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"Modeling For Inventory with Exponential Declining Demand, Variable Deterioration, Linear Holding Cost and Inflation without Shortages"

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Abstract: *In this paper, an inventory model with exponential declining demand and variable deterioration is to be optimized. The holding cost is a linear function of time. Shortages are not allowed. The items like grocery products, mobile phones, computer chips etc. are considered as deteriorating items as they have fixed life which decreases with time. A numerical example has been illustrated using MATLAB to describe the model and the sensitivity analysis of various parameters is carried out with graphical and tabular data.*
Keywords: *Variable deterioration; Exponential declining demand; Inflation; Inventory; Time dependent holding cost*

I. Introduction

The objective of the proposed model is to develop an inventory model having exponential declining demand, time varying linear holding cost and variable deterioration with inflation. The literature review of the present paper is given below.

Declining demand refer to a situation when potential buyer starts to purchase the product or item less often or stop purchasing it all together. Some products have a rising demand during the growth phase of their life cycle whereas the demand of some product decline due to introduction of more attractive products. Deteriorating items refer to the items that become decayed or damaged, evaporate or become invalid, loose their marginal value,and degrade through time. The process of deterioration can be categorized based on two categories of items. The first category refers to the items that become damaged, spoiled, decayed, evaporate or expired through time like flowers, vegetables, food grain, food stuffs, fruits, films, medicines and so on, while the other category refers to the items that loss their parts or their total values through time because of the introduction of new technology or the alternatives like fashion and seasonal goods, electronic equipment, computer chips and mobile phones and so on. For the first category, the items have a short natural life cycle whereas in the second category, the items have a short market life.

Harris[10] was the first researcher who designed the inventory model in 1915. Wilson[23]generalized Harris' model and gave a formula to obtain (EOQ) economic order quantity. T.M.Within[24] established the inventory model on fashion items deteriorating at the end of the shortage period in 1957 and this model extended by Ghare and Schrader[8]who concluded in their study that the consumption of the deteriorating items was closely relative to a negative exponential function of time and established the classical EOQ model with constant deterioration rate and without shortages. Dave U and Patel[8] studied firstly the inventory model on deteriorating items with linear increasing demand without shortages in 1981,Raafat[15]did survey of literature on continuously deteriorating inventory model while Shah and Shah[12]did survey of deteriorating items. Wee[21-22],Chang and Dye[4]and Mishra *et al.* [12] developed model for deteriorating items with time varying demand and partial backlogging. Chung and Ting[5] developed model for linear demand and partial backlogging in 1993.Abad[1-2] developed model for optimizing the cost under partial backlogging and reseller under partial backlogging in 2001 . Some of the recent work in this field has been established by Goyal and Giri[9] in 2001. Ouyang and Cheng[13]developed an inventory model for deteriorating items with partial backlogging and demand declining exponentially in 2005. Shah[16] studied the case of pricing and ordering strategy for retailer's with Weibull Deterioration rate and declining demand with trade credit. In 2010 Tripathy and Mishra[20] discussed policy for Weibull Deterioration Items, quadratic demand with permissible delay in payment. Li *et al.* [11] gave the up to date review on deteriorating inventory model. On the basis of demand variations, Singh and Pattnayak[17,18,19] also studied an inventory model for deteriorating items with linear and quadratic demand, variable deterioration and partial backlogging and allowed permissible delay in payment. Amutha and Chandrasekaran[3] also studied an inventory model on deteriorating items with quadratic demand and time dependent holding cost in 2013. In 2014 Dass and Singh[6]established an inventory model with time varying holding cost and declining exponential demand. Usually, the demand rate and inventory holding cost are considered to be constant.

In the present paper an inventory model with exponentially declining demand, linearly varying holding cost and inflation without shortages has been established. The objective is to minimize the total cost of an inventory system based on inventory holding cost, ordering cost and deterioration cost. In practical life, the demand rate and the inventory holding cost for deteriorating goods may be time-dependent. As time is an important factor in the inventory system the demand rate and inventory holding cost are considered to be time-dependent. An EOQ inventory model for deteriorating items with exponentially declining demand when the deterioration rate varies with time (linear function of time) with time-dependent linear holding cost with inflation and shortages are not allowed has been established.In the present paper, an effort has been made to analyze an EOQ model for deteriorating items by considering the time-dependent exponential declining demand rate and time-dependentlinear inventory holding cost with inflation and without shortages. The proposed model is based on the inventory items like fashion and seasonal goods, electronic equipment, computer chips and mobile phones and so on, as they experience fluctuations in the demand rate. The mathematical model has been derived under exponential declining demand rate. The rest part of the paper is arranged as follows: Section 1 describes review of the literature on the effects of deterioration and time-dependent demand rate and model proposed in relative to previous work. Section 2 details the model assumptions. Section 3 describes thenotation used in model. Section 4gives formulation of the model as a cost minimization problem. In Section 5, a mathematical solution is given to illustrate the model. Section 6details an algorithm for optimizing the solution .Section 7 summarizesthe conclusion of this work and suggest directions for future research. Numerical Solution and Sensitivity analysis of various parameters is taken in Section 8.

II. Assumptions

The following assumptions are made in developing the model.

- The inventory system considers a single item only.
- The demand rate is deterministic and is an exponential declining function of time.
- The deterioration rate is variable "vt".
- The inventory system is considered over a finite time horizon.
- Lead time is zero.
- Shortages are not allowed.

III. Notations

The following notations use for inventory model.

A :Setup cost.

$D(t)$:Exponential decliningdemand in a period [0,T]

$\theta(t)$:Deteriorating cost, which is variable function with $vt, v > 0$

Qo :Initial ordering quantity

TD : Total demand in a cycle period [0, T]

DU :Deteriorating unit in a cycle period [0, T]

C_d : Deteriorating cost per unit

DC :Deteriorating cost

HC : Holding cost

$TC(T)$: Total inventory cost

T^* :Optimal length size

Q_0^* : Optimal initial order quantity

$TC^*(T)$: Optimal total cost in the period [0, T]

IV. Mathematical Formulation

Consider the inventory model of deteriorating items with declining demand rate. As the inventory reduces due to demand rate as well as deterioration rate during the interval, the differential representing the inventory status is governed by [0,T]

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(t)\,, 0 \le t \le T \qquad \ldots(1)$$

Where $\theta(t) = vt$, $v > 0$ and $D(t) = \lambda e^{-\alpha t}$, λ , $\alpha > 0$

V. Mathematical Solution

The solution with boundary condition $I(T)=0$, of the Equation

$$\frac{dI(t)}{dt}+vtI(t)=-\lambda e^{-at},0\le t\le T \quad \ldots (2)$$

$$I(t)=\lambda e^{-\frac{vt^2}{2}}\left[(T-t)-\frac{\alpha}{2}(T^2-t^2)+\frac{v}{6}(T^3-t^3)\right] \quad \ldots(3)$$

Where use the expansion $e^{-\alpha t}\approx 1-\alpha t+\frac{(\alpha t)^2}{2}$ ..., α is small and positive.

So the initial order quantity is obtained by putting the boundary condition in Equation (3) $I(0)=Q_0$. Therefore,

$$Q_0=\lambda\left[T-\frac{\alpha}{2}T^2+\frac{v}{6}T^3\right] \quad \ldots (4)$$

The total demand during the cycle period [0, T] is

$$TD=\int_0^T D(t)dt$$

$$=\int_0^T \lambda e^{-\alpha t}dt$$

$$=\frac{\lambda}{\alpha}\left[1-e^{-\alpha T}\right]=\frac{\lambda}{\alpha}\left[\alpha T-\frac{(\alpha T)^2}{2}\right] \quad \ldots (5)$$

Then the number of deterioration units is

$$DU=Q_0-TD$$

$$=\lambda\left[T-\frac{\alpha}{2}T^2+\frac{v}{6}T^3\right]-\lambda\left[T-\frac{\alpha T^2}{2}\right]$$

$$=\lambda\frac{v}{6}T^3 \quad \ldots (6)$$

The deterioration cost for the cycle [0, T]

$$DC=C_d * \text{(Number of deterioration units)}$$

$$=C_d\lambda\frac{v}{6}T^3 \quad \ldots (7)$$

Holding Cost for the cycle [0,T] is

$$HC=\int_0^T (p+qt)e^{-rt}I(t)dt$$

$$=\lambda\int_0^T (p+qt)e^{-rt}e^{-\frac{vt^2}{2}}\left[(T-t)-\frac{\alpha}{2}(T^2-t^2)+\frac{v}{6}(T^3-t^3)\right]dt$$

$$=\lambda\int_0^T (p+qt)\left(1-rt-\frac{vt^2}{2}\right)\left[(T-t)-\frac{\alpha}{2}(T^2-t^2)+\frac{v}{6}(T^3-t^3)\right]dt$$

$$= \lambda[p\frac{T^2}{2} - \{p(2\alpha - r) - q\}\frac{T^3}{6} + \{p(2v + 3\alpha r) - q(3\alpha + 2r)\}\frac{T^4}{24}$$
$$- \{p(6rv - 8\alpha v) - q(12v + 8r\alpha - 3rv + 4v\alpha)\}\frac{T^5}{120} - pv^2\frac{T^6}{72} - qv^2\frac{T^7}{112}] \quad \ldots (8)$$

The total inventory cost TC (T)= Ordering cost (A) + Deterioration cost (DC)
+ Holding cost (HC)

Therefore the total variable cost per unit time

$$TC(T) = \frac{A}{T} + C_d\lambda\frac{v}{6}T^2 + \lambda[p\frac{T}{2} - \{p(2\alpha - r) - q\}\frac{T^2}{6}$$
$$+ \{p(2v + 3\alpha r) - q(3\alpha + 2r)\}\frac{T^3}{24} - \{p(6rv - 8\alpha v) - q(12v + 8r\alpha - 3rv + 4v\alpha)\}\frac{T^4}{120} \quad \ldots (9)$$
$$- pv^2\frac{T^5}{72} - qv^2\frac{T^6}{112}$$

The necessary and sufficient conditions for minimize cost a given value *T* are

$\frac{dTC(T)}{dT} = 0$ And $\frac{d^2TC(T)}{dT^2} > 0$ then differentiation with respect to T of (9), we get

$$\frac{dTC(T)}{dT} = -\frac{A}{T^2} + C_d\lambda\frac{v}{3}T + \lambda[p\frac{1}{2} - \{p(2\alpha - r) - q\}\frac{T}{3} + \{p(2v + 3\alpha r) - q(3\alpha + 2r)\}\frac{T^2}{12}$$
$$- \{p(6rv - 8\alpha v) - q(12v + 8r\alpha - 3rv + 4v\alpha)\}\frac{T^3}{30} - 5pv^2\frac{T^4}{72} - 3qv^2\frac{T^5}{56} = 0 \quad \ldots (10)$$

And the again differentiation with respect to T of (10), we get

$$\frac{d^2TC(T)}{dT^2} = 2\frac{A}{T^3} + C_d\lambda\frac{v}{3} - \lambda[\{p(2\alpha - r) + q\}\frac{1}{3} - \{p(2v + 3\alpha r) - q(3\alpha + 2r)\}\frac{T}{6}$$
$$+ \{p(6rv - 8\alpha v) - q(12v + 8r\alpha - 3rv + 4v\alpha)\}\frac{T^2}{10} - 5pv^2\frac{T^3}{18} - 15qv^2\frac{T^4}{56} \quad \ldots (11)$$

VI. Algorithm

To find out the solution following algorithm used

Step1: Find derivative $\frac{dTC(T)}{dT}$ and put $\frac{dTC(T)}{dT} = 0$

Step2: Solve equation (10) for T

Step3: Find the derivative $\frac{d^2TC(T)}{dT^2}$ and check $\frac{d^2TC(T)}{dT^2} > 0$ for T^* optimal length

Step4: Find optimal total cost and initial order quantity

VII. Conclusion

In the present paper, we developed an inventory model for variable deteriorating item with inflation, exponential declaring demand and without shortages give analytical solution, numerical solution and the effect of parameters of the model that minimize the total inventory cost. The deterioration factor taken into consideration in this model, as almost all items undergo either direct spoilage or physical decay in the course time, deterioration is natural feature in the inventory system. The model is very practical for the industries in which the demand rate is depending upon the time and holding cost is linear function with inflation. This model can further be extended by taking more realistic assumptions such as finite replenishment rate, probabilistic demand rate etc.

VIII. Numerical Solution

Consider an inventory system with the following parameter in proper units A = 1000, λ=50, α=0.02, v=0.05, p=0.5, q=0.3, C_d =5, and parameter r=0.01. The computer output of the program by using Mat lab software is T^* = 3.5904, Q_o * = 195.58 and TC^* = 369.33. The effect of changes in the parameter of the inventory model also can be study by using Mat lab.

Figure1:- Total cost (TC) vs. time (T)

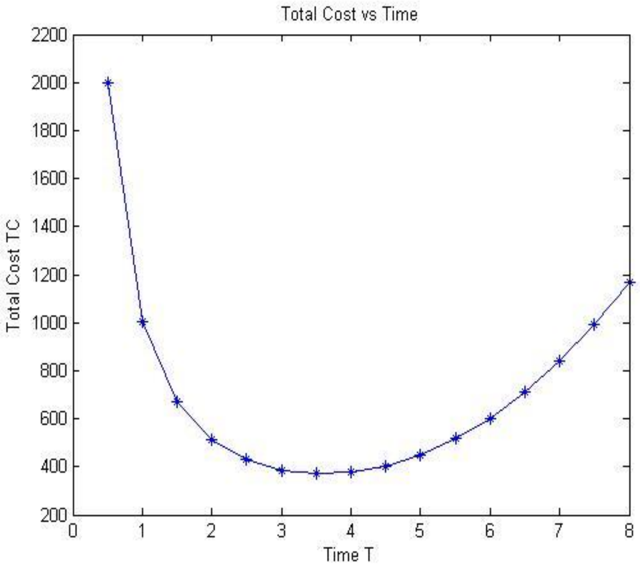

Table 1:- Total cost (TC) vs. time (T)

T	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**	**T* = 3.5904**	**4.0**
TC	**2000.10**	**1001.60**	**672.70**	**514.90**	**429.80**	**385.60**	**TC* = 369.33**	**403.40**
Q_o	**24.99**	**50.17**	**75.84**	**102.33**	**129.95**	**159.00**	Q_o ***= 195.58**	**222.67**

T	**4.5**	**5.0**	**5.5**	**6.0**	**6.5**	**7.0**	**7.5**	**8.0**
TC	**449.90**	**515.90**	**602.00**	**709.00**	**838.00**	**990.20**	**1166.90**	**1369.40**
Q_o	**257.91**	**295.83**	**336.76**	**381.00**	**428.86**	**480.67**	**536.72**	**597.33**

Figure2:- Variation of parameter v in TC

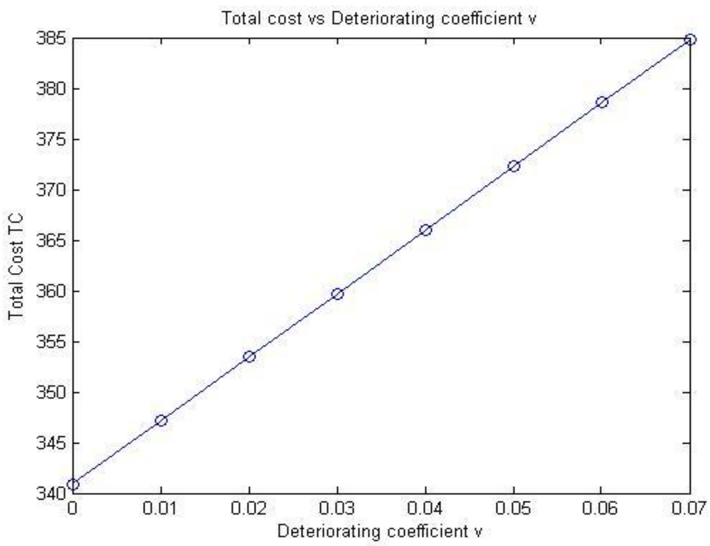

Table 2:-Variation of parameter v in TC

v	TC
0.00	342.15
0.01	347.59
0.02	353.04
0.03	358.48
0.04	363.93
0.05	369.37
0.06	374.83
0.07	380.28
0.08	385.74
0.09	391.20
0.10	396.66

Figure3:- Variation of parameter r in TC

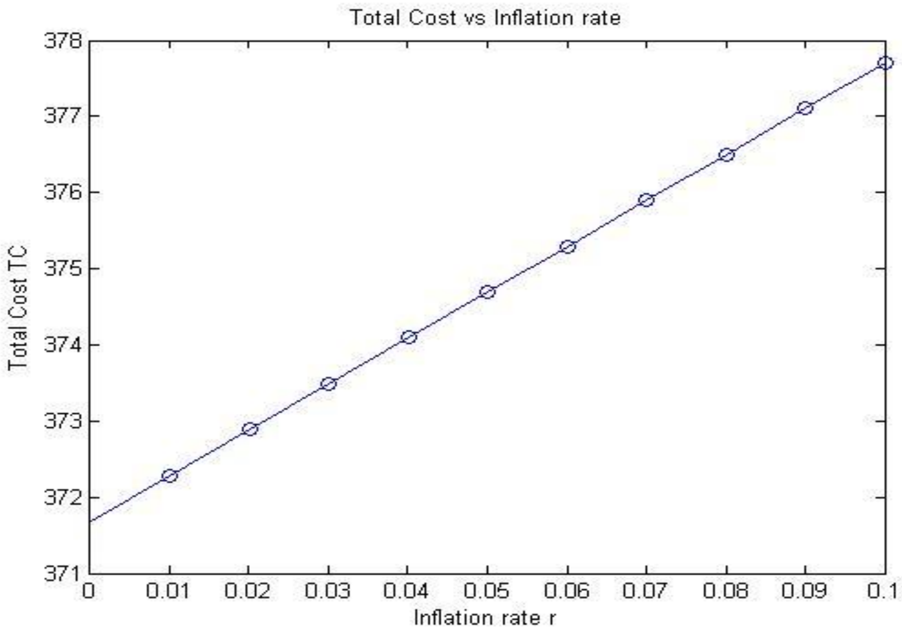

Table 3:- Variation of parameter r in TC

r	TC
0.00	368.85
0.01	369.37
0.02	369.90
0.03	370.43
0.04	370.96
0.05	371.48
0.06	372.01
0.07	372.53
0.08	373.06
0.09	373.58
0.10	374.11

Table 4:- Sensitivity Analysis

S. NO.	Variable	Variation	v	r	p	q	Q_0	TC
1	α	0.00	0.05	0.02	0.5	0.3	198.80	370.42
		0.01	0.05	0.02	0.5	0.3	195.58	369.37
		0.02	0.05	0.02	0.5	0.3	192.36	368.24
		0.03	0.05	0.02	0.5	0.3	189.14	367.16
		0.04	0.05	0.02	0.5	0.3	185.91	366.07
	Variable	**Variation**	α	r	p	q	Q_0	TC
2	v	0.03	0.01	0.02	0.5	0.3	187.87	358.48
		0.04	0.01	0.02	0.5	0.3	191.72	363.93
		0.05	0.01	0.02	0.5	0.3	195.58	369.37
		0.06	0.01	0.02	0.5	0.3	199.44	374.83
		0.07	0.01	0.02	0.5	0.3	203.30	380.28
	Variable	**Variation**	α	v	p	q	Q_0	TC
3	r	0.00	0.01	0.05	0.5	0.3	195.58	368.85
		0.01	0.01	0.05	0.5	0.3	195.58	369.37
		0.02	0.01	0.05	0.5	0.3	195.58	369.90
		0.03	0.01	0.05	0.5	0.3	195.58	370.43
		0.04	0.01	0.05	0.5	0.3	195.58	370.96
	Variable	**Variation**	α	v	r	q	Q_0	TC
4	p	0.3	0.01	0.05	0.02	0.3	195.58	330.94
		0.4	0.01	0.05	0.02	0.3	195.58	350.14
		0.5	0.01	0.05	0.02	0.3	195.58	369.37
		0.6	0.01	0.05	0.02	0.3	195.58	388.53
		0.7	0.01	0.05	0.02	0.3	195.58	407.72
	Variable	**Variation**	α	v	r	p	Q_0	TC
5	q	0.1	0.01	0.05	0.02	0.5	195.58	390.68
		0.2	0.01	0.05	0.02	0.5	195.58	380.00
		0.3	0.01	0.05	0.02	0.5	195.58	369.37
		0.4	0.01	0.05	0.02	0.5	195.58	350.66
		0.5	0.01	0.05	0.02	0.5	195.58	347.99

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