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Axisymmetric Steady Flow of Blood through a Stenosed Arterial Tube

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Abstract: *The objective of this paper is to develop a mathematical model for studying the non-Newtonian flow of blood in presence of mild stenosis. The variation of flux, dimensionless resistance to flow and skin friction with the variation of stenosis height for different values of yield stress have been incorporated here by using Casson fluid model. The numerical results are shown in graphical form.*
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Keywords: *Casson flow, flow rate, wall shear stress, resistance to flow, Skin friction.*

I. Introduction

Blood consists of suspension of fluid particles in an aqueous solution of proteins and electrolytes called plasma, which is composed of 90% of water and 7% protein. There are 5×10^9 cells in a millilitre of healthy human blood, out of which 95% are red cells whose main function is to transport oxygen from lungs to all parts of the body and removal of carbon dioxide formed by the metabolic process in the body to the lungs. So the knowledge of blood flow problems is very much important for better understanding the anatomy of human system. Many researchers have presented mathematical models to get insight the physiological system of human body. It is well known from medical survey that cardiovascular diseases are responsible for more than 90% of death. Among the cardiovascular diseases, the familiar ones, such as stroke and hypertension, brain haemorrhage are closely related to blood flow characteristics. Blood flow characteristics are changed abruptly by the arterial diseases. Among the arterial diseases the important one is stenosis, which is formed by the deposition of fatty substances, like fats/lipids , cholesterol and abnormal growth of the connective tissue. For this reason normal blood flow is disturbed abnormally and as a result various types of cardiovascular diseases occur. Many authors [Young [1], Young and Tsai [2], Shukla et. al. [3], Sarkar and Jayaraman [4], Pralhad and Schultz [5], Jung et. al [6], Mishra et. al. [7], Sankar et. al. [8], Medhavi et. al. [9], Singh et. al. [10]] have tried to study the blood flow related problems for better understanding the knowledge of cardiovascular diseases. Recently the study of the stenosis on blood flow has become quite interest to many biomedical researchers [Chakraborty and Mandal [11], Srivastav et. al. [12], Sivastav and Srivastava [13], Maiti [14], Biswas et. al. [15],] both from the theoretical and experimental point of view.

In last few decades many theoretical and numerical studies have been conducted by many Mathematicians [Chaturani and Ponnalagorsamy [16],Nanda and Bose [17], Halder [18],] to study the non-Newtonian behaviour of blood. Some researchers have studied the power law fluid model of blood by giving reason that under certain conditions, blood behaves like a power law fluid. Casson [19] examined the validity of Casson model in studies the flow characteristics of blood and reported that at low shear rate the yield stress for blood is nonzero. Some authors [Maruthiprasad and Radhakrishnamacharya [20], Maruthiprasad et. al [21], Siddiqui et. al [22], Misra, et. al. [23]] have analysed mathematical models by considering blood as Herschel-Bulkley type non-Newtonian fluid. Blair and Spanner [24] reported that blood behaves like a casson fluid in the case of moderate shear rate flows.

In the present study I propose to discuss the effects of stenosis on Casson flow of blood through an constricted arterial segment.

II. The problem and its solution

Let us consider the steady flow of blood through an axially symmetric but radially non-symmetric constricted artery.

The geometry of bell-shaped stenosis is given by

$$\frac{R(z)}{R_0} = 1 - \frac{\delta}{R_0}\exp\left(-\frac{m^2\varepsilon^2 z^2}{R_0^2}\right), \quad \text{.............................(1)}$$

where R_0 stands for the radius of the arterial tube outside the stenosis, R(z) is the radius in the stenotic region, δ is the depth of the stenosis, m is a parametric constant and ε characterises the relative length of the constriction, defined as the ratio of the radius to half-length of stenosis.

$$\varepsilon = \frac{R_0}{L_0}$$

The geometry of stenosis can be written as (cf. Fig. 1)

$$\frac{R(z)}{R_0} = 1 - ae^{-bz^2}, \qquad \text{.................................(2)}$$

where

$$a = \frac{\delta}{R_0} \text{ and } b = \frac{m^2\epsilon^2}{R_0^2}$$

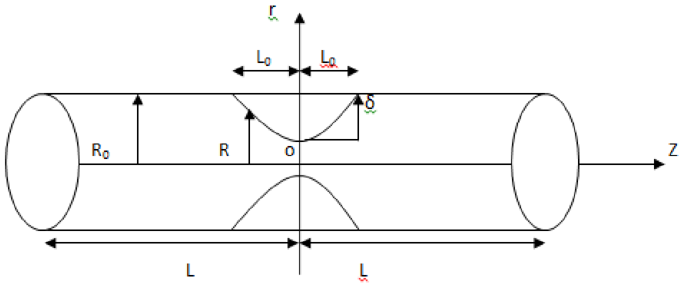

Fig. 1. Geometry of the arterial segment with stenosis

The equation governing the flow is given by

$$-\frac{dp}{dz} = \frac{1}{r}\frac{d(r\tau)}{dr}, \qquad \text{................................(3)}$$

in which τ represents the shear stress of blood considered as Casson fluid and p, the pressure at any point. The relationship between shear stress and shear rate is given by

$$-\frac{du}{dr} = f(\tau) = \frac{1}{k}(\sqrt{\tau} - \sqrt{\tau_c})^2; \ \tau \geq \tau_c$$
$$= 0; \qquad \tau < \tau_c, \ \text{...... ....(4)}$$

Where u stands for the axial velocity of blood, τ_c is the yield stress and k is the coefficient of viscosity.
The boundary conditions are

u = 0 at r = R(z) (no slip condition) ………………(5)

τ is finite at r = 0 (regularity condition) ....(6)

Integrating (3) and using the boundary condition (6) we get

$$\tau = -\frac{r}{2}\frac{dp}{dz}, \qquad \text{.......................(7)}$$

The skin-friction τ_R is given by

$$\tau = -\frac{R}{2}\frac{dp}{dz}, \text{ where } R = R(z) \qquad \text{...................................(8)}$$

The volumetric flow rate i.e., the flux is given by

$$Q = \int_0^R 2\pi r u \, dr \qquad \text{..........................(9)}$$

Integrating (9) and using no slip condition (5) we get

$$Q = 2\pi \int_0^{\tau_R} \left(-\frac{du}{dr}\right)\frac{r^2}{2} dr$$
$$= \pi \int_0^{\tau_R} \frac{1}{k}(\sqrt{\tau} - \sqrt{\tau_c})^2 . r^2 dr \qquad \text{.........................(10)}$$

From (7) and (8) we get

$$\frac{\tau}{\tau_R} = \frac{r}{R}$$

From which we get

$$r = \frac{R\tau}{\tau_R} \text{ and } dr = \frac{R}{\tau_R} d\tau$$

Thus we get

$$Q = \frac{\pi R^3}{k\tau_R^3}\int_0^{\tau_R} \tau^2(\tau - 2\sqrt{\tau\tau_c} + \tau_c)\, d\tau$$
$$= \frac{\pi R^3 \tau_R}{k}\left[\frac{1}{4} + \frac{\tau_c}{3\tau_R} - \frac{4}{7}\sqrt{\frac{\tau_c}{\tau_R}}\right] \qquad \text{………………(11)}$$

When $\frac{\tau_c}{\tau_R} \ll 1$, replacing $\frac{1}{3}$ by $\frac{16}{49}$ in the 2nd term of the above equation we get

$$Q = \frac{\pi R^3 \tau_R}{k}\left[\frac{1}{2} - \frac{4}{7}\sqrt{\frac{\tau_c}{\tau_R}}\right]^2$$

From which we get

$$\tau_R = \frac{64}{49}\tau_c + \frac{4kQ}{\pi R^3} + \frac{32}{7}\sqrt{\frac{\tau_c kQ}{\pi R^3}} \quad ....(12)$$

Thus

$$\frac{dp}{dz} = -\frac{128}{49}\frac{\tau_c}{R} - \frac{8kQ}{\pi R^4} - \frac{64}{7}\sqrt{\frac{\tau_c kQ}{\pi R^5}} \quad ....(13)$$

Integrating (13) along the length of the artery and using the conditions p = p_1 at z = $-L$
And p = p_2 at z = L we obtain

$$p_2 - p_1 = -\frac{128}{48}\frac{\tau_c}{R_0}\int_{-L}^{L}(\frac{R}{R_0})^{-1}\,dz - \frac{8kQ}{\pi R_0^4}\int_{-L}^{L}(\frac{R}{R_0})^{-4}dz - \frac{64}{7}\sqrt{\frac{\tau_c kQ}{\pi R_0^5}}\int_{-L}^{L}(\frac{R}{R_0})^{-5/2}\,dz, \quad ....……………..(14)$$

where $\frac{R}{R_0}$ can be obtained by using equation (4)

Thus the resistance to flow λ defined by

$$\lambda = \frac{p_2 - p_1}{Q}$$

$$= -\frac{256\tau_c}{49R_0Q}[(L-L_0) + \int_0^{L_0}(\frac{R}{R_0})^{-1}\,dz] - \frac{16k}{\pi R_0^4}[(L-L_0) + \int_0^{L_0}(\frac{R}{R_0})^{-4}\,dz]$$

$$-\frac{128}{7}\sqrt{\frac{\tau_c k}{\pi Q R_0^5}}[(L-L_0) + \int_0^{L_0}(\frac{R}{R_0})^{-5/2}\,dz] \quad …………………………..(15)$$

In the absence of stenosis the resistance to flow λ_N may be expressed as

$$\lambda_N = (-\frac{256\tau_c}{49R_0Q} - \frac{16k}{\pi R_0^4} - \frac{128}{7}\sqrt{\frac{\tau_c k}{\pi Q R_0^5}} \quad ……………(16)$$

In dimensionless form, the resistance to flow may be expressed as

$$\overline{\lambda} = \frac{\lambda}{\lambda_N} = 1 - \frac{L_0}{L} + \frac{1}{L}\frac{(f_1I_1 + f_2I_2 + f_3I_3)}{(f_1 + f_2 + f_3)} \quad .... (17)$$

Where

$$I_1 = \int_0^{L_0}(\frac{R}{R_0})^{-1}\,dz\;,\; I_2 = \int_0^{L_0}(\frac{R}{R_0})^{-4}\,dz\;,\; I_3 = \int_0^{L_0}(\frac{R}{R_0})^{-5/2}\,dz$$

and

$$f_1 = -\frac{256\tau_c}{49R_0Q}\;,\; f_2 = -\frac{16k}{\pi R_0^4},\; f_3 = -\frac{128}{7}\sqrt{\frac{\tau_c k}{\pi Q R_0^5}}$$

III. Numerical Discussions

To illustrate the flow analysis the results are shown graphically with the help of MATLAB-7.6.
To attain the numerical results for flux, resistance to flow and skin-friction, some parameters have been taken constant with the values

L = 1; k = 4,7; m = 1; $\varepsilon = 1$; Q = 0.002

Figures 2,3 give the variation of flow rate for different values of yield stress and z, with the variations of $\frac{\delta}{R_0}$. It is observed that Q decreases with the increase of $\frac{\delta}{R_0}$ and yield stress,but the reverse effect occurs when z increases. Figure 4 describes the effect of yield stress on resistance to flow against $\frac{\delta}{R_0}$. It is found that for fixed values of k, resistance to flow increases with the increase of $\frac{\delta}{R_0}$ but decreases when yield stress increases.Figures 5 and 6 depict the variation of skin-friction for different values of yield stress with the variation of z. It is observed that skin-friction increases with the increase of z up to the value zero and then decreases. Skin-friction decreases with the increase of yield stress for fixed values of k and z.

IV. Conclusions

Blood flow through an artery mainly depends on the pressure gradient and resistance to flow. It is clear that resistance to flow increases for irregular growth of stenos is whose consequences cause several diseases like hypertension, stroke, heart disease and brain haemorrhage. We cannot ignore this problem occurred in blood flow through human arteries while we present it by a model. The present mathematical analysis for modelling the blood flow in a human rigid constricted artery may be helpful for the development of new diagnostics tools which may predict diseases much before their clinical symptoms appear.

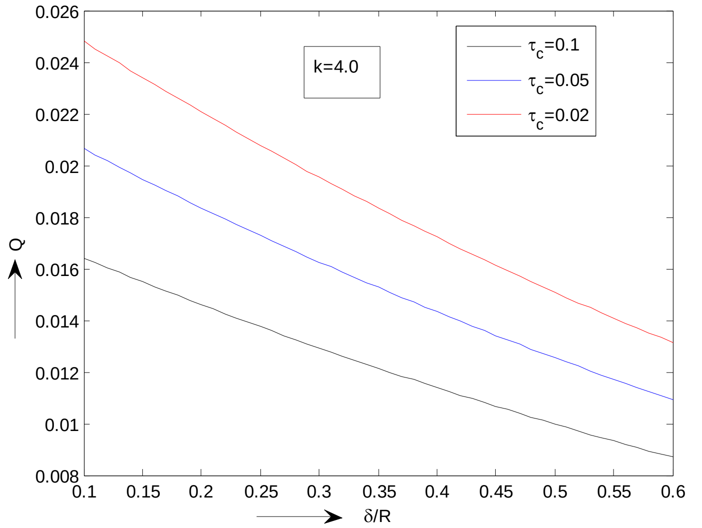

Figure -2

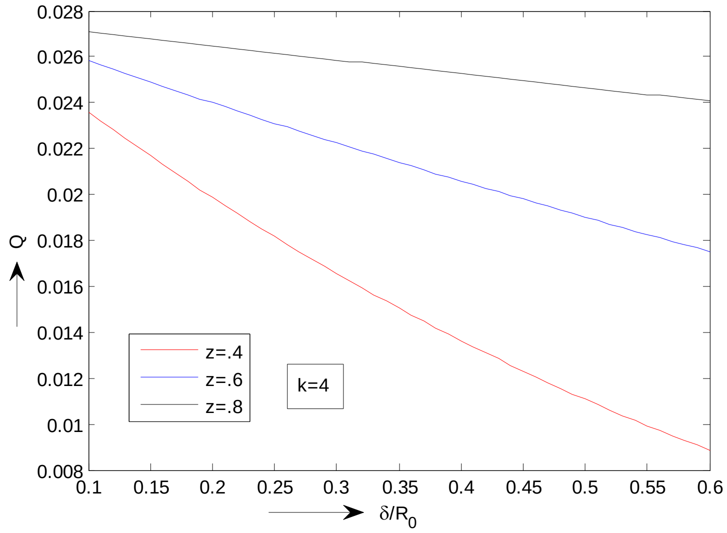

Figure -3

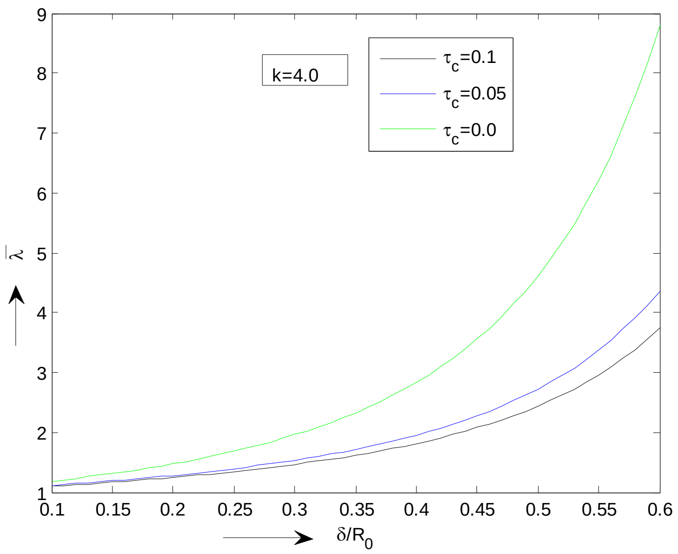

Figure -4

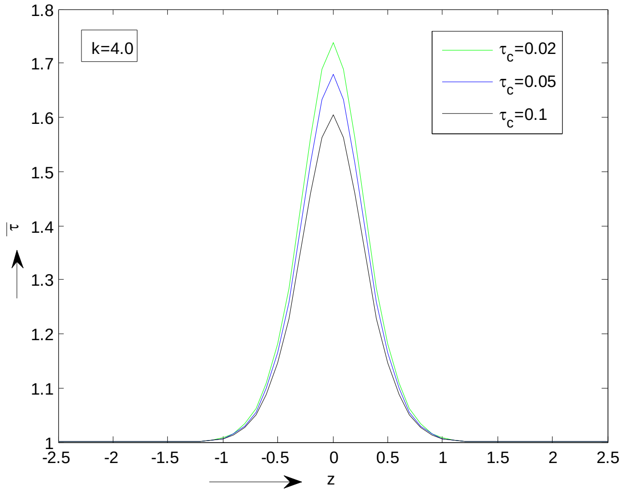

Figure-5

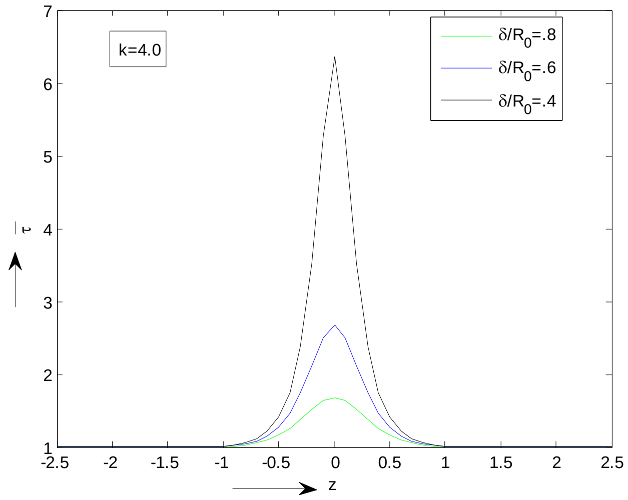

Figure -6

References

[1]. Young, D. F. : Effects of a time-dependent stenosis on flow through a tube, J. Engg. Ind., Traans, ASME (1968), vol. 90, 248-254.

[2]. Young, D.F. and Tsai, F. Y. : Flow characteristics in model of arterial stenosis-Steady flow, Journal of Biomechanics, (1973), vol.6, 395-410.

[3]. Shukla, J.B., Parihar, R.S. and Rao, B.R.P. :Effects of stenosis on non-Newtonian flow through an artery with mild stenosis, Bull. Math. Biol. (1980),vol.42,283-294.

[4]. Sarkar, A. and Jayaraman, G. : Correction to flow rate-pressure drop in coronary angioplasty: steady streaming effect , J. Biomech.,(1998), vol.31, 781-791.

[5]. Pralhad, R. N. and Schultz, D. H. : Modeling of arterial stenosis and its applications to blood diseases, Math. Biosci.,(2004), vol.190, 203-220.

[6]. Jung, H., Choi, J. W. and Park, C. G. : Asymmetric flow of non-Newtonian fluids in symmetric stenosed artery, Korea-Aust. Rheol, Journal, (2004), vol.16, 101-108.

[7]. Mishra, B. K. and Verma, N. : Effects of porous parameter and stenosis on the wall shear stress for the flow of blood in human body, Res. J. Medicine and Medical Sciences, (2007), vol.2, 98-101.

[8]. Sankar, A. R., Gunakala, S. R.and Comissiong, D. M. G.: A two-layered suspension blood flow through a composite stenosis, J. of Math. Res., (2013), vol.5 (4), 26-38.

[9]. Medhavi, A. , Srivastav, R. K., Ahmad, Q. S. and Srivastava, V. P. : Two-phase arterial blood flow through a composite stenosis, e-Journal of Science and Technology, (2012), vol.7(4), 83-94.

[10]. Singh, A.K., Singh, D. P. : Blood flow obeying Casson fluid equation through an artery with radially non-symmetric mild stenosis, American Journal of Mathematics and Mathematical Sciences, (2012), vol.1(1), 81-86.

[11]. Chakravarty, S. and Mandal, P. K. : Two-dimensional blood flow through tapered arteries under stenotic conditions, Int. J. Non-Linear Mech., (2001), vol. 36, 731-741.

[12]. Srivastav, R. K., Ahmad, Q. S. and Khan, A. W. : Blood flow through an overlapping stenosis in catheterized artery with permeable wall, e-Journal of Science and Technology, (2013), vol.8(2), 43-53.

[13]. Srivastav, R. K., Srivastava, V. P. : On two fluid blood flow through stenosed artery with permeable wall, Appl. Bionics. And Biomechanics, (2014), vol.11, 39-45.

[14]. Maiti, A. K. : Effect of stenosis on Bingham-Plastic flow of blood through an arterial tube , Int. J. Mathematics Trends and Technology,(2014), vol.13, 50-57.

[15]. Biswas, D. and Laskar, R. B. : Steady flow of blood through a stenosed artery: A non-Newtonian fluid model, Assam University Journal of Sci and Tech. (2011), vol.-7(11), 144-153.

[16]. Chaturani, P. and Ponnalagorsamy, R. : A study of non-Newtonian aspects of blood flow through stenosed arteries and its applications in arterial diseases, Biorheology, (1986), vol.- 22, 521-531.

[17]. Nanda, S., Bose, R. K. : A mathematical model for blood flow through a narrow artery with multiple stenosis, Journal of Applied Mathematics and Fluid Mechanics, (2012),vol.4(3), 233-242.

[18]. Haldar, K. : Effects of the shape of stenosis on the resistance to blood flow through an artery, Bull. Math. Biol.,(1985), vol.- 47, 545-550.

[19]. Casson, N. : Rheology of disperse systems in flow equation for pigment oil suspensions of the printing ink tube, Rhelogy of Disperse Systems, C. C. Mill, Ed., Pergamon Press, London, UK, (1959), 84-102.

[20]. Maruthiprasad, K. and Radhakrishnamacharya, G. : Flow of Herschel-Bulkley fluid through an inclined tube of non-uniform cross section with multiple stenosis. Arch. Mech., Warszawa, (2008), vol. 60(2),161-172.

[21]. Maruthiprasad, K., Vijaya, B. and Umadevi, C. : A mathematical model of Herschel-bulkley fluid through an over lapping stenosis, IOSR, Journal of mathematics, (2014), vol. 10(2), ver-II, 41-46.

[22]. Siddiqui, S. U., Verma, N. K. and Gupta, R.S.: A mathematical model for pulsatile flow of Herschel-Bulkley fluid through an stenosed arteries, Journal of science and technology, (2010),vol-4(5), 49-66.

[23]. Misra, J.C. and Shit, G. C. : Blood flow through an arteries in a Pathological state: A theoretical study, Int. J. Engg. Sci.(Elsevier),(2006), vol.- 44, 662-671.

[24]. Scott Blair, G.W. and Spanner, D. C. : An introduction to Biorheology, Elsevier Scientific Publishing Company, Amsterdam, Oxford and New York, (1974).