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Perfect Domination In Intuitionistic Fuzzy Graphs

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Abstract: *Let G= (V,E) be an intutitionistic fuzzy graph. Mahioub M. Q. Shubatah introduced the concept of perfect domination in intuitionistic fuzzy graphs. In this paper we study some theorem in perfect dominating sets of IFG. Also define connected perfect domination in intutionistic fuzzy graph and perfect domination in constant intuitionistic fuzzy domination number are discussed. Obtain some interesting results for this new parameter in IFG and constant IFG.*
Keywords*: Intuitionistic fuzzy graph, constant Intuitionistic fuzzy graph, perfect domination number, connected perfect domination number in IFG.*

I. Introduction

Preliminaries
In this section, some basic definitions relating to IFGs are given.
Definition 2.1. [12] Let E be the universal set. A *fuzzy set A* in E is represented by $A = \{(x, \mu_A(x)) : \mu_A(x) > 0, x \in\in E\}$, where the function $\mu_A : E \to [0, 1]$ is the membership degree of element x in the fuzzy set A.
Definition 2.2. [1] Let a set E be fixed. An *Intuitionistic Fuzzy set*(IFS) A in E is an object of the form $A = \{(x, \mu_A(x), \gamma_A(x))/x \in E\}$, where the function $\mu A : E \to [0, 1]$ and $\gamma_A : E \to [0, 1]$ determine the degree of memership and the degree of non - memership of the element $x \in E$, respectively and for every $x \in E$, $0 \le \mu_A(x) + \gamma_A(x) \le 1$.
Definition 2.3. [6] An *Intuitionistic Fuzzy Graph* (IFG) is of the form $G = (V,E)$, where
(i) $V = \{v_1, v_2, \ldots v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degree of membership and non - membershp of the element $v_i \in V$ respectively and $0 \le \mu_1 (v_i)+ \gamma_1 (v_i) \le 1$,for every $v_i \in V$ $(i = 1, 2, \ldots , n)$.
(ii) $E \subset V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that
$\mu_2 (v_i, v_j) \le min[\mu_1(v_i), \mu_1(v_j)]$
$\gamma_2 (v_i, v_j) \le max[\gamma_1(v_i), \gamma_1(v_j)]$
and $0 \le \mu_2 (v_i, v_j) + \gamma_2 (v_i, v_j) \le 1$ for every $(v_i, v_j) \in E$.
Here the triple $(v_i, \mu_{1i}, \gamma_{1i})$ denotes the degree of membership and degree of non - membership of the vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \gamma_{2ij})$ denotes the degree of membership and degree of non - membership of the edge relation $e_{ij} = (vi, v_j)$ on V .

Definition 2.4[9] Let $G = (V,E)$ be an IFG. Then the *cardinality* of G is defined to be $|G| = \left|\sum_{v_i \in V} \frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2} + \sum_{(v_i,v_j) \in E} \frac{1+\mu_2 (v_i,v_j)- \gamma_2 (v_i,v_j)}{2}\right|$
Definition 2.5[9] Let $G = (V,E)$ be an IFG, then the *vertex cardinality* of V defined by $|V| = \sum_{v_i \in V} \frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2}$ for all $v_i \in V$
Definition 2.6[9] Let $G = (V,E)$ be an IFG , then the *edge cardinality* of E defined by $|E|= \sum_{(v_i,v_j) \in E} \frac{1+\mu_2 (v_i,v_j)- \gamma_2 (v_i,v_j)}{2}$ for all $(vi, vj) \in E$.
Definition 2.7[9] The number of vertices (the cardinality of V) is called the *order* of an IFG, $G = (V,E)$, and is denoted by $O(G) = \sum_{v_i \in V} \frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2}$ for all $v_i \in V$

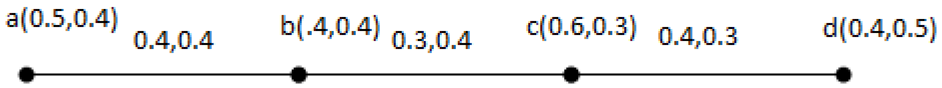
Definition 2.8[9] The number of edges (the cardinality of E) is called the *size* of an IFG, $G = (V,E)$, and is denoted by $S(G) = \sum_{(v_i,v_j) \in E} \frac{1+\mu_2 (v_i,v_j)- \gamma_2 (v_i,v_j)}{2}$ for all $(vi, vj) \in E$.
Definition 2.9[9] The *degree* of a vertex v in an IFG, $G = (V,E)$ is defined to be sum of the weights of the strong edges incident at v. It is denoted by $d_G(v)$.
The minimum degree of G is $\delta(G) = \min\{ d_G(v) \mid v \in V \}$.

The maximum degree of G is $\Delta(G)$ = max{ $d_G(v)$ | $v \in V$ }.
Definition 2.10[10] Two vertices v_i and v_j are said to be *neighbors* in IFG if either one of the following conditions hold
(1) $\mu_2(v_i, v_j) > 0, \gamma_2(v_i, v_j) > 0$,
(2) $\mu_2(v_i, v_j) = 0, \gamma_2(v_i, v_j) > 0$,
(3) $\mu_2(v_i, v_j) > 0, \gamma_2(v_i, v_j) = 0, v_i, v_j \in V$.
Definition 2.11 [10] A *path* in an IFG is a sequence of distinct vertices $v_1, v_2, \ldots v_n$ such that either one of the following conditions is satisfied:
(1) $\mu_2(v_i, v_j) > 0, \gamma_2(v_i, v_j) > 0$ for some *i* and *j*,
(2) $\mu_2(v_i, v_j) = 0, \gamma_2(v_i, v_j) > 0$ for some *i* and *j*,
(3) $\mu_2(v_i, v_j) > 0, \gamma_2(v_i, v_j) = 0$ for some *i* and *j*.
Definition 2.12 [10] The *length* of a path $P = v_1v_2 \ldots \ldots \ldots v_{n+1}$ $(n > 0)$ is *n*.
Definition 2.13[10] If v_i, v_j are vertices in $G = (V,E)$ and if they are connected by means of a path then the strength of that path is defined as $(min_{i,j}\mu_{2ij}, max_{i,j}\gamma_{2ij})$ where $min_{i,j}\mu_{2ij}$ is the $\mu-$ strength of the weakest arc and $max_{i,j}\gamma_{2ij}$ is the $\gamma-$ strength of the strongest arc.
Definition 2.14[10] If $v_i, v_j \in V \subseteq G$, the $\mu-$ strength of connectedness between v_i and v_j is $\mu_2^\infty(v_i, v_j) = \sup\{\mu_2^k(v_i, v_j) | k = 1, 2, \ldots, n\}$ and $\gamma-$strength of connectedness between v_i and v_j is $\gamma_2^\infty(v_i, v_j)$ = $\inf\{\gamma_2^k(v_i, v_j) \mid k = 1, 2, \ldots, n\}$
If *u*, *v* are connected by means of paths of length *k* then $\mu_2^k(u,v)$ is defined as $\sup\{\mu_2(u, v_1) \wedge \mu_2(v_1, v_2) \wedge \mu_2(v_2, v_3) \wedge \ldots \wedge \mu_2(v_{k-1}, v) \mid (u, v_1, v_2 \ldots v_{k-1}, v \in V)\}$ and $\gamma_2^k(u, v)$ is defined as $\inf\{\gamma_2(u, v_1) \vee \gamma_2(v_1, v_2) \vee \gamma_2(v_2, v_3) \ldots \vee \gamma_2(v_{k-1}, v) \mid (u, v_1, v_2 \ldots v_{k-1}, v \in V)\}$.
Definition 2.15 [10] Two vertices that are joined by a path is called *connected*.
Definition 2.16 [10] An IFG,$G = (V,E)$ is said to be complete IFG if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$ for every $v_i, v_j \in V$.
Definition 2.17 [10] The *complement* of an IFG, $G = (V,E)$ is an IFG, $\bar{G} = (\bar{V}, \bar{E})$, where
(i) $\bar{V} = V$,
(ii) $\overline{\mu 1i} = \mu_{1i}$ and $\overline{\gamma_{1i}} = \gamma_{1i}$,, for all $i = 1, 2, \ldots, n$,
(iii) $\overline{\mu 2ij} = \min(\mu_{1i}, \mu_{1j}) - \mu_{2ij}$ and
$\overline{\gamma_{2ij}} = \max(\gamma_{1i}, \gamma_{1j}) - \gamma_{2ij}$ for all $i, j = 1, 2, \ldots, n$.
Definition 2.18[10] Let *u* be a vertex in an IFG $G = (V,E)$, then $N(u) = \{v : v \in V \& (u, v)$ *is a strong arc*$\}$ is called *neighborhood* of *u*.
Definition 2.19[10] A vertex $u \in V$ of an IFG $G = (V,E)$ is said to be an *isolated vertex* if $\mu_2(u, v) = 0$ and $\gamma_2(u, v) = 0$ for all $v \in V$. That is $N(u) = \varphi$. Thus, an isolated vertex does not dominate any other vertex in *G*.
Definition 2.20[10] An arc (u, v) is said to be a *strong arc*, if $\mu_2(u, v) \geq \mu_2^\infty(u, v)$ and $\gamma_2(u, v) \geq \gamma_2^\infty(u,v)$.
Definition 2.21[10] Let $G = (V,E)$ be an IFG on *V* . let $u, v \in V$, we say that *u dominates v* in *G* if there exists a strong arc between them. A subset *S* of *V* is called a *dominating set* in *G* if for every $v \in V - S$, there exists $u \in S$ such that *u* dominates *v*.
Definition 2.22[10] A dominating set *S* of an IFG is said to be *minimal dominating set* if no proper subset of *S* is a dominating set. Minimum cardinality among all minimal dominating set is called *lower domination number* of *G*, and is denoted by $d(G)$. Maximum cardinality among all minimal dominating set is called *upper domination number* of *G*, and is denoted by $D(G)$.
Definition 2.23[10] Two vertices in an IFG, $G = (V,E)$ are said to be *independent* if there is no strong arc between them. A subset *S* of *V* is said to be *independent set* of *G* if $\mu_2(u, v) < \mu_2^\infty(u, v)$ and $\gamma_2(u, v) < \gamma_2^\infty(u, v)$ for all $u, v \in S$.
Definition 2.24[10] A dominating set D in an an intuitionistic fuzzy graph G = (V,E) is said to be independent dominating set in G if D is independent.
Definition 2.25[10] An independent dominating set D in an an intuitionistic fuzzy graph G =(V,E) is said to be minimal independent dominating set in G if for every v∈ D, D−{v} is not dominating set in G. The minimum cardinality among all minimal independent dominating set in G is called the independence domination number of G and is denoted by i(G)
Definition 2.26[10] Let $G = (V,E)$ be an IFG without isolated vertices. A set *D* is a *total dominating set* if for every vertex $v \in V$, there exists a vertex $u \in S$, $u \neq v$, such that *u* dominates *v*. The minimum cardinality of a total dominating set is called *lower total domination number* of *G*, and it is denoted by $t(G)$. The maximum cardinality of a total dominating set is called *upper total domination number* of *G*, and it is denoted by $T(G)$.

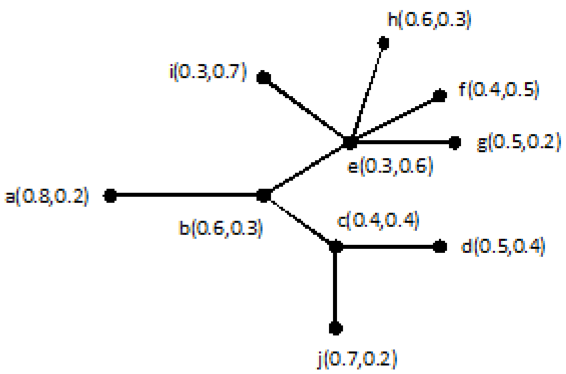
Definition 2.27[7]A dominating set D in an intuitionistic fuzzy graph G = (V,E) is called perfect dominating set in G if for each vertex v∈ V − D, there exists exactly one vertex u∈ D such that u dominates v. A perfect dominating set D in an intuitionistic fuzzy graph G = (V,E) is said to be minimal perfect dominating set if for each v∈ D, D − {v} is not a perfect dominating set in G. The minimum cardinality among all minimal perfect dominating sets in G is called the perfect domination number of G and is denoted by γp(G) or simply γp. A perfect dominating set D with smallest cardinality equal to γp(G) is called the minimum perfect dominating set and is denoted by γp − set.

II. Main Results

Definition 3.1 A perfect fuzzy dominating set D in an intuitionistic fuzzy graph G = (V,E) is said to be a **connected perfect dominating set(CPDS)** if the induced subgraph <D> is connected.
The minimum cardinality of a connected perfect dominating set in IFG is called the **connected perfect domination number** of G and is denoted by γ_{cpif} (G).
Theorem3.2: Every CPDS in IFG is a Total dominating set in intuitionistic fuzzy graph.
Proof: Let D be a minimal connected perfect dominating set of intuitionistic fuzzy graph G. Suppose v∈ $V - D$ is not dominated by some vertex u in D and <D> disconnected. We know that by definition of connected perfect dominating set is if every vertex v∈ $V - D$ is perfect dominated by exactly one vertex u in D and if induced subgraph <D> is connected. So, we have every vertex v∈ V is dominated by some vertex u in D and which is a contradiction. Therefore D is a total dominating set in intuitionistic fuzzy graph.
Theorem3.3: For any fuzzy tree such that G* is a tree T with more than 3 vertices, γ_{cpif} (G) = $p - \sigma(u^*)$ where u^* is the pendent vertices of IFG.
Proof: From by example, consider a tree T with 4 vertices, here {b,c} is the connected perfect dominating set. So, γ_{cpif} (G)= 1.15.
Also, $p - \sigma(u^*)$ = 2.15-1=1.15. Therefore γ_{cpif} (G)=$p - \sigma(u^*)$



Consider a tree 10 vertices, here {b,c,e} is the connected perfect dominating set. So, γ_{cpif} (G)=1.5. Also, $p - \sigma(u^*)$ = 5.65-4.15=1.4. Therefore γ_{cpif} (G)=$p - \sigma(u^*)$



Theorem3.4: If G is an IFG such that G* is a tree then every fuzzy cutnode of G is a CPDS for n≥ 4.
Proof: Since G is a intuitionistic fuzzy tree such that G* is a tree, every internal nodes of G are fuzzy cut nodes of G say D. then for all x∈V-D is adjacent to exactly only one vertex in D, which implies for all x∈V-D dominate exactly only one vertex in D. since every arc of a fuzzy tree is strong. Hence D is a perfect dominating set. Obviously D is a connected perfect dominating set.(since G* is a tree)

Theorem3.5: Every CPDS of an IFG is a DS of an IFG.
Proof: Let D be a minimal connected perfect dominating set in IFG. Let u,v ∈ D. Suppose u is not dominated by some vertex v in D. we know that if every vertex v∈ $V - D$ is dominated by exactly one vertex in D and the induced subgraph <D> is connected. Then we get every vertex v∈ $V - D$, there exists u∈ D such that u dominates v which is contradiction to our assumption. Therefore u dominates v. That is D is a dominating set.

Theorem3.6: Every CPDS of an IFG is a CDS of an IFG.

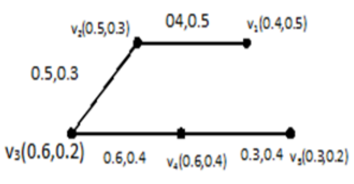
Proof: Let D be a minimal connected perfect dominating set IFG. Suppose v is not dominated by vertex u in D and the induced fuzzy graph<D> is disconnected. We know that by definition of connected perfect dominating set is if every vertex v$\in V-D$ is perfect dominated by exactly one vertex u in D and the induced fuzzy subgraph <D> is connected. So, we have every vertex v$\in V-D$ is dominated by some vertex u in D which is a contradiction. Therefore D is connected dominating set.

Theorem3.7: A CPDS in an IFG G=(V,E) is a minimal CPDS if for each vertex v$\in$ D then the following conditions are holds. (i) N(v)$\cap$D$\neq \emptyset$ (ii) for every vertex u$\in$V-D such that N(u) $\cap$D= {v}
Proof:Let D be a CPDS and for each v$\in D$ satisfies two conditions. Now we show that D is a minimal CPDS of intuitionistic fuzzy graph. Suppose D is not minimal, then there exists a vertex v$\in D$such that D-{v} is a connected perfect dominating set. As D-{v} is a connected perfect dominating set, u dominates to exactly one vertex in u$\in$ D-{v}.Also u$\in$ D-{v} is a connected perfect dominating set, every vertex u$\in$V-D is dominate to exactly one vertex D-{v}. That is (i) N(v)$\cap$D$\neq \emptyset$ condition (i) holds. If u$\neq$v we get (ii)
Theorem3.8: A CPDS of an intuitionistic fuzzy graph G = (V,E) is not independent.
Proof: Let G = (V,E) be an intuitionistic fuzzy graph. Suppose D be an independent set of IFG, if there exists no strong arc between them then for every vertex v in V-D is need not be an independent dominating set. Therefore D is a CPDS.

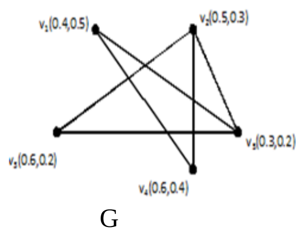
Theorem3.9: For any complete IFG G=(V,E) then perfect dominating set D of IFG is a singleton
Proof: Here we have two cases:
Case(i): Let G be a complete intuitionistic fuzzy graph. Suppose we take all the vertices are having the equal membership values, then we have $\gamma_{pif}(G)= \sigma$…………………………..(1) (since $\sigma(u) = \sigma(v) = \sigma$)
Case(ii): Let G be a complete intuitionistic fuzzy graph. Suppose we take all the vertices are having different membership values. Here every vertex is perfect dominate to all other vertices.
If perfect dominating set is the smallest membership values of the vertices then, $\gamma_{pif}(G)$ = min {$|u|$: u$\in V$}……………….(2) From (1) and (2) we get perfect dominating set D of IFG is a singleton.

Theorem3.10: An independent set is a maximal independent set of IFG, G = (V,E) if and only if it is independent and PDS.
Proof: Assume *D* is both independent and perfect dominating. Suppose *D* is not maximal independent, then there exists a vertex $v\in V-D$, the set $D\cup \{v\}$ is independent. If $D\cup\{v\}$ is independent then no vertex in *D* is exactly one vertex such that u is a strong neighbor to v. Hence *D* cannot be a perfect dominating set, which is a contradiction. Hence *D* is a maximal independent set. Conversely, Let *D* be a maximal independent set in an IFG, and hence for every vertex $v \in V-D$, the set $D \cup\{v\}$ is not independent. For every vertex $v\in V-D$, there is exactly one vertex $u\in D$ such that *u* is a strong neighbor to *v*. Thus *D* is a perfect dominating set. Hence *D* is both perfect dominating and independent set.
Theorem3.11: Every maximal independent set in an IFG G=(V,E) is a minimal PDS.
Proof: Let D be a maximal independent set in a IFG. By the above theorem D is a perfect dominating set.
Suppose D is not a minimal perfect dominating set, then there exists exactly one vertex $u\in D$ for
which D $-\{u\}$ is a perfect dominating set. But if $D-\{u\}$ dominates $V-\{D-(v)\}$, then there exists exactly one vertex in $D-\{v\}$ must be strong neighbor to *v*. This contradicts the fact that *D*is an independent set of *G*. Therefore, D must be a minimal perfect dominating set.

Remark: Let D be a PDS in complete IFG but $\overline{D}$ need not be a PDS in IFG.
Remark: Let D be a PDS in complete bipartite IFG then $\overline{D}$ is a PDS in IFG.
Remark: Let D be a PDS in IFG then $\overline{D}$ be a complementary of PDS in IFG. The converse need not be true for all graphs For example,



G:



G

From this figure, Let D be a perfect dominating set in IFG of G then $\overline{D}$ need not be a perfect dominating set in IFG of $\bar{G}$

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