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$\hat{\rho}$ - Closed sets in Topological Space

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Abstract: *In this paper, we introduce and study sets called $\hat{\rho}$-closed set and $\hat{\rho}$-open set. Also we have investigated some of their basic properties.*
Keywords: *$\hat{\rho}$ – closed set, $\hat{\rho}$ – open set*

I. Introduction

Levine [6], Mashhour et. al.,[9] introduced semi-open sets, preopen sets in topological spaces respectively. The complement of a semi-open (resp. preopen) set is called a semi-closed (resp. preclosed) set. Levine [5] introduced generalized closed (briefly g-closed) sets. Maki et al [8] introduced the concepts of generalized preclosed sets. In this paper, we define and study a set called $\hat{\rho}$ – closed set.

II. Preliminaries

Throughout this paper (X, τ) represents non-empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure of A and interior of A in X respectively.

2.1 Definition

Let (X, τ) be a topological space. A subset A of the space X is said to be

1. a semi-open set [6] if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.
2. a preopen set [9] if $A \subseteq int(cl(A))$ and a preclosed set if $cl(int(A)) \subseteq A$.
3. an α-open set [10] if $A \subseteq int(cl(int(A)))$ and a α-closed set if $cl(int(cl(A))) \subseteq A$.
4. a semi-preopen set [1] (β-open) if $A \subseteq cl(int(cl(A)))$ and a semi-preclosed set

(β-closed) if $int(cl(int(A))) \subseteq A$.

2.2 Definition ([9])

Let (X, τ) be a topological space and $A \subseteq X$.

1. The Pre-closure of A, denoted by pcl(A), is the intersection of all preclosed sets containing A.
2. The Pre-interior of A, denoted by pint(A), is the union of all preopen subsets of A.

2.3 Lemma ([1])

For any subset A of X, the following relations hold.

1. $Scl(A) = A \cup int(cl(A))$
2. $\alpha cl(A) = A \cup cl(int(cl(A)))$
3. $Pcl(A) = A \cup cl(int(A))$
4. $Spcl(A) = A \cup int(cl(int(A)))$

2.4 Definition

A subset A of a space (X, τ) is called

1. a generalized closed set [5] (g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open,
2. a generalized semi-closed set [2] (gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open,
3. a generalized preclosed set [8] (gp-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open,
4. a generalized α-closed set (gα-closed) [7] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open,
5. a generalized semi-preclosed set [4] (gsp-closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

The complements of the above mentioned sets are their respective open sets.

III. Basic Properties Of $\hat{\rho}$ – Closed Set

We introduce the following definition.

3.1 Definition
A subset A of a space (X, τ) is said to be ρ̂ - closed set in (X, τ) if Pcl (A) ⊆ Int (U) whenever A ⊆ U and U is semi open in (X, τ).

3.2 Theorem Every ρ̂ - closed set is pre closed.
Proof: Let A be a ρ̂ - closed set in X. Then Pcl (A) ⊆ Int (U) whenever A ⊆ U. Now, A ∪ cl(int(A)) ⊆ Int (U). Therefore, cl(int(A)) ⊆ A ∩ Int(U). Since A ⊆ U, cl(int(A)) ⊆ A. Hence, A is pre closed.
The converse of the above theorem need not be true as it is seen from the following example.

3.2.1 Example Let X= {a, b, c} and τ = { ϕ, X, {a}}. Then the set A = {b} is preclosed but not ρ̂ – closed in (X, τ).

3.3 Theorem Every ρ̂ - closed set is semipre closed.
Proof: Let A be a ρ̂ - closed set in X. Since every preclosed set is semipre closed and by theorem 1, it follows that A is semipre closed.
The converse of the above theorem need not be true as it is seen from the following example.

3.3.1 Example Let X= {a, b, c} and τ = { ϕ, {a}, {a,b}, X }. Then the set A = {c} is semipre closed but not ρ̂ – closed in (X, τ).

3.4 Theorem Every ρ̂ - closed set is gp-closed.
Proof: Let A be a ρ̂ - closed set in X. Let A ⊆ U and U is open in X. Every open set is semi open and thus A is ρ̂ – closed. Therefore Pcl (A) ⊆ Int (U) = U. Hence A is gp-closed.
The converse of the above theorem need not be true as it is seen from the following example.

3.4.1 Example Let X= {a, b, c} and τ = { ϕ, X, {a}}. Then the set A = {a, b} is gp-closed but not ρ̂ – closed in (X, τ).

3.5 Remark ρ̂-closed sets are independent concepts of semi-closed sets and α-closed sets as we illustrate by means of the following examples.

3.5.1 Example
1. Let X= {a, b, c} and τ = { ϕ, {a}, {a,b}, X }. Then the set A = {c} is both semi-closed and α-closed but not ρ̂-closed in (X, τ).
2. Let X= {a, b, c} and τ = { ϕ, X, {a, b}}. Then the set A = {b, c} is ρ̂-closed but neither semi-closed nor α-closed in (X, τ).

3.6 Remark The union (intersection) of two ρ̂-closed sets need not be ρ̂-closed.

3.6.1 Example Let X = {a, b, c, d} and τ = { ϕ, {c}, {a, b}, {a, b, c}}.
1. Let A = {a} and B = {b}. Here A and B are ρ̂-closed sets. But A ∪ B = {a, b} is not ρ̂-closed.
2. Let A = {b, c, d} and B = {a, c, d}. Here A and B are ρ̂-closed sets. But A ∩ B = {c, d} is not ρ̂-closed.

3.7 Theorem If a set A is ρ̂-closed, then Pcl(A) – A contains no nonempty closed set.
Proof: Let F ⊆ Pcl(A) – A be a nonempty closed set. Then F ⊆ Pcl(A) and A ⊆ X – F. Since X – F is semi open, then A is ρ̂-closed. Therefore Pcl(A) ⊆ Int(X – F) = X – cl(F), Cl(F) ⊆ X – Pcl(A). And so F ⊆ X – Pcl(A), F ⊆ Pcl(A) ∩ (X – Pcl(A)) = {∅}. Hence Pcl(A) – A contains no nonempty closed set.
The converse of the above theorem need not be true as it is seen from the following example.

3.7.1 Example Let X= {a, b, c} and τ = { ϕ, X, {a}}. Let A = {b}, then Pcl(A) – A contains no nonempty closed set. But A is not ρ̂-closed in (X, τ).

3.8 Theorem If a set A is ρ̂-closed, then Pcl(A) – A contains no nonempty semi-closed set.
Proof: Let F be a nonempty semi-closed set such that F ⊆ Pcl(A) – A. Then F ⊆ Pcl(A) and A ⊆ X – F. We have Pcl(A) ⊆ Int(X – F), Pcl(A) ⊆ X – cl(F), Cl(F) ⊆ X – Pcl(A). Therefore F ⊆ Pcl(A) ∩ (X – Pcl(A)) = {∅}. Hence Pcl(A) – A contains no nonempty semi-closed set.
The converse of the above theorem need not be true as it is seen from the following example.

3.8.1 Example Let X= {a, b, c} and τ = { ϕ, {a}, {a,b}, X }. Let A = {c}, then Pcl(A) – A contains no nonempty semi-closed set. But A is not ρ̂-closed in (X, τ).

3.9 Theorem An open set A of (X, τ) is gp-closed if and only if A is ρ̂-closed.
Proof: Let A be an open and gp-closed set. Let A ⊆ U and U be semi-open in X. Since A is open, A = Int(A) ⊆ Int(U). Observe that Int(U) is open and thus A is gp-closed. Hence Pcl(A) ⊆ Int(U) and A is an ρ̂-closed set.
Conversely, by Theorem 3, every ρ̂-closed set is gp-closed.

3.10 Remark From the above discussions and known results we have the following implications A→B represents A implies B but not conversely and A ↮ B represents A and B are independent of each other. See Figure 1.

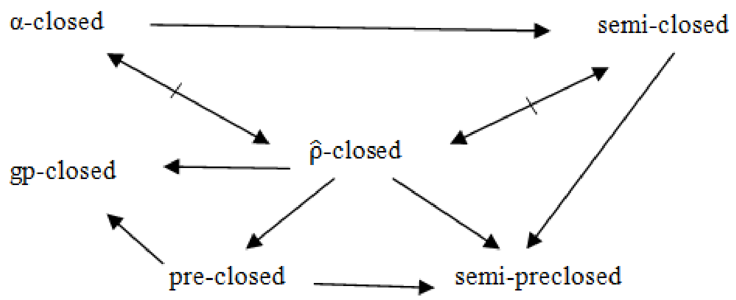

Figure 1: Implications

IV. ρ̂-Open Sets

4.1 Definition
A subset A of (X, τ) is said to be ρ̂-open in (X, τ) if its complement X – A is ρ̂-open in (X, τ).

4.2 Theorem A subset A of a topological space (X, τ) is an ρ̂-open set if and only if cl(K) ⊆ pint(A) whenever K ⊆ A and K is semi-closed.
Proof:
Necessity. Let A be an ρ̂-open set in (X, τ). Let K ⊆ A and K be semi-closed. Then X – A is ρ̂-closed and it is contained in the semi-open set X – K. Therefore Pcl(X – A) ⊆ Int(X – K), X – pint(A) ⊆ X – cl(K). Hence cl(K) ⊆ pint(A).
Sufficiency. If K is semi-closed set such that cl(k) ⊆ pint(A) whenever K ⊆ A. It follows that X – A ⊆ X – K and X – pint(A) ⊆ X – cl(K). Therefore Pcl(X – A) ⊆ Int(X – K). Hence X – A is ρ̂-closed and A becomes an ρ̂-open set.

4.3 Theorem If A ⊆ K is ρ̂-closed the Pcl(A) – A is ρ̂-open.
Proof: Let A be an ρ̂-closed. Then by Theorem 5, Pcl(A) – A contains no nonempty semi-closed set. Therefore ϕ = K ⊆ Pcl(A) – A and ϕ = K is semi-closed. Clearly, cl(K) ⊆ pint(Pcl(A) – A). Hence by theorem 7, Pcl(A) – A is ρ̂-open.

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