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Identities Involving Generalized H-Function Of Two Variables

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Abstract: *The aim of this research paper is to establish some new identities involving generalized H–function of two variables.*

I. Introduction

The generalized H–function of two variables is given by Shrivastava, H. S. P. [2] and defined as follows:

$$H^{m_1, n_1:m_2, n_2;m_3, n_3}_{p_1, q_1:p_2, q_2;p_3, q_3}\left[\begin{matrix} x \\ y \end{matrix} \middle| \begin{matrix} (a_j; \alpha_j, A_j)_{1, p_1}:(c_j, \gamma_j)_{1, p_2}:(e_j, E_j)_{1, p_3} \\ (b_j; \beta_j, B_j)_{1, q_1}:(d_j, \delta_j)_{1, q_2}:(f_j, F_j)_{1, q_3} \end{matrix}\right]$$

$$= \frac{-1}{4\pi^2}\int_{L_1}\int_{L_2} \phi_1(\xi, \eta)\, \theta_2(\xi)\theta_3(\eta)x^{\xi}\, y^{\eta}\, d\xi\, d\eta, \qquad (1)$$

where

$$\phi_1(\xi, \eta) = \frac{\prod_{j=1}^{n_1}\Gamma(1-a_j+\alpha_j\xi+A_j\eta)\prod_{j=1}^{m_1}\Gamma(b_j-\beta_j\xi-B_j\eta)}{\prod_{j=n_1+1}^{p_1}\Gamma(a_j-\alpha_j\xi-A_j\eta)\prod_{j=1}^{q_1}\Gamma(1-b_j+\beta_j\xi+B_j\eta)},$$

$$\theta_2(\xi) = \frac{\prod_{j=1}^{m_2}\Gamma(d_j-\delta_j\xi)\prod_{j=1}^{n_2}\Gamma(1-c_j+\gamma_j\xi)}{\prod_{j=m_2+1}^{q_2}\Gamma(1-d_j+\delta_j\xi)\prod_{j=n_2+1}^{p_2}\Gamma(c_j-\gamma_j\xi)},$$

$$\theta_3(\eta) = \frac{\prod_{j=1}^{m_3}\Gamma(f_j-F_j\eta)\prod_{j=1}^{n_3}\Gamma(1-e_j+E_j\eta)}{\prod_{j=m_3+1}^{q_3}\Gamma(1-f_j+F_j\eta)\prod_{j=n_3+1}^{p_3}\Gamma(e_j-E_j\eta)},$$

x and y are not equal to zero, and an empty product is interpreted as unity p_i, q_i, n_i and m_j are non negative integers such that $p_i \geq n_i \geq 0$, $q_i \geq 0$, $q_j \geq m_j \geq 0$, $(i = 1, 2, 3; j = 2, 3)$. Also, all the A's, α's, B's, β's, γ's, δ's, E's, and F's are assumed to the positive quantities for standardization purpose.

The contour L_1 is in the ξ-plane and runs from $-i\infty$ to $+i\infty$, with loops, if necessary, to ensure that the poles of $\Gamma(d_j - \delta_j\xi)$ $(j = 1, ..., m_2)$ lie to the right, and the poles of $\Gamma(1 - c_j + \gamma_j\xi)$ $(j = 1, ..., n_2)$, $\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)$ $(j = 1, ..., n_1)$ to the left of the contour.

The countor L_2 is in the η-plane and runs from $-i\infty$ to $+i\infty$, with loops, if necessary, to ensure that the poles of $\Gamma(f_j - F_j\eta)$ $(j = 1, ..., m_3)$ lie to the right, and the poles of $\Gamma(1 - e_j + E_j\eta)$ $(j = 1, ..., n_3)$, $\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)$ $(j = 1, ..., n_1)$ to the left of the contour.

The generalized H-function of two variables given by (1) is convergent if

$$U = \sum_{j=1}^{n_1}\alpha_j + \sum_{j=1}^{m_1}\beta_j + \sum_{j=1}^{n_2}\gamma_j + \sum_{j=1}^{m_2}\delta_j$$

$$-\sum_{j=n_1+1}^{p_1}\alpha_j - \sum_{j=m_1+1}^{q_1}\beta_j - \sum_{j=n_2+1}^{p_2}\gamma_j - \sum_{j=m_2+1}^{q_2}\delta_j; \qquad (2)$$

$$V = \sum_{j=1}^{n_1} A_j + \sum_{j=1}^{m_1} B_j + \sum_{j=1}^{n_3} E_j + \sum_{j=1}^{m_3} F_j - \sum_{j=n_1+1}^{p_1} A_j - \sum_{j=m_1+1}^{q_1} B_j - \sum_{j=n_3+1}^{p_3} E_j - \sum_{j=m_3+1}^{q_3} F_j, \quad (3)$$

where | arg x | < ½ Uπ, | arg y | < ½ Vπ.

In the present investigation we require the following formula:

From Rainvile [1]:

$$z\Gamma(z) = \Gamma(z+1) \quad (4)$$

II. Identities

$$H^{m_1,n_1;m_2+2,n_2;m_3,n_3}_{p_1,q_1;p_2+1,q_2+2;p_3,q_3}\left[{\zeta \atop \eta}\middle|{(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2},(1-k,\nu):(e_j,E_j)_{1,p_3} \atop (b_j,\beta_j;B_j)_{1,q_1}:(1,h),(2-k,\nu),(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}}\right]$$

$$= (1-k)H^{m_1,n_1;m_2+1,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2+1;p_3,q_3}\left[{\zeta \atop \eta}\middle|{(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3} \atop (b_j,\beta_j;B_j)_{1,q_1}:(1,h)(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}}\right]$$

$$-H^{m_1,n_1;m_2+1,n_2+1;m_3,n_3}_{p_1,q_1;p_2+1,q_2+2;p_3,q_3}\left[{\zeta \atop \eta}\middle|{(a_j,\alpha_j;A_j)_{1,p_1}:(0,\nu),(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3} \atop (b_j,\beta_j;B_j)_{1,q_1}:(1,h),(d_j,\delta_j)_{1,q_2},(1,\nu):(f_j,F_j)_{1,q_3}}\right] \quad (5)$$

provided that where | arg ξ | < ½ Uπ, | arg η | < ½ Vπ, where U and V are given in (2) and (3) respectively.

$$H^{m_1,n_1;m_2+1,n_2+1;m_3,n_3}_{p_1,q_1;p_2+1,q_2+1;p_3,q_3}\left[{\zeta \atop \eta}\middle|{(a_j,\alpha_j;A_j)_{1,p_1}:(k-1,\nu),(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3} \atop (b_j,\beta_j;B_j)_{1,q_1}:(1,h),(d_j,\delta_j)_{1,q_2},(k,\nu):(f_j,F_j)_{1,q_3}}\right]$$

$$= (1-k)H^{m_1,n_1;m_2+1,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2+1;p_3,q_3}\left[{\zeta \atop \eta}\middle|{(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3} \atop (b_j,\beta_j;B_j)_{1,q_1}:(1,h)(d_j,\delta_j)_{1,q_2}:(f_j,F_j)_{1,q_3}}\right]$$

$$+H^{m_1,n_1;m_2+1,n_2+1;m_3,n_3}_{p_1,q_1;p_2+1,q_2+2;p_3,q_3}\left[{\zeta \atop \eta}\middle|{(a_j,\alpha_j;A_j)_{1,p_1}:(0,\nu),(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3} \atop (b_j,\beta_j;B_j)_{1,q_1}:(1,h),(d_j,\delta_j)_{1,q_2},(1,\nu):(f_j,F_j)_{1,q_3}}\right] \quad (6)$$

provided that where | arg ξ | < ½ Uπ, | arg η | < ½ Vπ, where U and V are given in (2) and (3) respectively.

Proof:

To prove (5), consider left hand side of (5), after using (1), to obtain

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\,\theta_2(\xi)\theta_2(\eta)\frac{\Gamma(1-h\xi)\Gamma(2-k-\nu\xi)}{\Gamma(1-k-\nu\xi)}x^{\xi}y^{\eta}\,d\xi d\eta$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\,\theta_2(\xi)\theta_2(\eta)\,\Gamma(1-h\xi)(1-k-\nu\xi)x^{\xi}y^{\eta}\,d\xi d\eta$$

[On using (4)]

$$= \frac{(1-k)}{(2\pi\omega)^2}\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\,\theta_2(\xi)\theta_2(\eta)\Gamma(1-h\xi)x^{\xi}y^{\eta}\,d\xi d\eta$$

$$-\frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\,\theta_2(\xi)\theta_2(\eta)\frac{\Gamma(1-h\xi)\Gamma(\nu\xi+1)}{\Gamma(\nu\xi)}x^{\xi}y^{\eta}\,d\xi d\eta$$

which in the light of (1) provides right hand side of (5).

Similarly the result (6) can be established.

References

[1]. Rainville, E. D.: Special Functions, Macmillan, NewYork, 1960.
[2]. Srivastava, H. S. P.: H-function of two variables I, Indore Univ., Res. J Sci. 5(1-2), p.87-93, (1978).