity and v_0 is the suction velocity, a is the wavelength between plates, k_p the permeability of porous matrix, K is thermal conductivity, u is dynamic velocity, ρ is density, θ is the temperature at any point of fluid flow. Pr is the Prandtl number, M the Hartmann number, Ec the Eckert number and Re the Reynold number, G_r the Grashoff number.

Solutions:

It is observed that above governing equations are nonlinear coupled. Accordingly, we assume, for small $\in$ ($\ll$ 1), a very small constant quantity:

$u(y,z) = u_0(y) + \in u_1(y,z) + o(\in^2)$
$\theta(y,z) = \theta_0(y) + \in\theta_1(y,z) + o(\in^2)$…….(.7)

Substituting equation (7) into the equation (4), (5) and comparing like power of $\in$, neglecting higher power of $\in$, we get:

$$\frac{d^2 u_0}{dy^2} + R_e\frac{du_0}{dy} - \left(M^2 + \frac{1}{k_p}\right)u_0 = -G_r {R_e}^2\theta_0 \quad .. \quad (8) \qquad \text{....... (8)}$$

$$\frac{\partial^2 u_1}{\partial y^2} + R_e\frac{\partial^2 u_1}{\partial z^2} + R_e\frac{\partial u_1}{\partial y} - \left(M^2 + \frac{1}{k_p}\right)u_1 = -G_r {R_e}^2\theta_1 \text{ …. (9)}$$

$$\frac{d^2\theta_0}{dy^2} + p_r R_e\frac{d\theta_0}{dy} = -E_c p_r\left[\left(\frac{du_0}{dy}\right)^2 + M^2 {u_0}^2\right] \text{ ….. (10)} \qquad \text{....... (10)}$$

$$\frac{\partial^2\theta_1}{\partial y^2} + \frac{\partial^2\theta_1}{\partial z^2} + p_r R_e\frac{\partial\theta_1}{\partial y} = -2E_c p_r\frac{du_0}{dy}\frac{\partial u_1}{\partial y} - 2E_c M^2 p_r u_0 u_1 \text{.................(11)}$$

The corresponding boundary conditions are:

$u_0 = -1$, $u_1 = 0$, $\theta_0 = 1$, $\theta_1 = cos\pi z$ $at\ y = 0$
$u_0 = 1$, $u_1 = 0$, $\theta_0 = 0$, $\theta_1 = 0$ $at\ y = 1$ ..(12)

Solving equations (8) to (11) using the homotopy perturbation technique with boundary conditions (12).Now from the equation (8) and equation (10):

Construct homotopy, [Ref. 38, 39, 40]

H= L (u_0)-L (U_i) + p [L (U_i) + N (u_0) - F(r)] = 0 …..(13)
H = L (θ_0) - L (θ_i) + p [L (θ_i) + N (θ_0) - F(r)] = 0 … (14)

Where, L(u_0), L(θ_0) and N(u_0), N(θ_0) are the Linear and Non-linear term of u_0, θ_0 respectively and U_i, θ_i is the initial value of u_0, θ_0 :

Let, $u_0(y) = u_{00} + p\, u_{01} + \ldots\ldots\ldots$
$\theta_0(y) = \theta_{00} + p\,\theta_{01} + \ldots\ldots\ldots$ (15)

Substituting the equations (8) and (10) into the equations (13) and (14) respectively, and solve it using the equation (15) with the following converted boundary condition from the equation (12), we get:

$u_{00} = -1$, $u_{01} = 0$, $\theta_{00} = 1$, $\theta_{01} = 0$ $at\ y = 0$
$u_{00} = 1$, $u_{01} = 0$, $\theta_{00} = 0$, $\theta_{01} = 0$ $at\ y = 1$

And the solutions of equation (8) and (10) subject to above boundary conditions are:

$$u_0(y) = \beta_1 e^{a_1 y} + \beta_2 e^{b_1 y} - \alpha_1 Sin\frac{\pi}{2}y + \alpha_2 Cos\frac{\pi}{2}y \text{ ……………….. (16)}$$

$$\theta_0(y) = \cos\frac{\pi}{2}y + \frac{(\alpha_4 - \alpha_3)}{1 - e^{-p_r Re}}(1 - e^{-p_r R_e y}) + \alpha_3\left(1 - \cos\frac{\pi}{2}y\right) + \alpha_5(1 - cos2\pi y) - \alpha_4 y + \alpha_6 sin2\pi y.. \quad (17)$$

To solve equation (9) and (11) for u_1 and θ_1, we assume that:

$u_1(y,z) = u_{11}(y)\cos\pi z$……………………….(18)
$\theta_1(y,z) = \varphi(y)\cos\pi z$……………………….. (19)

Substituting the $u_1(y,z)$ and $\theta_1(y,z)$ from equations (18) and (19) into the equations (9) and (11), we get:

$\frac{d^2u_{11}}{dy^2}+R_e\frac{du_{11}}{dy}-(M_1^2+\pi^2)u_{11}=-G_rR_e{}^2\varphi$ …… (20)

$\frac{d^2\varphi}{dy^2}+p_rR_e\frac{d\varphi}{dy}-\pi^2\varphi=-2E_cp_r\frac{du_0}{dy}\frac{du_{11}}{dy}-2E_cM^2p_ru_0u_{11}$………… (21)

The corresponding boundary conditions are:

$u_{11}=0\,,\quad u_1=0\,,\ \varphi=1\,,\ \theta_1=cos\pi z\quad at\ y=0$

$u_{11}=0\,,\quad u_1=0\,,\ \varphi=0\,,\ \theta_1=0\quad at\ y=1$ ... (22)

Solving equations (20) and (21) under the boundary conditions (22) and using the homotopy perturbation technique, we get:

$u_{11}(y,z)=\left(\frac{\alpha_7e^{c_3}+\alpha_8}{e^{c_4}-e^{c_3}}\right)(e^{c_4y}-e^{c_3y})-\alpha_7e^{c_3y}+\alpha_7Cos\frac{\pi}{2}y-\alpha_8Sin\frac{\pi}{2}y$…….(23)

$\varphi(y,z)=\frac{[\{e^{a_2}(1+\gamma_2+\gamma_4)+(\gamma_1-\gamma_3)+\gamma_5(e^{a_2}+e^{b_1})+\gamma_6(e^{a_2}+e^{a_1})\}(e^{b_2y}-e^{a_2y})]}{(e^{a_2}-e^{b_2})}+[(1+\gamma_2+\gamma_4+\gamma_5+\gamma_6)e^{a_2y}]-\gamma_1Sin\frac{3\pi}{2}y-\gamma_2Cos\frac{3\pi}{2}y-\gamma_3Sin\frac{\pi}{2}y-\gamma_4Cos\frac{\pi}{2}y-\gamma_5e^{b_1y}Cos\pi y-\gamma_6e^{a_1y}Cos\pi y-\gamma_7e^{b_1y}Sin\pi y-\gamma 8ea1ySin\pi y$………………. (24)

Where:

u(y,z) = u_0(y) +∈ u_1(y,z) + o(∈^{2})

θ(y,z) = θ_0(y) + ∈θ_1(y,z) + o(∈^{2})

So

u(y,z) = u_0(y) +∈ u_{11}(y,) cosπz + o(∈^{2})…………(25)

θ(y,z) = θ_0(y) + ∈ φ(y,) cosπz + o(∈^{2})…………..(26)

Corresponding constants are:

$M_1{}^2=\left(M^2+\frac{1}{k_p}\right),\ a_1=\frac{-R_e+\sqrt{R_e{}^2+4M_1{}^2}}{2}$,

$b_1=\frac{-R_e-\sqrt{R_e{}^2+4M_1{}^2}}{2}$,

$\alpha_1=\frac{8\pi G_rR_e{}^3}{4R_e{}^2\pi^2+(4M_1{}^2+\pi^2)^2},\alpha_2=\frac{4G_rR_e{}^2(4M_1{}^2+\pi^2)}{4R_e{}^2\pi^2+(4M_1{}^2+\pi^2)^2},\ \alpha_3=1,$

$\alpha_4=\frac{E_c(\pi^2+M^2)}{2R_e},\ \alpha_5=\frac{E_cp_r(\pi^2-M^2)4\pi^2}{2(4\pi^2p_r{}^2R_e{}^2+16\pi^4)}$,

$\alpha_6=\frac{E_cp_r(\pi^2-M^2)2\pi p_rR_e}{2(4\pi^2p_r{}^2R_e{}^2+16\pi^4)}$,

$\alpha_7=\frac{G_rR_e{}^2\left(M_1{}^2+\frac{5\pi^2}{4}\right)}{R_e{}^2\left(\frac{\pi}{2}\right)^2+\left(M_1{}^2+\frac{5\pi^2}{4}\right)^2},\ \alpha_8=\frac{\frac{\pi}{2}G_rR_e{}^3}{R_e{}^2\left(\frac{\pi}{2}\right)^2+\left(M_1{}^2+\frac{5\pi^2}{4}\right)^2}$,

$c_3=\frac{-R_e+\sqrt{R_e{}^2+4(M_1{}^2+\pi^2)}}{2}$,

$c_4=\frac{-R_e-\sqrt{R_e{}^2+4(M_1{}^2+\pi^2)}}{2}\ ,a_2=\frac{-p_rR_e-\sqrt{p_r{}^2R_e{}^2+4\pi^2}}{2}$,

$b_2=\frac{-p_rR_e+\sqrt{p_r{}^2R_e{}^2+4\pi^2}}{2}$

$\beta_1=-\left[\frac{\alpha_1+(1+\alpha_2)e^{b_1}+1}{e^{b_1}-e^{a_1}}\right],\ \beta_2=\left[\frac{(1+\alpha_2)e^{a_1}+1+\alpha_1}{e^{b_1}-e^{a_1}}\right]$

$\gamma_1=\beta_3+\beta_4$, $\gamma_2=\beta_5+\beta_6$, $\gamma_3=\beta_7+\beta_8$, $\gamma 4=\beta_9+\beta_{10}$, $\gamma 5=\beta_{11}+\beta_{12}$, $\gamma_6=\beta_{13}+\beta_{14}$, $\gamma_7=\beta_{15}+\beta_{16}$, $\gamma_8=\beta_{17}+\beta_{18}$

$\beta_3=\frac{E_cp_r\alpha_1\left(\frac{\pi^2}{2}-M^2\right)\left(\frac{-3\pi}{2}\right)p_rR_e}{p_r{}^2R_e{}^2\left(\frac{3\pi}{2}\right)^2+\left(\frac{13\pi^2}{4}\right)^2}$

$\beta_4=\frac{E_cp_r\alpha_2\left(\frac{\pi^2}{2}-M^2\right)\left(\frac{13\pi^2}{4}\right)}{p_r{}^2R_e{}^2\left(\frac{3\pi}{2}\right)^2+\left(\frac{13\pi^2}{4}\right)^2}$,

$\beta_5=\frac{E_cp_r\alpha_1\left(\frac{\pi^2}{2}-M^2\right)\left(\frac{13\pi^2}{4}\right)}{p_r{}^2R_e{}^2\left(\frac{3\pi}{2}\right)^2+\left(\frac{13\pi^2}{4}\right)^2}$,

$\beta_6=\frac{E_cp_r\alpha_2\left(\frac{\pi^2}{2}-M^2\right)\left(\frac{3\pi}{2}p_rR_e\right)}{p_r{}^2R_e{}^2\left(\frac{3\pi}{2}\right)^2+\left(\frac{13\pi^2}{4}\right)^2}$,

$$\beta_7 = -\frac{E_c p_r \alpha_2 \left(\frac{\pi^2}{2} + M^2\right)}{{p_r}^2 {R_e}^2 \left(\frac{\pi}{2}\right)^2 + \left(\frac{5\pi^2}{4}\right)^2}\left(\frac{5\pi^2}{4}\right)$$

$$\beta_8 = \frac{E_c p_r \alpha_1 \left(\frac{\pi^2}{2} + M^2\right)\left(\frac{-\pi}{2}\right) p_r R_e}{{p_r}^2 {R_e}^2 \left(\frac{\pi}{2}\right)^2 + \left(\frac{5\pi^2}{4}\right)^2}$$

$$\beta_9 = \left[\frac{E_c p_r \alpha_1 \left(\frac{\pi^2}{2} + M^2\right)}{{p_r}^2 {R_e}^2 \left(\frac{\pi}{2}\right)^2 + \left(\frac{5\pi^2}{4}\right)^2}\right]\frac{5\pi^2}{4}$$

$$\beta_{10} = -\left[\frac{E_c p_r \alpha_2 \left(\frac{\pi^2}{2} + M^2\right)}{{p_r}^2 {R_e}^2 \left(\frac{\pi}{2}\right)^2 + \left(\frac{5\pi^2}{4}\right)^2}\right]\frac{\pi}{2} p_r R_e$$

$$\beta_{11} = \frac{2E_c p_r \beta_2 b_1 \pi\left(p_r R_e b_1 + {b_1}^2 - 2\pi^2\right)}{(2b_1 + p_r R_e)^2\pi^2 + \left(p_r R_e b_1 + {b_1}^2 - 2\pi^2\right)^2}$$

$$\beta_{12} = \frac{-2E_c M^2 \beta_2 p_r \{\pi(2b_1 + p_r R_e)\}}{(2b_1 + p_r R_e)^2\pi^2 + \left(p_r R_e b_1 + {b_1}^2 - 2\pi^2\right)^2}$$

$$\beta_{13} = \frac{2E_c p_r \beta_1 a_1 \pi({a_1}^2 + p_r R_e a_1 - 2\pi^2)}{\pi^2(p_r R_e - 2a_1)^2 + ({a_1}^2 p_r R_e a_1 - 2\pi^2)^2}$$

$$\beta_{14} = \frac{-2\pi E_c M^2 p_r \beta_1 (p_r R_e + 2a_1)}{\pi^2(p_r R_e + 2a_1)^2 + ({a_1}^2 + p_r R_e a_1 - 2\pi^2)^2}$$

$$\beta_{15} = \frac{2\pi^2 E_c p_r \beta_2 b_1 (2b_1 + p_r R_e)}{(2b_1 + p_r R_e)^2\pi^2 + \left(p_r R_e b_1 + {b_1}^2 - 2\pi^2\right)^2}$$

$$\beta_{16} = \frac{2E_c M^2 p_r \beta_2\left(p_r R_e b_1 + {b_1}^2 - 2\pi^2\right)}{(2b_1 + p_r R_e)^2\pi^2 + \left(p_r R_e b_1 + {b_1}^2 - 2\pi^2\right)^2}$$

$$\beta_{17} = \frac{(2E_c p_r \beta_1 a_1 \pi^2)(p_r R_e + 2a_1)}{\pi^2(p_r R_e + 2a_1)^2 + ({a_1}^2 - p_r R_e a_1 - 2\pi^2)^2}$$

$$\beta_{18} = \frac{2E_c M^2 p_r \beta_1 ({a_1}^2 + p_r R_e a_1 - 2\pi^2)}{\pi^2(p_r R_e + 2a_1)^2 + ({a_1}^2 - p_r R_e a_1 - 2\pi^2)^2}$$

Skin Friction coefficient at the heated plate (y=0) is given by

$(\tau_w) = \left(\frac{\partial u}{\partial y}\right)_{y=0}$

The rate of heat transfer in terms of the Nusselt Numbers at the heated plate (y=0) is given by

$(N_u) = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$

Can be calculated by Presenting Table (1) and (2):

Table-1: Skin friction coefficient for different values of the magnetic parameter (M) and Reynold number (Re).

R_e	M	τ_{w1}(z=1/3	τ_{w2}(z=1/4)
2	0.1	10.533	9.8907
3	0.1	21.204	19.912
2	0.2	10.523	9.876

Table-2: Nusselt Numbers for the different values of magnetic parameter (M) and Prandtl number (Pr), dissipation parameter (Ec) and suction parameter (Re):

E_c	Re	M	Pr	N_u (z=1/3)	N_u (z=1/4)
.001	2	.1	0.7	1.8756	1.8837
.01	2	.1	07	13.7265	13.7547
.01	2	.1	0.7	1.8551	1.8631
.01	2	.5	0.7	1.8546	1.8626
.01	3	.1	0.7	2.3896	2.3983

III. Results And Discussion

The channel flow problem of viscous incompressible fluid filled with porosity between two vertical plates moving in opposite directions and heat transfer with span-wise co-sinusoidal in the presence of transverse magnetic field is examined in the present work. Viscous dissipation and constant suction at the heated plate are taken into consideration for generating inferences. The Bousinesque momentum equation and heat transfer equation are analytically solved by homotopy perturbation technique. The effect of velocity field and temperature field are obtained in terms of Skin friction and Nusselt number with the help of Table 1 and Table 2 respectively.

Computational and graphical results are carried out for different values of magnetic parameter M, convection parameter G_r, permeability parameter Kp, Reynold number R_e, Prandtl number Pr, Eckert number E_c. The effect of these parameters are explicitly shown by velocity and temperature profile figures from (1) to (6); the velocity starts to increase from the heated plate (y=0) and maximum velocity is not inclining in the middle of flow field but moves towards the heated plate and gets exhausted asymptotically at the prescribed velocity of the cold plate. On the other hand, maximum temperature is achieved at the heated plate (y=0) then falls exponentially and finally becomes zero at the cold plate.

Figure 1 and **Figure 2** depict the effect of magnetic parameter M, convection parameter G_r, permeability parameter Kp and suction parameter R_e on the velocity profile. It is also evident that an increase in G_r (Grashoff number) accompanies a rise of flow velocity. This is due to increasing G_r giving mount to the buoyancy effects resulting in more induce flow throughout the region. It is also observed from same described figures that the suction parameter (R_e) affects the main velocity of flow field which rises because suction parameter removes the obstacle dust particles. On the other hand, it is also found that the velocity decreases with increasing M, signifies the increase of resistive type force (Lorentz force) which tends to resist the fluid flow and thus retarding flow velocity. It is noted that increasing of porous medium enhances the velocity of flow so boundary layer thickness reduces because permeability parameter $\overline{K}_p$ is directionally proportionally to actually parameter Kp of porous medium so the resistance of porous medium decreases.

Figure 3: It is observed that decreasing the Prandtl number (Pr), when M = 3, R_e = 2, E_c = .01, G_r = 4 and Kp = 1 increases the thermal conductivity and therefore, heat is able to diffuse away from plate more swiftly than the higher value of Pr. Here, it is very interesting to state that for the higher value of Pr (for water), temperature of fluid flow decreases, but a certain point of the temperature field, diffusion of heat become invariable, hence tends to zero which is away from the heated plate and towards the cold plate resulting in a very low temperature profile.

Figure 4 illustrates the temperature profile for different values of the suction parameter (R_e) when M = 3, E_c = .01, G_r = 4, Kp = 1 and Pr =0 .7, the raising of suction parameter (Re) reduces the temperature at all the points of the flow field. It is attributed to the fact that suction parameter absorbs the heat so the temperature of the fluid region falls.

Figure 5: It encounters to conclude from this figure that the increasing value of the viscous dissipation parameter (Ec) is to enhance the temperature distribution in flow region. This is due to the heat energy accumulated in the liquid because of the internal frictional heating.

Figure 6 shows the temperature profile for different values of magnetic parameter (M). It reveals the temperature increases with increase in M which implies that the presence of transverse magnetic field sets in Lorentz force, which results in retarding force on the velocity field and therefore introduces additional friction heating (Joule heating), hence temperature of the fluid increases.

The Skin friction coefficient and the Nusselt numbers are presented in Table (1) and Table (2) respectively. The Skin friction coefficient for constant values of kp=1 and Gr=4 at the heated plate (y=0) for different values of the suction parameter (Re) and magnetic parameter (M) has been perceived in the table (1). The influence of suction parameter (Re) is to enhance the skin friction coefficient accelerating the velocity of flow at the boundary (y =0). This is because of the fact that suction removes the obstacles at the boundary. The skin friction for various values of magnetic parameter (M) implies that growing of magnetic parameter is to suppress the shear stress coefficient. The presence of a magnetic field decelerates the motion of the flow of the heated plate (y=0) which shows the retarding effect of magnetic field. Nusselt numbers decrease with decreasing Prandtl number (Pr) or decreasing the suction parameter (Re). The reason is that the thermal layer for low Pr or low Re is thick and consequently the rate of heat transfer decreases, while magnetic parameter (M) or dissipating parameter (Ec) reverses the consequence.

IV. Conclusion

MHD channel flow with heat transfer and viscous dissipation effects in porous media are discussed when both the plates are moving in opposite direction to each other.

The following results are concluded:

1) The velocity of a fluid increases due to increase in Grashoff number (Gr), suction parameter (Re) and permeability parameter (Kp). On the other hand, the magnetic parameter (M) decreases the velocity of flow field because of the retarding effect of Lorentz's force.
2) The temperature falls with increasing suction parameters (Re) or Prandtl number (Pr), while it increases due to increase in the magnetic parameter (M) and dissipating parameter (Ec).
3) Shear stress on the heated plate (y=0) decreases with an increase in magnetic field (M) but it shows the reverse effects in case of suction parameter (Re).
4) Rate of heat transfer at the heated plate (y=0) increases due to increase in the suction parameter (Re) or Prandtl number (Pr), whereas it decreases due to increase in the magnetic parameter (M) or dissipation parameter (Ec).

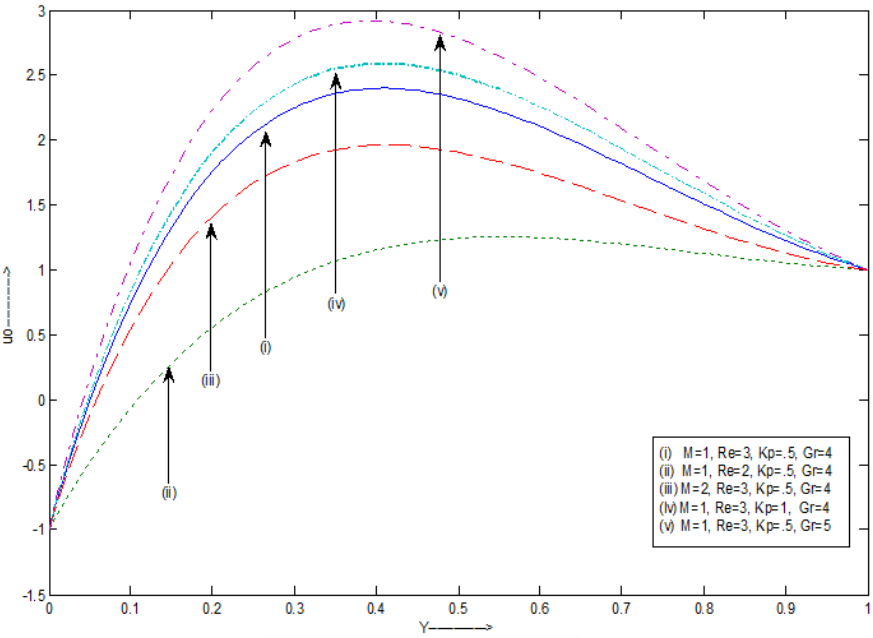

Figure1: Zeroth order velocity distribution versus Y

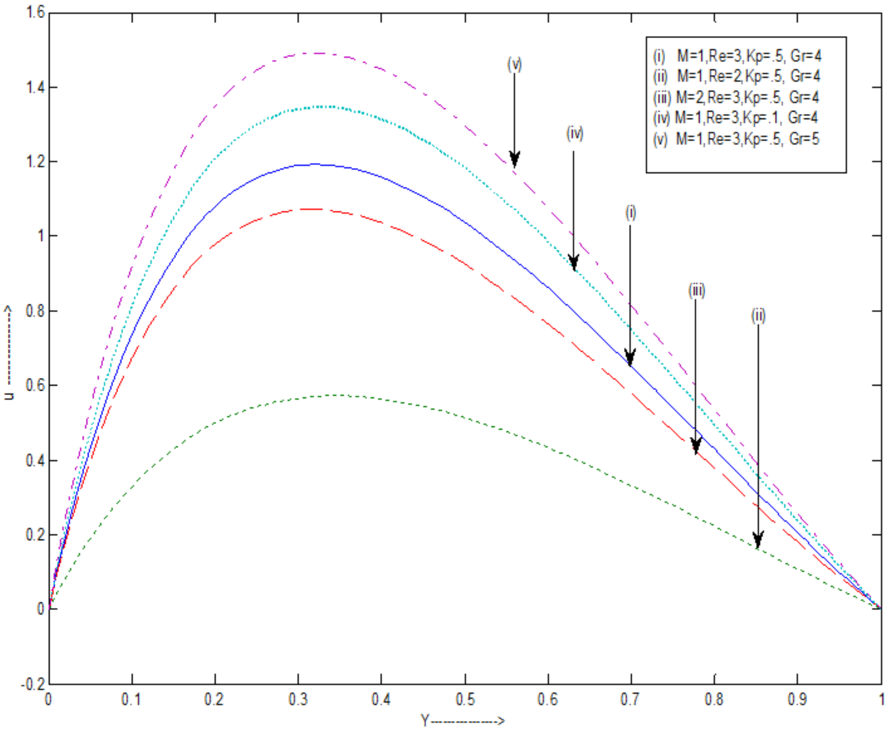

Figure-2: First order velocity distribution versus Y

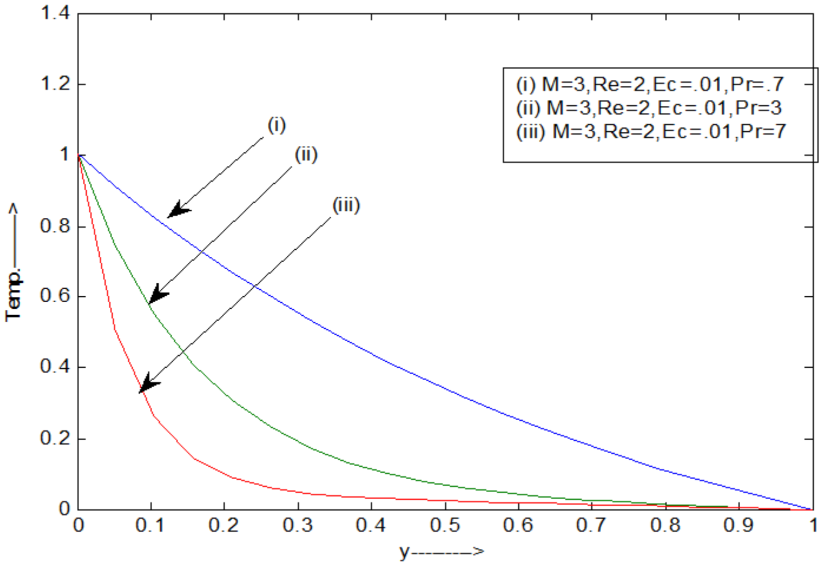

Figure 3: Temperature against y for different values of Pr with z=1/3,E=.01

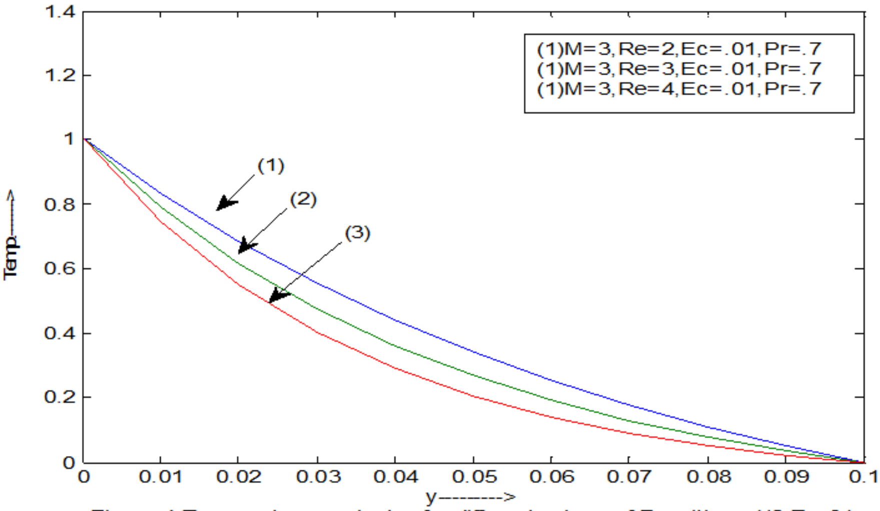

Figure 4:Temperature against y for different values of Re with z=1/3,E=.01

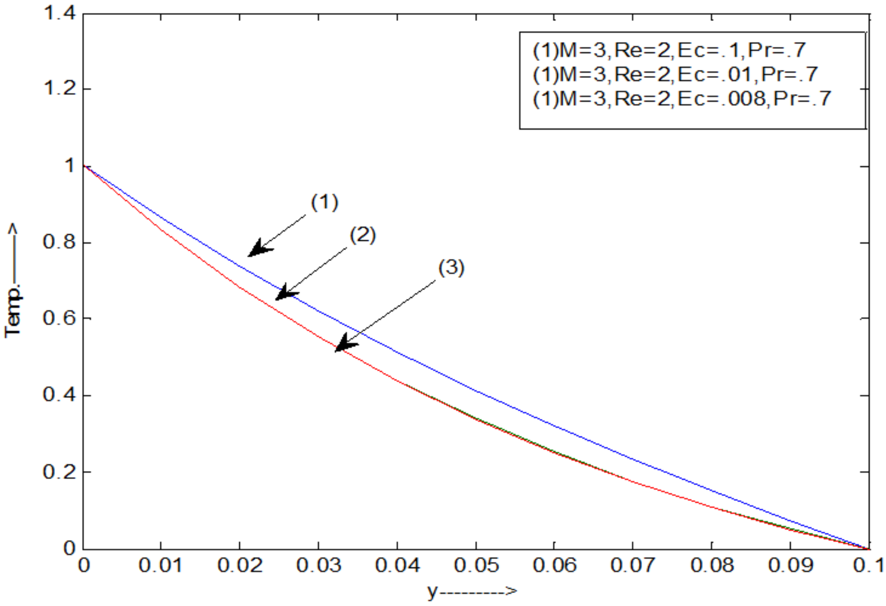

Figure 5 :Temperature against y for different values of Ec with z=1/3,E=.01

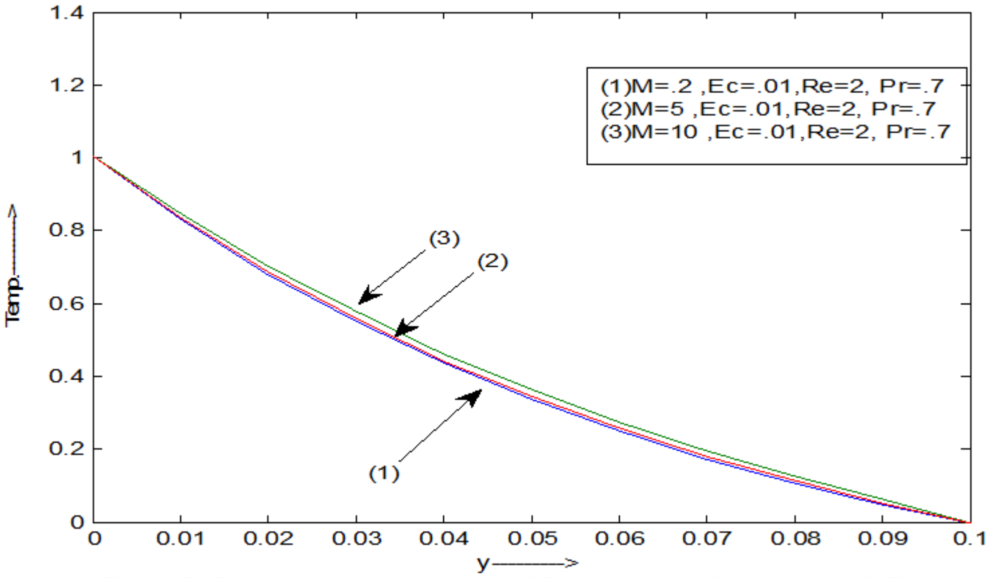

Figure 6: Temperature against y for different values of M with z=1/3,E=.01

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