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Improving the Pedagogical Content Knowledge (PCK) of Mathematics Teachers to Enhance Student Learning and Understanding

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Abstract: *The effective teaching and learning of mathematics could be attributed to many factors. Among them include the teachers' content knowledge, the application of teaching strategies and the knowledge and experience that students bring into the teaching and learning environment. The term Pedagogical Content Knowledge (PCK) is defined as teachers' interpretations and transformations of subject matter knowledge in the context of facilitating student learning and understanding. This paper aims at highlighting some of the key elements of PCK that may enhance the teaching and learning of mathematics, especially at foundation phase. Data for the study were collected from literature and the experiences of the researcher as a mathematics teacher at various levels of the education system also contributed to the study. The study found that the following elements of PCK may facilitate students' learning of mathematics: curriculum knowledge, knowledge of representations of subject matter, understanding of students' conceptions and/or misconceptions of the subject matter and general teaching strategies.*

Keywords: *mathematics, teaching and learning, students' knowledge, content knowledge.*

I. Introduction

The effective teaching and learning of mathematics at schools, especially at the foundation phase, have been a major concern in both developed and developing countries. This comes as a result of the large number of students who fail to obtain good results in the subject. The poor results of students in mathematics, especially at school level, have sometimes been attributed to lack of adequate content knowledge of teachers (McCarthy & Oliphant, 2013). For that matter most of the professional development initiatives for in-service mathematics teachers focus on increasing the content knowledge of the participants. In South Africa, for example, a large amount of time and other resources have been spent in an attempt to improve the content knowledge of the existing mathematics teachers. It is generally perceived that once a teacher has a good knowledge of the subject, he/she can be a great teacher. However, teaching is a complex activity which requires more than just knowing the subject in and out.

Even though a mathematics teacher needs to possess adequate knowledge of the subject, there are other attributes that contribute to the effective teaching and learning of the subject. Shulman (1986), acknowledged the importance of the combination of both content and teaching knowledge for effective teaching to take place. He described this as pedagogical content knowledge (PCK). According to Shulman (1986), PCK is a form of practical knowledge that is used by teachers to guide their actions in a highly conceptualised classroom setting.

The concept of PCK was initiated by Shulman in 1986 as the combination of content and pedagogical knowledge in teaching. Since then many authors, such as Gess-Newsome and Lederman (2001) have expanded the concept of PCK to include dynamic behaviours and dispositions that evolve over time. The study therefore seeks to highlight other key elements of PCK and its importance, especially in the teaching and learning of mathematics.

Content knowledge

Content knowledge describes the teachers' knowledge about the subject he/she is teaching. The content knowledge of a teacher is very critical in all disciplines. In mathematics, the content knowledge of a teacher includes knowledge of concepts, theories, ideas and established practices. It will be very disastrous and suicidal for a mathematics teacher not to have a comprehensive base of content knowledge. This may lead to students receiving incorrect information and eventually developing misconceptions about particular topics (National Research Council, 2000). Hoadley (2016) affirms a positive link between teacher content knowledge and student learning. An effective mathematics teacher therefore, needs to have a conceptual understanding of mathematics. Fennema and Frankie (1992) argue that if a teacher has a conceptual understanding of mathematics, this will influence classroom instruction in a positive way.

Pedagogical knowledge

Pedagogical knowledge of teachers addresses the teachers' deep knowledge about the processes and practices or methods of teaching and learning. This may include the overall educational purposes, values and aims (Koehler & Mishra, 2009). Pedagogical knowledge generally applies to understanding the students we teach and knowing how they learn, and general classroom skills. Good instructional strategies, coupled with the knowledge of lesson planning and forms of assessment generally form part of the teachers' pedagogical knowledge. It is therefore imperative that a mathematics teacher possesses a deep pedagogical knowledge so that he/she can understand how students construct knowledge and acquire skills. Koehler and Mishra (2009) state that pedagogical knowledge requires an understanding of cognitive, social and developmental theories of learning and how they apply to students in the classroom.

In a mathematics classroom, it is necessary for teachers to understand the importance of knowledge of mathematical representations, since mathematics is seen as a composition of a large set of highly related abstractions (Fennema & Frankie, 1992). Thus mathematics teachers need to know how to translate those abstractions into a form that will help students to relate the new lesson to what they already know.

Pedagogical content knowledge

In the teaching and learning of mathematics, this form of practical knowledge includes the knowledge of the common conceptions and misconceptions that students bring into a mathematics lesson.It also includes teachers' knowledge and understanding of common learning difficulties and preconditions of students. Turnuklu and Yesildere (2007) affirm that the knowledge of students' cognitions is seen as one of the important components of teacher knowledge. In addition to that mathematics teachers are to know aspects of the subject that are typically easy for students and which are more difficult. In this study therefore the concept of PCK is defined as follows: the content and pedagogical knowledge as well as the knowledge and experience of students in order to structure and represent academic content for teaching and learning. PCK is uniquely constructed by teachers and thus is the "special" form of a teacher's professional knowledge and understanding. PCK has also been described as craft knowledge and it comprises integrated knowledge representing teachers' accumulated wisdom with respect to their teaching practice. However, it should be understood that it is highly impossible to distinguish PCK from either subject-matter knowledge or general pedagogical knowledge. In support of the above, van Driel, Verloop and de Vos (1998), assert that a teacher's familiarity with a specific topic in combination with his or teaching experience, positively contributes to PCK. Furthermore, a general pedagogical knowledge may constitute a supporting framework for the development of PCK.

In a study by Anthony and Walshaw (2009), it was found that the nature of classroom mathematics teaching significantly affects the nature and outcome of student learning. This paper then seeks to highlight some of the key elements of PCK that may enhance the teaching and learning of mathematics at school level.

II. Literature study

The meaning of the concept PCK has evolved over the past three decades since its introduction by Shulman in 1986. PCK, according to Shulman, is a combination of content and teaching knowledge. Rather than being treated as mutually exclusive, teacher education programs need to combine the two knowledge fields. Consequently, the key elements of PCK are the knowledge of representation of subject matter on one hand and the understanding of specific learning difficulties and students conceptions on the other hand. In alluding to the importance of PCK, Rowan (2001) claims that the concept (PCK) builds on other forms of professional knowledge. Hence it is deemed to be a critical constitutive element in the knowledge base of teaching. Van Driel, Verloop and de Vos (1998) refer to PCK as a specific form of craft knowledge or teachers' practical knowledge. They describe craft knowledge as an integrated knowledge which represents teachers' accumulated wisdom with respect to their teaching practice. This knowledge, according to van Driel, Verloop and de Vos (1998), guides teachers' actions in practice. It encompasses teachers' knowledge and beliefs with respect to the various aspects such as pedagogy, students, subject matter and curriculum. Thus, whereas Shulman's (1986) PCK encompasses every category of knowledge which may be relevant for teaching, the definition of craft knowledge, according to van Driel, Verloop and de Vos (1998), is restricted to types of knowledge which actually guides the teachers' behaviour during classroom practice.

From the above, it could be argued that teaching is a complex human activity that requires an interweaving of many kinds of specialised knowledge. In teaching mathematics for example, teachers need to apply complex knowledge structures across different cases and contexts. This means for effective teaching of mathematics, one requires a flexible access to rich, well-organised and integrated knowledge from different domains (Koehler & Mishra, 2009). These include knowledge of student thinking and learning, knowledge of subject matter and, more recently, knowledge of modern technology. Building on Shulman's (1986) descriptions of PCK, Koehler and Mishra (2009) have come out with how teachers' understanding of educational technologies and PCK interact with one another to produce effective teaching with technology. Koehler and

Mishra argue that at the heart of good teaching with technology are three core components: content, pedagogy and technology, together with the relationship among and between them.

Cochran, De Ruiter and King (1993) revised the original model of Shulman's PCK to make it more consistent with constructivist perspective on teaching and learning. Cochran, De Ruiter and King (1993) describe a model of PCK that results from an integration of four major components. These are 1) subject matter knowledge, 2) pedagogical knowledge, 3) knowledge of students' ability and learning strategies, ages and developmental levels, attitudes, motivations and prior knowledge of the concept to be taught and 4) knowledge and understanding of social, political, cultural and physical environments in which students are supposed to learn.

III. Discussion

PCK generally is the intersection of teachers' knowledge in a subject and the pedagogical knowledge. It concerns the manner in which teachers relate their pedagogical knowledge to their subject matter. Therefore in order to develop a strong PCK, a teacher needs to have a good command of the subject content. Anthony and Walshaw (2009), affirm that the way mathematics teachers organise classroom instruction is very much dependent on what they know and believe about mathematics. They went on to say that a sound content knowledge enables teachers to represent mathematics as a coherent and connected system. Mathematics teachers who have a good knowledge of the subject are in a better position to assess their students' level of mathematical understanding. They are also in a better position to identify students' misconceptions and guide them appropriately.

In the teaching and learning of mathematics, teachers need to figure out how best students can be helped to grasp core mathematical concepts. In order to do this successfully they need substantial pedagogical content knowledge and a grounded understanding of students as learners. Mathematics teachers with substantial PCK will be very much aware of the possibility of students' conceptions and misconceptions on a particular topic. Consequently, Shulman (1986) affirms that effective teaching is described and evaluated in terms of PCK. In essence, PCK is unique to individual teachers.

In teaching and learning mathematics, there is a need for students to develop understandings of how a concept or skill is connected in multiple ways to other mathematical ideas. Anthony and Walshaw (2009) affirm that effective teachers support students to make connections by providing them with opportunities to engage in complex tasks and by encouraging them to explain their thinking and solution strategies. Watson and Mason (2006) believe that teachers can assist students to make connections by using carefully sequenced examples, including examples of students' own solution strategies, to illustrate key mathematical ideas. In order for mathematics teachers to help students to develop the understanding of connections between concepts, demands a good pedagogical content knowledge. Hence PCK is the understanding of what makes the learning of specific concepts easy or difficult.

A good knowledge of students' ability and learning strategies is very critical in teaching. This knowledge will inform the teacher what teaching strategies to apply in a lesson. The knowledge of students' ability will assist the mathematics teacher to plan his or her lessons in a way that allows students to build on their existing proficiencies and experiences. Thus in planning lessons, students' current knowledge and interests are put at the centre of instruction. There is therefore the need for an on-going assessment of students' competencies on mathematical reasoning for teachers to adjust their instruction to meet the learning needs of their students. Having a good knowledge of students' mathematical ability allows a mathematics teacher to use a range of assessment practices to make students' thinking visible and support students'' learning. It is therefore important that a wide range of formal and informal assessments are conducted to monitor learning progress, to diagnose learning and to determine what can be done to improve learning (Anthony & Walshaw, 2009). More information about students can also be gathered by watching and listening to them as they engage in group work and by talking to them. Mathematics teachers need to monitor their students' understanding and observe the strategies that they prefer. Steinberg, Empson & Carpenter (2004) affirm that classroom exchanges in the form of careful questioning provide a powerful way to assess students' current knowledge and ways of thinking. By integrating class discussion as a strategy for teaching and learning allows students to provide their teachers with information about what they know and what they need to know (Anthony & Walshaw, 2009).

One of the components of PCK demands that teachers have the knowledge and understanding of students' social and cultural background in addition to the physical environment within which they learn. For example to get some concepts across to students, a mathematics teacher may use a narrative or a story. The choice of relevant stories in most cases will be determined by the socio-cultural background of majority of the students.

PCK is developed through teachers' everyday work and therefore the amount of a teacher's pedagogical knowledge is relative to the teaching experience (Gess-Newsome & Lederman, 2001). However it encompasses both theory learned during teacher training as well as experiences gained from everyday school

activities. Studies by Carpenter, Fennema, Petersen & Carey, 1988 and Feiman-Nemser & Parker, 1990, have shown that new teachers have incomplete or superficial levels of pedagogical content knowledge. According to Carpenter et al (1988) a novice teacher tends to rely on unmodified subject matter knowledge (most often directly extracted from the curriculum) and may not have a coherent framework or perspective from which to present the information. Van Driek, Verloop and de Vos (1998) affirm that an experienced teachers' PCK may differ considerably from a novice teacher even when their subject-matter knowledge is similar and when they teach the same curriculum.

Other factors that may influence the PCK of a teacher are related to the teacher's personal attributes and disposition. Some of these attributes may include friendliness, being approachable and good interpersonal relationship. In fact, these attributes can assist the teacher to get more information from his/her students. Information about students' misconceptions, learning styles and reasons behind some of their responses can be obtained if teachers have good personal relationships with them. Hence it is important for teachers (especially mathematics teachers) to develop a good interpersonal relationship with students. It is not enough for mathematics teachers to know and understand students' limitations as far as the teaching of the subject is concerned, they also need to show interest that they are prepared to assist students. From my teaching experiences, I have come to realise that students are open to teachers who are approachable and friendly.

By engaging students in group work, an effective mathematics teacher would gather information about students. During group work, most students tend to be more active and participate effectively in discussions than when the whole class is involved. The group work will therefore allow the teacher to have a better assessment of such students and that more information could be obtained from each member of the group.

IV. Summary

The paper aimed at highlighting some of the elements of PCK that may enhance the teaching and learning of mathematics. The following domain of knowledge were identified for an effective teaching and learning of mathematics.

- Knowledge of what teaching strategies would be appropriate in a particular learning context
- Knowing how different aspects of a topic can be arranged to facilitate learning
- Knowledge of what makes concepts difficult or easy to learn
- Knowledge of students' prior knowledge
- Knowledge of teaching strategies that address students difficulties and misconceptions and foster meaningful understanding
- Knowledge of what students bring to the learning environment, such as prior conceptions and misconceptions, and potential misapplications of prior concepts.

V. Conclusion

This paper has acknowledged that effective mathematics teachers need to demonstrate some competencies such as subject knowledge competency. However, subject knowledge competency is only one of the many domains of knowledge that are critical to effective mathematics teaching. The concept of PCK has revealed that knowing the subject matter inside out, does not make one an effective teacher. In addition to knowing the subject matter, an effective teacher should be able to package the lesson in such a way that it builds on students' prior knowledge and meets their interest and understanding. The teacher's knowledge of how to get students understand a particular concept is one of the key elements of PCK. For example a mathematics teacher may use creative and interesting examples coupled with careful demonstrations to explain a concept to students. The paper therefore concludes that there is a need for mathematics teachers to improve their PCK in order to stimulate the interest of students in learning mathematics.

VI. Recommendations

The study recommends that mathematics teachers should always reflect on what they do in class and see how they can improve on their instructional strategy. It is also recommended that mathematics teachers explore new ways of teaching particular mathematics concepts that may appeal to the diverse student population. The paper further recommends that mathematics teachers have a cordial relationship with their students. This will encourage students to be open and disclose whatever information needed to help in your instructional strategy.

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