IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 12, Issue 6 Ver. II (Nov. - Dec.2016), PP 07-13
www.iosrjournals.org

Some Parameters of SM family of Graphs

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Abstract
Two classes of graphs were introduced with certain properties. The First one was the SM sum graph and other was the SM balancing Graph. These graphs together with its subgraphs, complement graphs are named here after as SM family of Graphs. Here we discuss some parameters of SM family of graphs. These are systematically arranged graphs. Since the systematic array of graphs are more significant in real life , science and technology, we are using these types of graphs to form a family of graphs with certain properties. Graphs with fixed parameters are required in real life applications in many fields. Also the topological indices like wiener indices are more important in life science and computer science. Most of the real life problems can be illustrated diagrammatically with a set of points joined together with lines or arcs. Here we are dealt with some of the graph parameters like wiener indices and Zagreb indices of these graphs.
AMS subject classification: $05C99$
keywords: Center and Radius of $n^{th}SM$ Balancing graphs and $n^{th}SM$ sum graphs, Zagreb indices and Wiener indices of SM graphs - n^{th} $SM(B_n)$ and n^{th} $SM(\Sigma_n)$ graphs.

Introduction

Graph theory is one of the most research focused branch of Mathematics. It has been witnessed a tremendous growth due to a number of applications in computer communication, IT networks, molecular Physics, Chemistry, Social network and biological sciences, computational linguistics and in many other areas. Graph Parameters have a worthy role in all these applications. Many Graph parameters have been introduced and applied in various fields. The mainly used graph parameters are various types of domination numbers, domatic numbers, independence number, covering number, matching number, Zagreb indices, Harary indices, wiener indices etc. In this paper we discuss some of these parameters and its applications. Two classes of graphs were introduced by us in earlier papers. Now we are exploring these graphs up to applications. Some preliminaries are given below.

1 n^{th} SM Balancing Graphs and n^{th} SM sum Graphs

Definition 1.1. *[8] Consider the set $T = \{3^m, 0 \le m \le n-1\}$ for a fixed positive integer $n \ge 2$. Let $I = \{-1, 0, 1\}$. Any positive integer $p \le \frac{1}{2}(3^n - 1)$ which is not a power of 3 can be expressed as $p = \sum_{j=1}^{n} \alpha_j y_j$ for some $\alpha_i \in I$ and $y_j \in T$. If $\alpha_i \ne 0$, then each y_j is called a balancing component of p.*

Definition 1.2. *[8] let T be the set $T = \{3^m, 0 \le m \le n-1\}$ for a fixed positive integer $n \ge 2$. Consider the simple directed graph G=(V,E), where the vertex set $V = \{v_1, v_2, \ldots, v_{\frac{1}{2}(3^n-1)}\}$ and adjacency of vertices defined by, two distinct vertices v_x is adjacent to v_{yj} if (1) holds and $\alpha = -1$ and two distinct vertices v_{yj} is adjacent to v_x if (1) holds and $\alpha = 1$. This directed graph G is called the $n^{th}SMD$ Balancing Graph, $SMD(B_n)$. The underlying undirected graph is called n^{th} SM Balancing Graphs, $SM(B_n)$.*

Definition 1.3. *If $p < 2^n$, is a positive integer which is not a power of 2, then $p = \sum_1^n x_i$, with $x_i =$ 0 or 2^m, for some $0 \le m \le n-1$ and x_is are distinct. Here we call each $x_i \ne 0$ as an additive component of p.*

Definition 1.4. *For a fixed $n \geq 2$, define a simple graph $SM(\sum_n)$, called n^{th} SM sum graph, with vertex set $\{v_1, v_2, \ldots, v_{2^n-1}\}$ and adjacency of vertices defined by, v_i and v_j are adjacent if either i is an additive component of j or j is an additive component of i.*

Definition 1.5. *[3] If $G(V, E)$ is a graph, then wiener index, $w(G)$ is defined as the sum of distances between all unordered pairs of vertices of G. ie., $w(G) = \sum_{\{u,v\} \subseteq V} d(u, v)$, where $d(u, v)$ is the distance between u and v.*

Definition 1.6. *[3] If $G(V, E)$ is a graph, then hyper wiener index, $ww(G)$ is defined as* $ww(G) = \frac{1}{2} \sum_{\{u,v\} \subseteq V} [d(u, v)^2 + d(u, v)]$

Definition 1.7. *[8] Let $G = SM(\sum_n)$ or $SM(B_n)$ with vertex set V be an n^{th} SM graph. The Adi -R-set of degrees, denoted by A_n^R is defined as $A_n^R = \{degv_{i(x)}, v_i \in V\}$, where x is the number of times each $degv_i$ repeats.*

2 Parameters of complement of $SM(\sum_n)$ and $SM(B_n)$

Here we are considering the complement of graphs $SM(\sum_n)$ and $SM(B_n)$. In most of the cases graph parameters are useful to study the nature of graph up to isomorphism. The complement $\overline{G}$ of a graph G is the graph with the same vertices as of G and with the property that two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. Some parameters of the complement of these graphs are discussed here.

Definition 2.1. *Consider the graph $SM(\sum_n)$. If x_i, $i = 1, 2, \ldots n$, are the additive components of x, where $x < 2^n$, is a positive integer which is not a power of 2, then $C_x = \{x_i, i = 1, 2, 3, ...n\}$ where $x_i \in P$, is called a "component set" associated with the number x. Also we say that x is formed by C_x.*

Lemma 2.2. *If $G = \overline{SM(\sum_n)}$, is the complement of $SM(\sum_n)$, $n \geq 3$ and $P = \{2^m : 0 \leq m \leq n-1\}$, then*

$$d(v_i, v_j) = \begin{cases} 2 & \text{, if } i \text{ is an additive component of } j \text{ or } j \text{ is an additive component of } i \\ 1 & \text{, otherwise} \end{cases}$$

Proof. Let $G = \overline{SM(\sum_n)}$, $P = \{2^m : 0 \leq m \leq n-1\}$, $V = \{v_1, v_2, \ldots, v_{2^n-1}\}$.
case 1: If i is an additive component of j or j is an additive component of i, In this case v_i and v_j are non adjacent in G. Then v_i and v_j have a common neighbour v_t where t is formed by $(C_i - \{j\}) \bigcup (P - C_i)$ or $(C_j - \{i\}) \bigcup (P - C_j)$ and it follows that $d(v_i, v_j) = 2$.
case 2:

(i) Now let $i, j \in P$. In this case v_i and v_j are adjacent in G, then it follows that $d(v_i, v_j) = 1$

(ii) Also when $i, j \notin P$, v_i and v_j are adjacent in G, then it follows that $d(v_i, v_j) = 1$

(iii) When $i \in P$ and $j \notin P$, i is not an additive component of j or $j \in P$ and $i \notin P$, j is not an additive component of i. In these cases, v_i and v_j are adjacent in G, then it follows that $d(v_i, v_j) = 1$.

□

Corollary 2.3. *Suppose $G = \overline{SM(\sum_n)}$ graph ,$n \geq 3$, then $diam(G) = 2$*

Proof. By lemma 2.2 , the corollary follows. □

Proposition 2.4. *Let $G = \overline{SM(\sum_n)}$ be the complement of n^{th} SM sum graph. $n \geq 3$. Let $d_r(v_i, v_j)$ denotes the number of unordered pairs of vertices for which $d(v_i, v_j) = r$. Then:*

$$d_r(v_i, v_j) = \begin{cases} n.(2^{n-1} - 1) & , \textit{if } r = 2 \\ \dfrac{(2^n - 2)(2^n - 1)}{2} - n.(2^{n-1} - 1) & , \textit{if } r = 1 \end{cases}$$

Proof. when $r = 2$, by lemma 2.2, i is an additive component of j or j is an additive component of i. In this case the number of unordered pairs is equal to $n.(2^{n-1} - 1)$.
Also we have $|V| = 2^n - 1$.
Now let us consider the remaining cases, *ie*, the cases in which $r = 1$. In this case the number of unordered pairs is equal to $\dfrac{(2^n - 2)(2^n - 1)}{2} - n.(2^{n-1} - 1)$. Hence the proof. □

Theorem 2.5. *Let $G = \overline{SM(\sum_n)}$, $n \geq 3$. Then $w(G) = 2^{2n-1} - 3.2^{n-1} + n.2^{n-1} - n + 1$. Also $ww(G) = \frac{1}{2}\left[2^{2n} - 3.2^n + 2 + n.2^{n+1} - 4n\right]$.*

Proof. By the definition of $w(G)$ and proposition 2.4,

$$\begin{aligned} w(G) &= \sum_{\{u,v\} \subseteq V} d(u, v) \\ &= 2n.(2^{n-1} - 1) + 1.[\frac{(2^n - 2)(2^n - 1)}{2} - n.(2^{n-1} - 1)] \\ &= 1.(2^{2n-1} - 3.2^{n-1} + 1) + n.(2^{n-1} - 1) \\ &= 2^{2n-1} - 3.2^{n-1} + n.2^{n-1} - n + 1 \end{aligned}$$

Now we get,

$$\begin{aligned} \sum_{\{u,v\} \subseteq V} (d(u, v))^2 &= 4n.(2^{n-1} - 1) + 1.[\frac{(2^n - 2)(2^n - 1)}{2} - n.(2^{n-1} - 1)] \\ &= 1.(2^{2n-1} - 3.2^{n-1} + 1) + 3n.(2^{n-1} - 1) \\ &= 2^{2n-1} - 3.2^{n-1} + 3n.2^{n-1} - 3n + 1 \end{aligned}$$

$$\begin{aligned} Therefore,\ ww(G) &= \frac{1}{2}[w(G) + \sum_{\{u,v\} \subseteq V} (d(u, v))^2] \\ &= \frac{1}{2}[2^{2n-1} - 3.2^{n-1} + 3n.2^{n-1} - 3n + 1 + 2^{2n-1} - 3.2^{n-1} + n.2^{n-1} - n + 1] \\ &= \frac{1}{2}[2^{2n} - 3.2^n + 2 + n.2^{n+1} - 4n] \end{aligned}$$

Hence the theorem. □

Corollary 2.6. *Let $G = \overline{SM(\sum_n)}$, $n \geq 3$ .Let m be the number of edges of G. Then $w(G) = m + 2n.(2^{n-1} - 1)$.*

Proof. We have, $m = \dfrac{(2^n - 2)(2^n - 1)}{2} - n.(2^{n-1} - 1)$. Hence the proof. □

Definition 2.7. *Consider the graph $SM(B_n)$. $T = \{3^m : 0 \leq m \leq n - 1\}$. If x_i, $i = 1, 2, ..n$, are the balancing components of the positive integer x, where $x \leq \frac{1}{2}(3^n - 1)$ which is not a power of 3, then $B_x = \{x_i, i = 1, 2, 3, ...n\}$ where $x_i \in T$ is called a "component set" associated with the number x. Also we say that x is formed by B_x.*

Lemma 2.8. *If $G = \overline{SM(B_n)}$,is the complement of $SM(B_n)$. $T = \{3^m : 0 \leq m \leq n - 1\}$, $n \geq 3$, then*

$$d(v_i, v_j) = \begin{cases} 2 & , \textit{if } i \textit{ is a balancing component of } j \textit{ or } j \textit{ is a balancing component of } i \\ 1 & , \textit{otherwise.} \end{cases}$$

Proof. Let $G = \overline{SM(B_n)}$, $T = \{3^m : 0 \le m \le n-1\}$, $V = \{v_1, v_2, \ldots, v_t\}$. , where $t = \frac{1}{2}(3^n - 1)$
case 1: Consider the case when i is a balancing component of j or j is a balancing component of i, In this case v_i and v_j are non adjacent in G. Assume j is a balancing component of i. Then v_i and v_j have a common neighbour v_k where k is formed by $(B_i - \{j\}) \bigcup (T - B_i)$ and it follows that $d(v_i, v_j) = 2$.
case 2:

(i) $i, j \in T$. In this case v_i and v_j are adjacent in G, then it follows that $d(v_i, v_j) = 1$.

(ii) $i, j \notin T$, v_i and v_j are adjacent in G,and therefore $d(v_i, v_j) = 1$.

(iii) When $i \in T$ and $j \notin T$, i is not a balancing component of j or $j \in T$ and $i \notin T$, j is not a balancing component of i. In these cases, v_i and v_j are adjacent in G, then it follows that $d(v_i, v_j) = 1$.

□

Corollary 2.9. *Suppose* $G = \overline{SM(B_n)}$ *graph , then*

$$diam(G) = 2$$

Proof. By lemma 2.8 , the corollary follows. □

Proposition 2.10. *Let* $G = \overline{SM(B_n)}$ *be an* n^{th} *SM sum graph. Let* $d_r(v_i, v_j)$ *denotes the number of unordered pairs of vertices for which* $d(v_i, v_j) = r$. *Let* $t = \frac{1}{2}(3^n - 1)$, $n \ge 3$, *Then:*

$$d_r(v_i, v_j) = \begin{cases} n.(3^{n-1} - 1) & , \text{if } r = 2 \\ \dfrac{(t-1)t}{2} - n.(3^{n-1} - 1) & , \text{if } r = 1 \end{cases}$$

Proof. when $r = 2$, by lemma 2.8, i is a balancing component of j or j is a balancing component of i. In this case the number of unordered pairs is equal to $n.(3^{n-1} - 1)$. Also we have $|V| = t$. Now let us consider the remaining cases, *ie* the cases in which $r = 1$. In this case the number of unordered pairs is equal to $\dfrac{t(t-1)}{2} - n.(3^{n-1} - 1)$. Hence the proof. □

Theorem 2.11. *Let* $G = \overline{SM(B_n)}$, $n \ge 3$. *Then* $w(G) = n.3^{n-1} - n + \frac{3^{2n}}{8} - \frac{3^n}{2} + \frac{3}{8}$. *Also* $ww(G) = 2n.3^{n-1} - 2n + \frac{3^{2n}}{8} - \frac{3^n}{2} + \frac{3}{8}$.
Proof. Let $t = \frac{1}{2}(3^n - 1)$. By the definition of $w(G)$ and proposition 2.10,

$$\begin{aligned} w(G) &= \sum_{\{u,v\} \subseteq V} d(u, v) \\ &= 2n.(3^{n-1} - 1) + \frac{(t-1)t}{2} - n.(3^{n-1} - 1) \\ &= n.3^{n-1} - n + \frac{3^{2n}}{8} - \frac{3^n}{2} + \frac{3}{8} \end{aligned}$$

$$\begin{aligned} \textit{Now we find, } \sum_{\{u,v\} \subseteq V} (d(u, v))^2 &= 4n.(3^{n-1} - 1) + \frac{(t-1)t}{2} - n.(3^{n-1} - 1) \\ &= 3n.3^{n-1} - 3n + \frac{3^{2n}}{8} - \frac{3^n}{2} + \frac{3}{8} \end{aligned}$$

Therefore,

$$\begin{aligned} ww(G) &= \frac{1}{2}[w(G) + \sum_{\{u,v\}\subseteq V} (d(u,v))^2] \\ &= \frac{1}{2}[n.3^{n-1} - n + \frac{3^{2n}}{8} - \frac{3^n}{2} + \frac{3}{8} + 3n.3^{n-1} - 3n + \frac{3^{2n}}{8} - \frac{3^n}{2} + \frac{3}{8}] \\ &= 2n.3^{n-1} - 2n + \frac{3^{2n}}{8} - \frac{3^n}{2} + \frac{3}{8} \end{aligned}$$

Hence the theorem. □

Corollary 2.12. *Let $G = \overline{SM(B_n)}$, $n \geq 3$ .Let m be the number of edges of G. Then $w(G) = m + 2n.(3^{n-1} - 1)$.*

IIII. Centre and Radius of SM Graphs

The diameter of a connected simple graph G , denoted by $diam(G)$ is the maximum distance between two vertices. The eccentricity, $e(v)$, of a vertex is defined as the maximum distance from it to any other vertex. The radius of G, denoted by $rad(G)$ is the minimum eccentricity among all vertices of G. These parameters have been computed for many other graphs. Here we provide these parameters for the SM Graphs. The centre of a graph ,$C(G)$, is the subgraph induced by the set of vertices of minimum eccentricity. The periphery of G is $P(G) = \{v \in V : e(v) = diam(G)\}$.

Proposition 3.1. *If $G = SM(\sum_n)$, $n \geq 4$, $P = \{2^m : 0 \leq m \leq n-1\}$, then the eccentricity is given by*

$$e(v_i) = \begin{cases} 3 & , if\ i \in P \\ 2 & , if\ i = 2^n - 1 \\ 4 & , if\ i \neq 2^n - 1\ and\ i \notin P \end{cases}$$

Proof. Let $G = SM(\sum_n)$, $P = \{2^m : 0 \leq m \leq n-1\}$, $V = \{v_1, v_2, \ldots, v_{2^n-1}\}$.
case 1: If $i \in P$, then $j \in P\ or j \notin P$. Now there are 3 cases.

(i) If $j \in P$, then $d(v_i, v_j) = 2$.

(ii) If $j \notin P$ and i is not an additive component of j, then $d(v_i, v_j) = 3$

Therefore $e(v) = 3$
case 2: If $i = 2^n - 1$, that is $i = 1 + 2 + 4 + ... + 2^{n-1}$. Then $d(v_i, v_j) = 2\ or\ 1$. Therefore $e(v) = 2$
case 3: When $i \neq 2^n - 1$ and $i \notin P$, then there are 4 cases.

(a) $j \in P$ and is not an additive component of i . So $d(v_i, v_j) = 3$

(b) $j \in P$ and is an additive component of i . So $d(v_i, v_j) = 1$

(c) $j \notin P$ and, i and j have no common additive components . So $d(v_i, v_j) = 4$

(d) $j \notin P$ and, i and j have common additive components . So $d(v_i, v_j) = 2$

Therefore $e(v) = 4$. Hence proved. □

Corollary 3.2. *If $G = SM(\sum_n)$, $n = 3$, $P = \{2^m : 0 \leq m \leq n-1\}$, then the eccentricity is given by*

$$e(v_i) = \begin{cases} 2 & , if\ i = 2^n - 1 \\ 3 & , otherwise \end{cases}$$

Theorem 3.3. *Let $G = SM(\sum_n)$, $n \geq 4$ .Then the radius of G , $rad(G) = 2$. The centre of G is $C(G) = \{v_{2^n-1}\}$ and $P(G) = \{v_i,\ i \neq 2^n - 1\ and\ i \notin P\}$.*

Proof. The proof follows from the proposition 3.1. □

Proposition 3.4. *If $G = SM(B_n)$, $n \geq 4$, $T = \{3^m : 0 \leq m \leq n-1\}$, $V = \{v_1, v_2, \ldots, v_t\}$,where $t = \frac{1}{2}(3^n - 1)$, then*

$$e(v_i) = \begin{cases} 3 & , if\ i \in T \\ 2 & , if\ i = t \\ 4 & , if\ i \neq t\ and\ i \notin T \end{cases}$$

Proof. Let $G = SM(B_n)$, $T = \{3^m : 0 \leq m \leq n-1\}$, $V = \{v_1, v_2, \ldots, v_t\}$. ,where $t = \frac{1}{2}(3^n - 1)$
case 1: If $i \in T$, then $j \in T\ or j \notin T$. Now there are 3 cases.

(a) If $j \in T$, then $d(v_i, v_j) = 2$.

(a) If $j \in T$, then $d(v_i, v_j) = 2$.

(b) If $j \notin T$ and i is a balancing component of j, then $d(v_i, v_j) = 1$.

(c) If $j \notin T$ and i is not a balancing component of j, then $d(v_i, v_j) = 3$. Therefore $e(v) = 3$

case 2: If $i = t$, that is $i = 1 + 3 + 9 + ... + 3^{n-1}$. Then $d(v_i, v_j) = 2 \; or \; 1$. Therefore $e(v) = 2$.
case 3: When $i \neq t$ and $i \notin T$, then there are 4 cases.

(i) $j \in T$ and is not a balancing component of i . So $d(v_i, v_j) = 3$.

(ii) $j \in T$ and is a balancing component of i . So $d(v_i, v_j) = 1$

(iii) $j \notin T$ and, i and j have no common balancing components . So $d(v_i, v_j) = 4$

(iv) $j \notin T$ and, i and j have common balancing components. So $d(v_i, v_j) = 2$

Therefore $e(v) = 4$. Hence proved ☐

Theorem 3.5. *Let $G = SM(B_n)$, $n \geq 4$ and $t = \frac{1}{2}(3^n - 1)$. Then the radius of G , $rad(G) = 2$. The centre of G is $C(G) = \{v_t\}$ and $P(G) = \{v_i, \; i \neq t \text{ and } i \notin T\}$.*

Proof. The proof follows from the proposition 3.4. ☐

IV. First Zagreb indices of SM graphs

There are many vertex- degree based indices and distance based indices. The Zagreb indices are the examples of this kind of indices which are studied and given focus in recent research fields. The Zagreb indices was introduced by Gutman and Trinajstic [2]. Let $G = (V, E)$ be a simple graph. The degree of a vertex $v \in V$ is denoted by $d(v)$. The first Zagreb index was defined as $M_1(G) = \sum_{v \in V} d(v)^2$. The first Zagreb eccentricity index was defined as $M^*(G) = \sum_{v \in V} e(v)^2$, where $e(v)$ is the eccentricity of the vertex v.

Theorem 4.1. *Let $G = SM(\sum_n)$ be the n^{th} sum graph. For $n \geq 4$, first Zagreb index is given by $M_1(SM(\sum_n)) = \sum_{r=2}^{n} \binom{n}{r}.r^2 + n.(2^{n-1} - 1)^2$ and the first Zagreb eccentricity index is given by $M^*(SM(\sum_n)) = 2^{n+4} - 7n - 28$.*

Proof. For $SM(\sum_n)$, we have $A_n^R = \{2_{(\binom{n}{2})}, 3_{(\binom{n}{3})}, \ldots, n_{(\binom{n}{n})}, 2^{n-1} - 1_{(n)}\}$. $n \geq 4$
Therefore $M_1(SM(\sum_n)) = \sum_{v \in V} d(v)^2 = \sum_{r=2}^{n} \binom{n}{r}.r^2 + n.(2^{n-1} - 1)^2$.
By the proposition 3.1,

$$M^*(G) = \sum_{v \in V} e(v)^2 = 9n + 16(2^n - n - 2) + 4$$
$$= 2^{n+4} - 7n - 28$$

Theorem 4.2. *Let $G = SM(B_n)$ be the n^{th} SM balancing graph. $T = \{3^m : 0 \leq m \leq n - 1\}$. $n \geq 3$. First Zagreb index is given by $M_1(SM(B_n)) = \sum_{r=2}^{n} 2^{r-1}\binom{n}{r}.r^2 + n.(3^{n-1} - 1)^2$. First Zagreb eccentricity index is given by $M^*(SM(B_n)) = 8.3^n - 7n - 20$.*

Proof. We have $A_n^R = \{2_{(2\binom{n}{2})}, 3_{(2^2\binom{n}{3})}, \ldots, n_{(2^{n-1}\binom{n}{n})}, 3^{n-1} - 1_{(n)}\}$, for $n \geq 3$.
Therefore $M_1(SM(B_n)) = \sum_{v \in V} d(v)^2 = \sum_{r=2}^{n} 2^{r-1}\binom{n}{r}.r^2 + n.(3^{n-1} - 1)^2$.
By the proposition 3.4,

$$M^*(SM(B_n)) = \sum_{v \in V} e(v)^2 = 9n + 16\left[\frac{1}{2}(3^n - 1) - n - 1\right] + 4$$
$$= 8.3^n - 7n - 20$$

V. Conclusion

These n^{th} SM graphs form a special family of graphs with certain properties. Since the construction of these classes of graphs are based on different combinations of powers of 3 or 2, it will be useful in combinatorics, computer Science and IT related fields. A comprehensive study of these graphs may help in solving many real life network problems or other graph theory problems. It may be useful to study the nature and different graph theoretic parameters by the help of Zagreb indices of n^{th} SM Balancing graphs and n^{th} SM sum graphs. These graphs have some similarity in nature and structure. So the similar way of approach can be done in the proof as well as in the formation of results regarding these family of graphs. Also the similar nature of distance based indices may help in further comparative study of these family of graphs which will lead to many combinatorial results.

References

1. M.R Farhani, M.K.Jamil,Rajesh Khanna, R.Pradeepkumar,"Zagreb eccentricity indices of-PAHk", IJSER, 7(2),2016,1132-1135.
2. Gutman.I and N Trinajstic, "Graph Theory and molecular orbitals ,Total π electron energy of alternant hydrocarbons", Chem.Phy.Lett.17 (1972)13-19.
3. Harary F, Graph theory, addison-wesly publishing company inc(1994)
4. Kinkal ch Das and ivan Gutman, "Estimating the winer index by means of number of vertices, Number of edges and diameters", math commun.comp.chem, 64(2010) 647-660
5. Liyan, Ying angLi, Wei Gao and Junsheng Li, "On the externel Hyper-wiener index of graphs", journal of Chemical and Pharmaceutical Research, 2014, 6(3) ; 477-481.
6. Parthasarathy K R, basic Graph theory, Tata Mc Graw Hill New York, 1994.
7. Ramakrishnan K, Dr. M Upendrakumar, "Research methodology on application of Graph theory to Computer Science in Bug Graph networks for big data", International Journal of Mathematics trends and technology - vol 25, number 1- september 2015.
8. Sreekumar.K.G and Dr.Manilal, "n^{th} SM balancing graph and some of its parameters", IJCMS,Vol 5,p 56-65
9. Susanth C and Sunny Joseph Kalayathankal, "operations on covering numbers of certain gaphs", ar XIV: 1506.0325 —V—[math.co] 10 June 2015.
10. Udovit Neipel L, Martin Knor, "Domonation in a graph and its reverse", discrete applied mathematics, 157(2009), 2973-2977.
11. Zahra Yarahmadia , Sirous Moradib, "Center and periphery of composite graphs",IJMC,Vol 5, supplemnt 1, dec 2014,p 34-45.