IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 5 Ver. IV (Sep. - Oct.2016), PP 57-61
www.iosrjournals.org

Prime Harmonious Labeling of Some New Graphs

P.Deepa[1], S. Uma Maheswari[2], K. Indirani[3]
[1]*Department of Mathematics, CMS College of Science And Commerce, Coimbatore, India*
[2]*Asst.professor, Department of Mathematics, CMS College of Science And Commerce, Coimbatore , India*
[3]*Head and Asso. Professor, Department of Mathematics, Nirmala College For Women, Coimbatore, India*

Abstract: *In this paper we have introduced prime harmonious labeling of some new graphs. We prove that the graphs Fan f_n, Star S_n, Comb graph $P_n.K_n$, Crown graph $C_n . K_{1,n}$, Bistar $B_{n,n}$, Middle graph $M(P_n)$ and paths are prime harmonious labeling.*

Keywords: *Prime labeling, Harmonious labeling, Prime harmonious labeling, Bistar $B_{n,n}$, Middle graph $M(P_n)$.*

I. Introduction

All graphs in this paper are finite, simple and undirected. The symbolsV(G) and E(G)will denote the vertex and edge set of the graph G. For standard terminology and notations we follow Gross and Yellon [5]. We will provide brief summary of definitions and other information which are necessary for the present investigations.

Definition 1.1: Let G = G(V,E) be a graph. A bijection f : V→ {1,2,3…….,|v|} is called prime labeling if for each edge e = {u, v} belong to E, we have GCD (f(u),f(v)) = 1. A graph which admits a prime labeling is called a prime graph.

Definition 1.2: Let G be a graph with q edges. A function f is called harmonious labeling of graph G if f : V→ {0,1,2,3…….,q-1} is injective and the induced functionf^* : E(G) → {0,1,2,….q} defined as f^* (e = uv) =(f(u) + f(v))(mod q) is bijective. A graph which admits a harmonious labeling is called a harmonious graph.

Definition 1.3: Let G be a graph with q edges. A function f is called prime harmonious labeling of graph G if f : V→ {0,1,2,3…….,2q-1} is injective and the induced function f^* : E(G) → {0,1,2,….2q} is defined by

$$f^*(e = uv) = \begin{cases} \mathrm{GCD}\left(f(u), f(v)\right) = 1 \quad \text{and} \\ f(e = uv) = (f(u) + f(v))(\bmod q) \end{cases}$$

A graph which satisfies the conditions of prime labeling and harmonious labeling is called a prime harmonious labeling. A graph which admits a prime harmonious labeling is called a prime harmonious graph and it is denoted by P_H.

Definition 1.4: The fan f_n(n ≥ 2) is obtained by joining all vertices of P_n(Path of n vertices)to a further vertex called the center and contains n+1 vertex and 2n-1 edges (i.e) $f_n = P_n + K_1$.

Definition 1.5: A star S_n is the complete bipartite graph $K_{1,n}$ is a tree with one internal node and n leaves.

Definition 1.6: The graph obtained by joining a pendent edge at each vertex of a path P_n is called a comb and is denoted by $P_n . K_n$ or P_n^+.

Definition 1.7: A cycle C_n is a simple graph in which the start and end vertices are same.

Definition 1.8: A crown graph is obtained by joining $K_{1,n}$ vertices at each vertex of the cycle C_n and it is denoted by
$C_n . K_{1,n}$.

Definition 1.9: The middle graph M(G) of a graph G is the graph whose vertex set is V(G) ∪ E(G) and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident with it.

Definition 1.10: Bistar is the graph obtained by joining the apex vertices of two copies of star $K_{1,n}$.

Definition 1.11: By a graph P_n^2 , we mean the graph obtained from $P_n$joining each pair of vertices at distance 2 in P_n .

II. Main Result

In this paper we prove that the Path P_n , $K_{1,n}$, Cycle ,Comb, Crown $C_n . K_{1,n}$ and the Fan f_n , Bistar $B_{n,n}$, Middle graph $M(P_n)$ are prime harmoniouslabeling.

Theorem 2.1: The Path P_n is a prime harmonious labeling.

Proof: Let u_1 , u_2 ,……..u_n and v_1 , v_2 ,……..v_n be the vertices of the path P_n .
Define f : V(G) → {1,2,3……… 2q-1}is injection.
Then f induces an edge labelingf^* : E(G) →{1,2,3……… 2q}is a bijection.

Case(1): If n is odd
Here the vertices are labeled such that GCD (f(u),f(v)) = 1and edges are labeled (e = uv) =(f(u) + f(v))(mod q).
Case(2): If n is even
We define f : V(G) → {1,2,3……… n} by P_n = n+1 where n = 0,1,2……Here also the same labeling pattern is followed as in the case 1. Hence we conclude that the path P_n admits a prime harmonious labeling.
Prime Harmonious labeling of path is shown in Figure 1

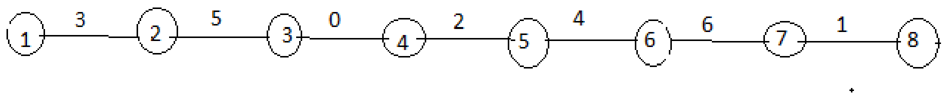

Figure 1

Theorem 2.2: Every comb graph P_n .K_1 is a prime harmonious labeling.
Proof: Let u_1 , u_2 ,……..u_n be the vertices of the path P_n .
Let v_1 , v_2 ,……..v_n be the vertices adjacent to each vertex of the path P_n .
Define f: V(P_n .K_1) → {1,2,3……… 2q-1} is injection.
Then f induces an edge labeling f^* : E(P_n .K_1) → {1,2…..2q}is a bijection.
Hence the graph (P_n . K_1)admits a prime harmonious labeling.
Prime Harmonious labeling of Comb P_8 .K_1 is shown in Figure 2

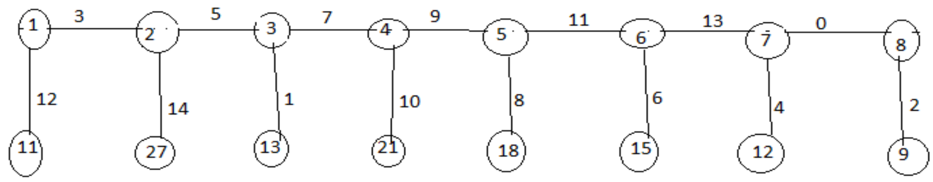

Figure 2

Theorem 2.3: The star $K_{1,n}$ is prime harmoniouslabeling.
Proof: Let V($K_{1,\,n}$)= { u_i /1≤ i ≤ n}
Let E($K_{1,\,n}$) = {e_i / i= 1,2,3…..n-1}
Define an injection f: V($K_{1,\,n}$) → {1,2,……..2q-1} is defined by
f(v) = 1
f(v_i) = 2 ∀ i = 1
f(v_{i+1}) = n+1 , i = 1,2,…….n.
Then f induces a bijection f^* : E($K_{1,\,n}$) → {1,2………….. 2q}.
In the view of above labeling pattern $K_{1,n,}$ admits a prime harmonious labeling Prime Harmonious labeling of Star $K_{1,7}$ is shown in Figure 3

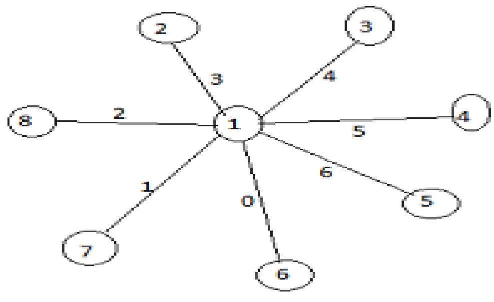

Figure 3

Theorem 2.4: The fan f_n is a prime harmonious labeling.
Proof: Let u be the center vertex of the fan f_n
Let v_1 , v_2 ,……..v_n be the vertices of leaves of fan f_n
Label the center vertex with 1 and path vertices are labeled by 2 to n. It contains n+1vertex and 2n-1 edges. (i.e) f_n= P_n +K_1
Joining all vertices of path P_n to the center vertex u.It satisfies the conditions of P_H . Therefore the fan f_n is a prime harmonious labeling.

Prime Harmonious labeling of fan f_7 is shown in Figure 4

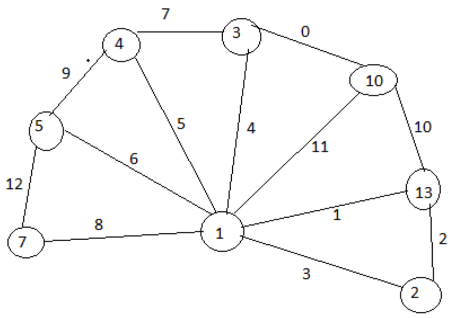

Figure 4

Theorem 2.5: The cycle C_n is prime harmonious labeling
Proof: We shall consider the following three cases
Case(1): If n is odd
Define $f : V(G) \rightarrow \{1,2,\ldots\ldots.2q\text{-}1\}$ by $f(v_i) = j-1$ where $i = 1,2,\ldots\ldots\ldots n$, $j= 2,3,\ldots\ldots.n$
Then f induces an edge label $f^* : E(G) \rightarrow \{1,2,3\ldots\ldots\ldots 2q\}$ is bijection.
Therefore the cycle C_n is a prime harmonious labeling if n is odd.
Case 2: If n is even (n ≤ 4)
Define $f : V(G) \rightarrow \{1,2,\ldots\ldots.2q\text{-}1\}$ by $f(v_1) = 1$, $f(v_1) = 2$, $f(v_1) =3$, $f(v_1) = 7$
Then f induces an edge label $f^* : E(G) \rightarrow \{1,2,3\ldots\ldots\ldots 2q\}$
Therefore the cycle C_n is a prime harmonious labeling if n is even (n ≤ 4).
Case 3: If n is even (n ≥ 4)
As n is even let n = 2m where m = 3,4…… m-1
Let v_1 , v_2 ,……..v_{2m-1} be the vertices of cycle $C_{2m.}$
Adding the vertices $v_1 + v_2$, $v_2 + v_3$,………………, $v_{2m-1} + v_1$ we obtain some edges of C_{2m} are equal, which is impossible.
Therefore the cycle C_{2m} (n ≥ 4) is not a prime harmonious labeling.
Prime Harmonious labeling of cycle C_9 is shown in Figure 5

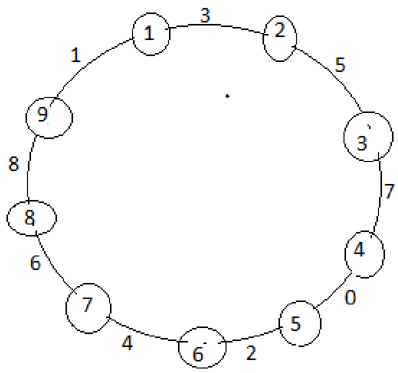

Figure 5

Theorem 2.6: The crown graph $C_n .K_{1,n}$ is a prime harmonious labeling
Proof: Let $\{v_1, v_2, v_3, \ldots\ldots\ldots v_n\}$ be the vertices of the cycle C_n.
Let $\{u_1, u_2, u_3, \ldots\ldots\ldots u_n\}$ be the vertices of $K_{1,n.}$
Case (1): When n is odd cycle
The size of the graph is p = 3n+3. Define a mapping $f : V(C_n .K_{1,n}) \rightarrow \{1,2,3\ldots\ldots\ldots (3n+3)\}$ by $f(v_i)= i+1$ where i= 0,1,2,…..n and $f : E(G) \rightarrow \{1,2\ldots\ldots\ldots..(3n+3)\}$
Case (2) : When n is even cycle
The size of the graph is p = 4n+4. Define a mapping $f : V(C_n .K_{1,n}) \rightarrow \{1,2,3\ldots\ldots\ldots (4n+4)\}$ by $f(v_i)= i+1$ where i= 0,1,2,…..n .
Then f induces a bijection $f^* : E(G) \rightarrow \{1,2,\ldots\ldots(4n+4)\}$
Therefore the crown graph $C_n .K_{1,n}$ is a prime harmonious labeling.
Prime Harmonious labeling of crown graph $C_4 .K_{1,4}$ is shown in Figure 6

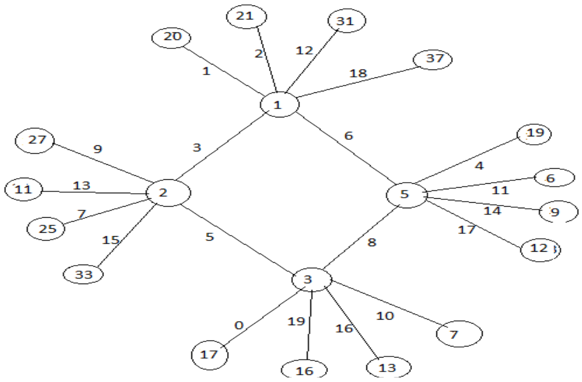

Figure 6

Theorem 2.7: The Bistar $B_{n,n}$ admits a prime harmonious labeling.
Proof: Consider the two copies of $K_{1,n}$.
Let $v_1, v_2, v_3, \ldots\ldots\ldots v_n$ and $u_1, u_2, u_3, \ldots\ldots\ldots u_n$ be the corresponding vertices of each copies of $K_{1,n}$ with apex vertex u and v.
Let $e_i = vv_i$, $e_i' = uu_i$ and $e = uv$ of bistar graph.
Note that $|V(B_{n,n})| = 2n + 2$ and $|E(B_{n,n})| = 2n + 1$.
Define the labeling $f : V(B_{n,n}) \rightarrow \{1,2,3\ldots\ldots\ldots |V|\}$
Then f induces an edge labeling $f^* : E(B_{n,n}) \rightarrow \{1,2,3\ldots\ldots\ldots 2n+1\}$ is bijection.
In view of above labeled pattern $B_{n,n}$admits a prime harmonious labeling.
Prime Harmonious labeling of Bistar B8 , 8 is shown in Figure 7

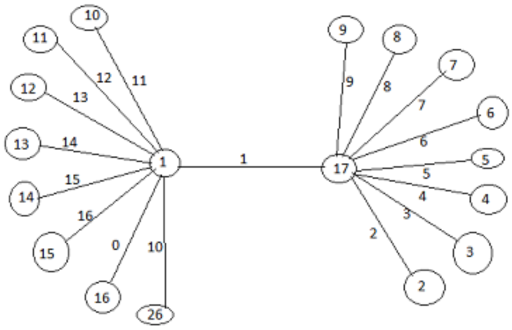

Figure 7

Theorem: 2.8 $M(P_n)$ is a prime harmonious labeling.
Proof: Let $v_1, v_2, v_3, \ldots\ldots\ldots v_n$ and $e_1, e_2, e_3, \ldots\ldots\ldots e_{n-1}$ are respectively be the vertices of P_n .
Add vertices $u_1, u_2, u_3, \ldots\ldots\ldots u_{n-1}$ corresponding to the edges $e_1, e_2, e_3, \ldots\ldots\ldots e_{n-1}$ in order to obtain middle graph.
Let G be the graph $M(P_n)$. Then $| V(G)| = 2n - 1$ and $|E(G)|= 3n - 4$
Define the mapping $f : V(M(P_n)) \rightarrow \{1,2,3\ldots\ldots\ldots |V|\}$
Then f induces an edge labeling $f^* : E(M(P_n)) \rightarrow \{1,2,3\ldots\ldots\ldots 3n-4\}$ is bijection and it is satisfies the conditions of prime harmonious labeling.
Prime Harmonious labeling of $M(P_8)$ is shown in Figure 8

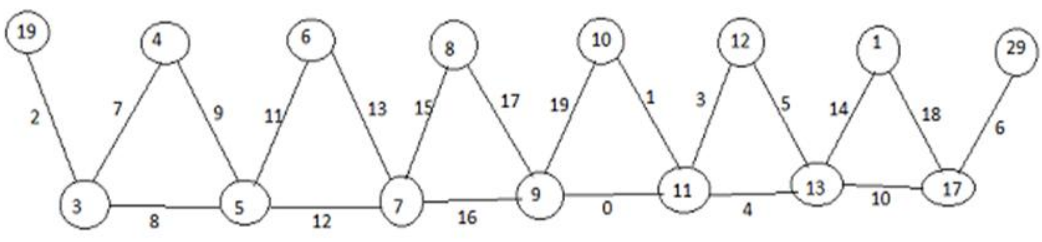

Figure 8

Theorem: 2.9: The graph P_n^2 is a prime harmonious labeling.
Proof: Let the vertices of P_n^2 be $\{ v_i : 1\leq i\leq n\}$ and the edges of P_n^2 be $\{v_iv_{i+1} : 1\leq i\leq n-1\} \cup\{v_iv_{i+2} : 1\leq i\leq n-2\}$.
We first label the vertices of P_n^2 as follows :
Define $f : V(P_n^2) \rightarrow \{1,2,3\ldots\ldots\ldots n\}$ by $f(v_i) = 2(i) +1$, $1\leq i\leq n$
Then the induced edge labels $f^*(E(G)) : f^*(v_iv_{i+1}) \cup f^*(v_iv_{i+2}) \rightarrow \{1,2\ldots\ldots..2n-3\}$ is bijection.
Therefore P_n^2 satisfies the condition of prime harmonious labeling.
Prime Harmonious labeling of $P_7^2$is shown in Figure 9

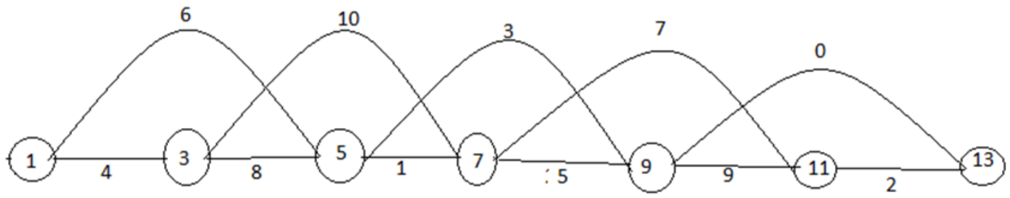

Figure 9

III. Conclusion

The harmonious labeling is one of the most important labeling techniques. Here we introduce the new concept of prime harmonious labeling of some new graphs and it is very interesting to investigate a graph family which admits prime harmonious labeling. In general, all the graphs are not prime harmonious labeling.

References

[1]. E. L. Aldred and B. D. Mckay, Graceful and harmonious labeling of trees, personal communication.
[2]. Dushyant Tanna, " Harmonious Labeling Of Certain Graphs"(*IJAERS) E-ISSN 2249-8974*
[3]. *Gallian , J.A.(2010).A dynamic survey of graph labeling, The Electronics Journal of Combinatorics, 17 (#DS6).*
[4]. R. L. Graham and N.J.A. Sloane, on additive bases and harmonious graphs, SIAM, J. Alg. Disc. Meth. 1, (1980), 382 – 404.
[5]. J. Gross and J. Yellon , "Graph Theory and its Application" CRC Press , Boca Raton, 1999.
[6]. Dr . Manonmani and R. Savithiri , Some New Results on K – Even Sequential Harmonious Labeling of Graphs.(LRG Govt. Arts College(W) , Tirupur- India)
[7]. A. Rosa, On certain Valuations of the vertices of a graph, Theory of Graphs (International Symposium, Rome, July 1966), Gordan ad Breach, N.Y. and Dunod Paris (1967) 349-355.
[8]. S.Uma Maheswari and Dr. K. Indirani, " Difference Speed Sequence Graph", (IJMTT) *–Volume 33 November 2-May 2016*
[9]. S.K. Vaidya and N .H. Shah, "Some New Families Of Prime Cordial Graphs," Journal of Mathematics Research.*Vol 3, No.4, 2011,pp 21-30.*