IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 4 Ver. III (Jul. - Aug.2016), PP 01-05
www.iosrjournals.org

Fuzzy Rayleigh Distribution Model for the Expected Salivary Excretion of Oxytocin in Humans

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Abstract:*The study isset to investigation the oxytocin was developed bypreset on-line solid-phase micro extraction coupled with liquid chromatography-tandem mass spectrometry in saliva samples using the fuzzy Rayleigh distribution. Parameter of Rayleigh distribution find out by the Maximum Likelihood Estimator.The fuzzy mean values and variance values are calculated for different alpha values. The result shows that the mean and variance values are increasing for lower alpha values and decreasing for upper alpha values.*
Keywords:*Oxytocin, Rayleigh distribution, Maximum Likelihood Estimator.*
2010 Mathematics Subject Classification: *97Mxx, 93A30, 60-XX*

I. Introduction

The Rayleigh distribution is named after the British physicist Lord Rayleigh (1842–1919), also known as Baron John William StruttRayleigh and Nobel Prize winner in physics 1904. Consequent to the exponential law, the Rayleigh distribution is the mainly far and wide renowned particularcase of the Weibull distribution. It comes up through the Weibull density when the shapeparameter is set equivalent to two. Similarly the square root of a chi-squared χ_v^2 random variable with v = 2, that is of an exponential random variable, follows the Rayleigh distribution[8].The Rayleigh distribution was firstly derived in association with an obstacleinacoustics, and has been used in modelling certain features of electronic waves and as thedistance distribution between individuals in a spatial Poisson process. Most frequentlyhowever it appears as a suitable model in life testing and reliability theory.Heading for additionalparticulars on the Rayleigh distribution the reader is referred to Johnson et al. (1994) [6]. Approximate Maximum Likelihood Estimator (MLE) of the Scale Parameter of the RayleighDistribution with Censoring sample was discussed by Balakrishnan.N [1].

Oxytocin is mammalian neurohypophysialnonapeptide hormone secreted by the posterior pituitary gland revealed to performance vital roles in numerous perceivingtasks. For example oxytocin behaves as a neuromodulator, and has been shown to be involved in stress, anxiety, trust, empathy, social recognition, orgasm, parturition, lactation, maternal behaviors, and mother-child and pair bonding [2-4], [7] and [10-12].Oxytocin is biotic fluids has been measured by radioimmunoassay, enzyme immunoassay, high performance liquid chromatography (HPLC), and liquid chromatography (LC) plus tandem mass spectrometry (MS/MS). In-tube solid-phase micro extraction (SPME) using an open tubular fused-silica capillary with an inner surface coating as the SPME device, is a simple method that can be easily coupled with LC[9].

The objective of this work is analyze the on-line in-tube solid-phase microextraction coupled with liquid chromatography-tandem mass spectrometry (online in tube SPME LC-MS/MS) method in fuzzy environment viaestimate the fuzzy expected values and fuzzy variance for salivary excretion of oxytocin through fuzzy Rayleigh distributionby finding the parameter of the Rayleigh distribution in the method of MLE.

II. Notations

β	–Scale Parameter of Rayleigh distribution.
$\bar{\beta}(\alpha)$	– Alpha cut of the scale parameter
$E(X)$	– Mean of X
$V(X)$	– Variance of X
$\bar{E}(X)$	–FuzzyMean of X
$\bar{V}(X)$	– Fuzzy Variance of X

III. Materials And Methods

Rayleigh Distribution:

The cumulative function of random variable X follows Rayleigh distribution is denoted by $X \sim R(x, \beta)$ and it is defined by

$$F(x) = P(X \le x) = 1 - e^{-\frac{1}{2}\left(\frac{x}{\beta}\right)^2}, \text{ where } 0 \le x \le \infty,\ \beta > 0.$$

The probability density function (pdf) is given by

$$f(x) = \frac{d}{dx} F(x)$$

$$= \frac{x}{\beta^2} e^{-\frac{1}{2}\left(\frac{x}{\beta}\right)^2}$$

The mean value of Rayleigh distributionis $E(X) = \int_0^\infty x\, f(x)\, dx = \beta\sqrt{\frac{\pi}{2}}$

The variance value of Rayleigh distribution is $V(X) = E(X^2) - [E(X)]^2 = \beta^2\left(2 - \frac{\pi}{2}\right)$.

Parameter Estimation by MLE

Here we extant the method of Maximum Likelihood Estimation as this technique gives simpler estimation as compared to the Method of moments and the Local frequency ratio method of estimation. Now we are estimate the parameter of the Rayleigh distribution from which the sample comes.Let $x_1, x_2, \cdots x_n$ be a random sample of *n* observations from the Rayleigh population with pdf

$$f(x) = \frac{x}{\beta^2} e^{-\frac{1}{2}\left(\frac{x}{\beta}\right)^2}, x > 0.$$

The Likelihood function for this sample is

$$L = \prod_{i=1}^{n} f(x) = \prod_{i=1}^{n} \frac{x}{\beta^2} e^{-\frac{1}{2}\left(\frac{x}{\beta}\right)^2} = \frac{\prod_{i=1}^{n} x_i}{\left(\beta^2\right)^n} . e^{-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^2}$$

$$\log L = \log\left(\frac{\prod_{i=1}^{n} x_i}{\left(\beta^2\right)^n}\right) - \frac{1}{2}\sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^2$$

$$\log L = \log\left(\prod_{i=1}^{n} x_i\right) - \log\left(\left(\beta^2\right)^n\right) - \frac{1}{2}\sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^2$$

$$\log L = \log\left(\prod_{i=1}^{n} x_i\right) - 2n\log\beta - \frac{1}{2}\sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^2$$

The likelihood equation is $\frac{\partial \log L}{\partial \beta} = 0$.

$$\Rightarrow -\frac{2n}{\beta} + \frac{\sum_{i=1}^{n} x_i^2}{\beta^3} = 0.$$

$$\widehat{\beta} = \sqrt{\frac{\sum_{i=1}^{n} x_i^{2}}{2n}} \quad ------ (1)$$

Fuzzy Rayleigh Distribution

In life time applications, fickleness is not the loneattribute of vagueness. In many fields of application, owing tothe fuzziness of environment and the negligence of observers, it is sometimes impossible to obtain exact annotationsoflifetime [9]. The acquired lifetime data may be "contaminated" and wooly most of the time. In addition, constrained byhuman being and other wherewithal in experiment, mainly for novelequipment's, unusually long-life equipment's, andnon-mass-production products, for which there is no comparative dependability information available, more often thannot, the lifetime is based upon subjective evaluation or rough estimate. That leads to the fuzziness of lifetime data. In the circumstance Rayleigh distribution consider with fuzzy rules.

Now consider the Rayleigh distribution with fuzzy parameter $\overline{\beta}$ that is swapped with β. The probability of a random variable X follows Fuzzy Rayleigh distribution is denoted by $X \sim \text{FR}\left(x;\overline{\beta}\right)$ the fuzzy probability density function of a random variable $X \sim \text{FR}\left(x;\overline{\beta}\right)$ is defined by

$$f\left(x,\overline{\beta}\right) = \left\{ f(x)[\alpha], \mu_{f(x)} \,\middle|\, f(x)[\alpha] = \left[f_{\min}(x)[\alpha], f_{\max}(x)[\alpha] \right], \mu_{f(x)} = \alpha \right\}$$

$$f_{\min}(x)[\alpha] = \inf\left\{ f(x,\beta)(\alpha) \,\middle|\, \beta \in \overline{\beta}(\alpha) \right\},$$

$$f_{\max}(x)[\alpha] = \sup\left\{ f(x,\beta)(\alpha) \,\middle|\, \beta \in \overline{\beta}(\alpha) \right\}.$$

$$f\left(x,\overline{\beta}\right) = \frac{x}{\overline{\beta}^{2}} e^{-\frac{1}{2}\left(\frac{x}{\overline{\beta}}\right)^{2}}, x > 0, \overline{\beta} \in \overline{\beta}(\alpha)$$

The Mean value of FR distribution is given by

$$\overline{E}(X) = \left\{ E(X)[\alpha], \mu_{E(X)} \,\middle|\, E(X)[\alpha] = E_{\min}(X)[\alpha], E_{\max}(X)[\alpha], \mu_{E(X)} = \alpha \right\}$$

$$E_{\min}(X)[\alpha] = \inf\left\{ E(X) \,\middle|\, \beta \in \overline{\beta}(\alpha) \right\}$$

$$E_{\max}(X)[\alpha] = \sup\left\{ E(X) \,\middle|\, \beta \in \overline{\beta}(\alpha) \right\}$$

$$\overline{E}(X) = \overline{\beta}\sqrt{\frac{\pi}{2}}, \; \overline{\beta} \in \overline{\beta}(\alpha).$$

Let the life time random variable is a fuzzy random variable with p.d.f. $f\left(x,\overline{\beta}\right)$, then the fuzzy variance is defined as follows

$$\overline{V}(X) = \left\{ V(X)[\alpha], \mu_{V(X)} \,\middle|\, V(X)[\alpha] = V_{\min}(X)[\alpha], V_{\max}(X)[\alpha], \mu_{V(X)} = \alpha \right\}$$

$$V_{\min}(X)[\alpha] = \inf\left\{ V(X) \,\middle|\, \beta \in \overline{\beta}(\alpha) \right\}$$

$$V_{\max}(X)[\alpha] = \sup\left\{ V(X) \,\middle|\, \beta \in \overline{\beta}(\alpha) \right\}$$

$$\overline{V}(X) = \left(\overline{\beta}\right)^{2}\left(2 - \frac{\pi}{2}\right)$$

IV. Result And Discussion

Let us consider the trial in Shujitsu University, School of Pharmacy, in Japan [8]. Tocalculate the salivary secretion ofoxytocin,2mgmL^{-1}oxytocin solution was directed byfour bouquets (containing *ca.* 1.47 mg of oxytocin) into the adenoidal caves of 59 male volunteers.Salivawas collected by rinsing the mouth of each subject with water, followed by thecollectionofsalivasamplesinSalisofttubes containing polypropylenepolyethylenesponge (Assist,Tokyo,Japan).After saliva samples were collected into Salisoft tubes containing polypropylene-polyethylene sponges, followed by ultracentrifugation with Amicon Ultra to eliminate the proteins. To eradicate salivary interfering substances such as mucin, the filtrate was extracted with MonoTip C18, a monolithic silica adsorbent packed into a micro-tip. The saliva samples were successfully analyzed without interference peaks using the established in-tube SPME LC−MS/MS method with MRM mode detection.Fig 4.1 shows the salivary excretion of oxytocin after intranasal oxytocin administration.

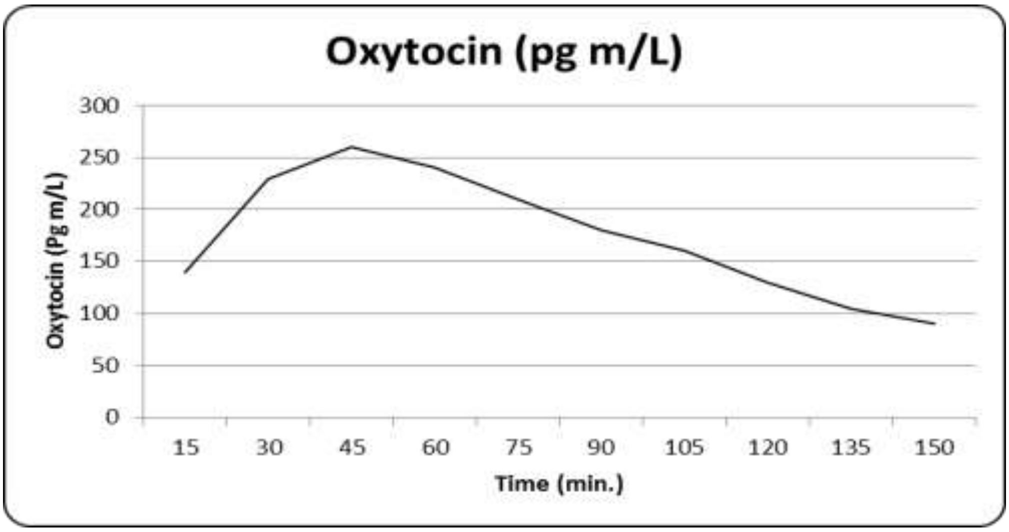

Fig. 4.1. Salivary excretion of Oxytocin

Based on the above observation sample the parameter of Rayleigh distribution by MLE (1) is $\beta = 65.812$. The corresponding fuzzy triangular numbers are [63.917, 65.812, 67.480]. The corresponding $\alpha - cut$ is

$\overline{\beta}(\alpha) = [63.967 + 1.845\alpha, 65.812, 67.480 - 1.668\alpha]$. The fuzzy mean values and fuzzy variance values for $\alpha - cuts$ are presented in Table 4.1.

Table 4.1. Mean and Variancevalues for alpha cuts

Alpha Values	$E_{\min}[X](\alpha)$	$E_{\max}[X](\alpha)$	$V_{\min}[X](\alpha)$	$V_{\max}[X](\alpha)$
0	80.171	84.574	1756.206	1954.401
0.05	80.286	84.469	1761.275	1949.573
0.1	80.402	84.365	1766.351	1944.751
0.15	80.518	84.260	1771.435	1939.935
0.2	80.633	84.156	1776.526	1935.124
0.25	80.749	84.051	1781.624	1930.320
0.3	80.864	83.946	1786.730	1925.522
0.35	80.980	83.842	1791.843	1920.730
0.4	81.096	83.737	1796.963	1915.944
0.45	81.211	83.633	1802.090	1911.164
0.5	81.327	83.528	1807.225	1906.389
0.55	81.443	83.424	1812.367	1901.621
0.6	81.558	83.319	1817.517	1896.859
0.65	81.674	83.215	1822.674	1892.102
0.7	81.789	83.110	1827.838	1887.352
0.75	81.905	83.006	1833.009	1882.608
0.8	82.021	82.901	1838.188	1877.869
0.85	82.136	82.797	1843.374	1873.137
0.9	82.252	82.692	1848.567	1868.410
0.95	82.367	82.588	1853.767	1863.690
1	82.483	82.483	1858.975	1858.975

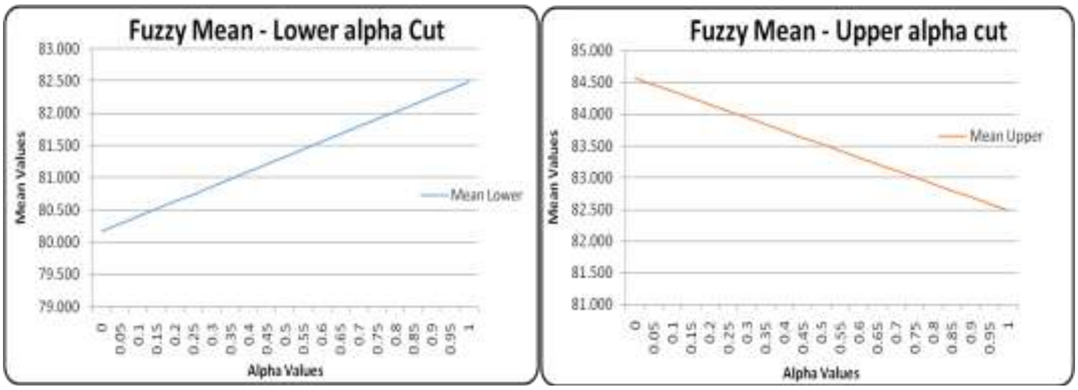

Fig. 4.2. Fuzzy Mean values of salivary excretion of Oxytocin for Lower & Upper alpha cuts

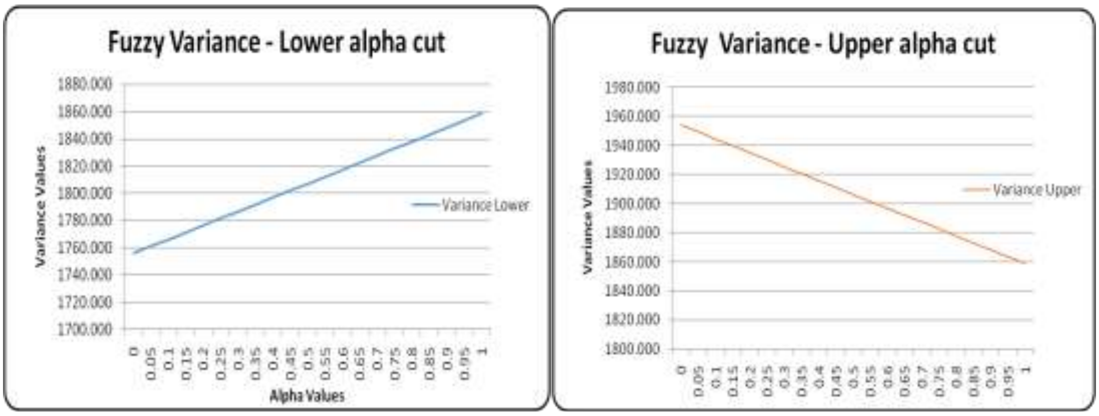

Fig. 4.3. Fuzzy Variance values of salivary excretion of Oxytocin for Lower & Upper alpha cuts

V. Conclusion

The parameter for Rayleigh distribution was calculated successfully by using MLE. The mean and variance values are estimated for the unremitting drawing out and concentration of oxytocin in saliva samplesanalysis using fuzzy Rayleigh distribution. Analyzing of fuzzy mean and variance shows that for lower alpha cuts has increasing expected salivary excretion than the upper alpha cuts. The fuzzy Rayleigh distribution model for investigation of oxytocin analyzed by online in tube SPME LC-MS/MS methodis very handy for drool examples and for impartial assessment of the biological belongings of oxytocin.

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