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Skolem Difference Fibonacci Mean Labelling of Special Class of Path Related Graphs

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Abstract: *The concept of Skolem difference mean labelling was introduced by K. Murugan and A. Subramanian [2]. The concept of Fibonacci labelling was introduced by David W. Bange and Anthony E. Barkauskas [1]in the form Fibonacci graceful. This motivates us to introduce Skolem difference Fibonacci mean labelling and is defined as follows: "A graph G with p vertices and q edges is said to have Skolem difference Fibonacci mean labelling if it is possible to label the vertices x∈ V with distinct elements f(x) from the set {1,2,...,F_{p+q}} in such a way that the edge e = uv is labelled with $\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting edge labels are distinct and are from {F_1, F_2,...,F_q}. A graph that admits* ***Skolem difference Fibonacci mean labelling*** *is called a Skolem difference Fibonacci mean graph". In this paper, we prove that (P_n : $K_{1,m}$), $P_m \ominus S_{2,n}$, $P_{n_{(m)}}$, (P_n : C_3) and $(P_n \otimes S_m)^k$ are Skolem difference Fibonacci mean graphs.*

Keywords: *Skolem difference mean labelling, Fibonacci labelling, Skolem difference Fibonacci mean labelling*

I. Introduction

A graph G with p vertices and q edges is said to have Skolem difference Fibonacci mean labelling if it is possible to label the vertices x∈ V with distinct elements f(x) from the set {1,2,...,F_{p+q}} in such a way that the edge e = uv is labelled with $\left|\frac{f(u)-f(v)}{2}\right|$ if $|f(u)-f(v)|$ is even and $\frac{|f(u)-f(v)|+1}{2}$ if $|f(u)-f(v)|$ is odd and the resulting edge labels are distinct and are from {F_1, F_2,...,F_q}. A graph that admits Skolem difference Fibonacci mean labelling is called a Skolem difference Fibonacci mean graph. It was found that standard graphs [7], H- class of graphs [8], some special class of graphs [9] and some special class of trees [10] are Skolem difference Fibonacci mean graphs.

II. Preliminaries

In this section, some basic definitions and preliminary ideas are given which is useful for proving theorems.

Definition 2.1[3]:

Let G be a graph with fixed vertex v and let (P_m : G) be the graph obtained from m copies of G and the path P_m : $u_1u_2...u_m$ by joining u_i with the vertex v of the i^{th} copy of G by means of an edge, for $1 \le i \le n$.

Definition 2.2[3]:

The graph $P_m \ominus S_n$ is the tree obtained from the path P_m by adding a star graph S_n to each of the pendant vertices of P_m. It has 2n+m vertices and 2n+m-1 edges.

Definition 2.3[3]:

Let T be any tree. Denote the tree obtained from T, by considering two copies of T and adding an edge between them, by $T_{(2)}$ and in general the graph obtained from $T_{(n-1)}$ and T by adding and edge between them is denoted by $T_{(n)}$. Note that $T_{(n)}$ is nothing but T.

Definition 2.4[3]:

Let G be any graph and S_m be a star with m spokes. We denote by $G \otimes S_m$ the graph obtained from G by identifying one vertex of G with any vertex of S_m other than the centre of S_m.

Definition 2.5[3]:

A regular bamboo tree is one point union of $(P_n \otimes S_m)^k$ where k is the number of copies of $P_n \otimes S_m$.

III. Main Results

Theorem 3.1
The graph $(P_n : K_{1,m})$ is Skolem difference Fibonacci mean graph for all $n > 1$ and $m \geq 1$.
Proof:
Let G be the graph $(P_n : K_{1,m})$.

Let $V(G) = \{u_i, v_i, v_{ij} \,/\, 1 \leq i \leq n \text{ and } 1 \leq j \leq m\}$

$E(G) = \{u_iu_{i+1} / 1 \leq i \leq n-1\} \cup \{u_iv_i / 1 \leq i \leq n\} \cup \{v_iv_{ij} / 1 \leq i \leq n \text{ and } 1 \leq j \leq m\}$

Then $|V(G)| = 2n+mn$ and $|E(G)| = 2n+mn-1$

Let $f: V \rightarrow \{1,2,\ldots,F_{4n+2mn-1}\}$ be defined as follows

$f(u_i) = 2F_{i+1},\ 1 \leq i \leq n$

$f(v_i) = 2F_{n+(m+1)(i-1)} + f(u_i),\ 1 \leq i \leq n$

$f(v_{ij}) = 2F_{n+(m+1)(i-1)+j} + f(v_i),\ 1 \leq i \leq n \text{ and } 1 \leq j \leq m$

$f^+(E) = \{f(u_iu_{i+1}) / i=1,2,\ldots,n-1\} \cup \{f(u_iv_i) / 1 \leq i \leq n\} \cup \{f(v_iv_{ij}) / 1 \leq i \leq n \text{ and } 1 \leq j \leq m\}$

$= \{f(u_1u_2), f(u_2u_3), \ldots, f(u_{n-1}u_n)\} \cup \{f(u_1v_1), f(u_2v_2),\ldots, f(u_nv_n)\} \cup \{f(v_1v_{11}), f(v_1v_{12}),\ldots, f(v_1v_{1m}), f(v_2v_{21}), f(v_2v_{22}),\ldots, f(v_2v_{2m}),\ldots, f(v_nv_{n1}), f(v_nv_{n2}),\ldots, f(v_nv_{nm})\}$

$= \left\{\left|\frac{f(u_1)-f(u_2)}{2}\right|, \left|\frac{f(u_2)-f(u_3)}{2}\right|,\ldots, \left|\frac{f(u_{n-1})-f(u_n)}{2}\right|\right\} \cup \left\{\left|\frac{f(u_1)-f(v_1)}{2}\right|, \left|\frac{f(u_2)-f(v_2)}{2}\right|,\ldots, \left|\frac{f(u_n)-f(v_n)}{2}\right|\right\} \cup \left\{\left|\frac{f(v_1)-f(v_{11})}{2}\right|, \left|\frac{f(v_1)-f(v_{12})}{2}\right|,\ldots, \left|\frac{f(v_1)-f(v_{1m})}{2}\right|, \left|\frac{f(v_2)-f(v_{21})}{2}\right|, \left|\frac{f(v_2)-f(v_{22})}{2}\right|,\ldots, \left|\frac{f(v_2)-f(v_{2m})}{2}\right|,\ldots, \left|\frac{f(v_n)-f(v_{n1})}{2}\right|, \left|\frac{f(v_n)-f(v_{n2})}{2}\right|,\ldots, \left|\frac{f(v_n)-f(v_{nm})}{2}\right|\right\}$

$= \left\{\left|\frac{2F_2-2F_3}{2}\right|, \left|\frac{2F_3-2F_4}{2}\right|,\ldots, \left|\frac{2F_n-2F_{n+1}}{2}\right|\right\} \cup \left\{\left|\frac{f(u_1)-2F_n-f(u_1)}{2}\right|, \left|\frac{f(u_2)-2F_{n+m+1}-f(u_2)}{2}\right|,\ldots, \left|\frac{f(u_n)-2F_{n+(m+1)(n-1)}-f(u_n)}{2}\right|\right\}$
$\cup \left\{\left|\frac{f(v_1)-2F_{n+1}-f(v_1)}{2}\right|, \left|\frac{f(v_1)-2F_{n+2}-f(v_1)}{2}\right|,\ldots, \left|\frac{f(v_1)-2F_{n+m}-f(v_1)}{2}\right|, \left|\frac{f(v_2)-2F_{n+m+2}-f(v_2)}{2}\right|, \left|\frac{f(v_2)-2F_{n+m+3}-f(v_2)}{2}\right|,\ldots, \left|\frac{f(v_2)-2F_{n+2m+1}-f(v_2)}{2}\right|,\ldots, \left|\frac{f(v_n)-2F_{n+(m+1)(n-1)+1}-f(v_n)}{2}\right|, \left|\frac{f(v_n)-2F_{n+(m+1)(n-1)+2}-f(v_n)}{2}\right|,\ldots, \left|\frac{f(v_n)-2F_{n+(m+1)(n-1)+m}-f(v_n)}{2}\right|\right\}$

$= \{F_1, F_2,\ldots, F_{n-1}\} \cup \{F_n, F_{n+m+1},\ldots, F_{2n+mn-m-1}\} \cup \{F_{n+1}, F_{n+2},\ldots, F_{n+m}, F_{n+m+2}, F_{n+m+3},\ldots, F_{n+2m+1},\ldots, F_{2n+mn-m}, F_{2n+mn-m+1},\ldots, F_{2n+mn-1}\}$

$= \{F_1, F_2,\ldots, F_{n-1}, F_n, F_{n+1}, F_{n+2},\ldots, F_{n+m}, F_{n+m+1}, F_{n+m+2}, F_{n+m+3},\ldots, F_{n+2m+1},\ldots, F_{2n+mn-m-1}, F_{2n+mn-m}, F_{2n+mn-m+1},\ldots, F_{2n+mn-1}\}$

$= \{F_1, F_2, \ldots, F_{2n+mn-1}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2, \ldots, F_{2n+mn-1}$.
Hence, the graph $(P_n : K_{1,m})$ is Skolem difference Fibonacci mean graph for all $n > 1$ and $m \geq 1$.

Example 3.2:
The Skolem difference Fibonacci mean labelling of the graph $(P_4 : K_{1,3})$ is

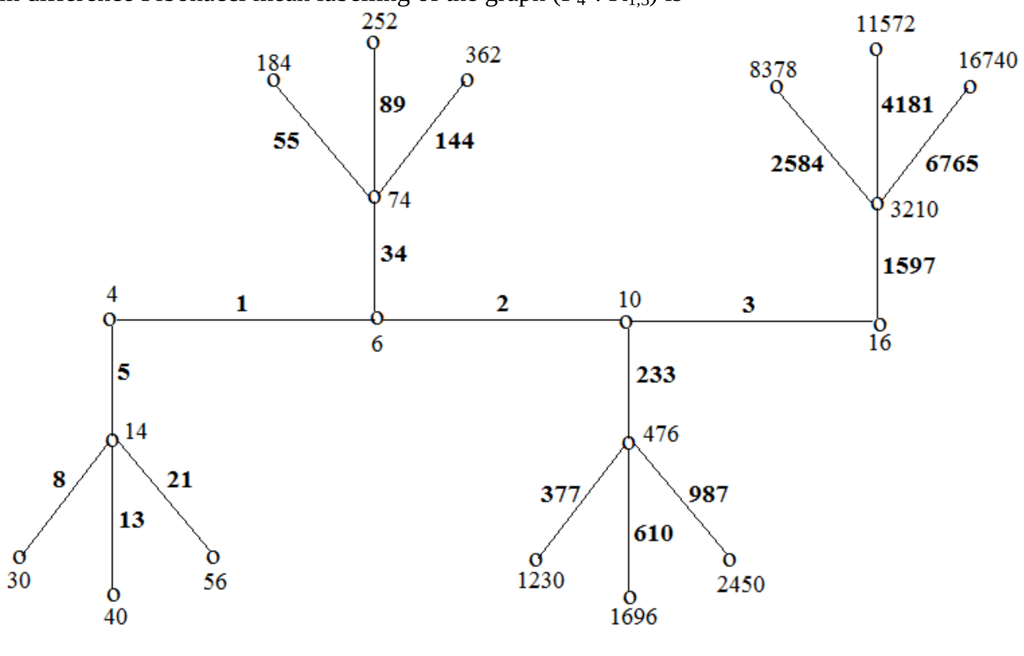

Figure 1

Theorem 3.3
$P_m \ominus S_{2,n}$ is Skolem difference Fibonacci mean graph for all m, $n \geq 2$.
(Recall that $S_{2,n}$ is the graph obtained from $K_{1,n}$ by subdividing each edge exactly once)

Proof:

Let G be the graph $P_m \ominus S_{2,n}$.

Le u_0 be the centre of the first subdivided star $S_{2,n}$ and v_0 be the centre of the second subdivided star $S_{2,n}$. Let u_0 and v_0 be identified with the vertices w_1 and w_m respectively.

Let $V(G) = \{u_i, u_i^1, v_i, v_i^1, w_j / 1 \leq i \leq n \text{ and } 1 \leq j \leq m\}$

$E(G) = \{u_iu_i^1, v_iv_i^1, u_0w_2, w_{m-1}v_0, w_jw_{j+1} / 1 \leq i \leq n \text{ and } 2 \leq j \leq m\text{-}2\} \cup \{u_0u_i, v_0v_i / 1 \leq i \leq n\}$

Then $|V(G)| = 4n+m$ and $|E(G)| = 4n+m-1$

Let f: V (G) → $\{1, 2,..., F_{8n+2m-1}\}$ be defined as follows

$f(w_1) = f(u_0) = 1$

$f(w_m) = f(v_0) = 2F_{2n+m-1} + f(w_{m-1})$

$f(u_i) = 2F_i+1,\ 1 \leq i \leq n$

$f(u_i^1) = 2F_{n+i} + f(u_i),\ 1 \leq i \leq n$

$f(w_j) = 2F_{2n+j-1} + f(w_{j-1}),\ 2 \leq j \leq m-1$

$f(v_i) = 2F_{2n+m+i-1} + f(v_0),\ 1 \leq i \leq n$

$f(v_i^1) = 2F_{3n+m+i-1} + f(v_i),\ 1 \leq i \leq n$

$f^+(E) = \{f(u_iu_i^1), f(v_iv_i^1), f(u_0w_2), f(w_{m-1}v_0), f(w_jw_{j+1}) / 1 \leq i \leq n \text{ and } 2 \leq j \leq m-2\} \cup \{f(u_0u_i), f(v_0v_i) / 1 \leq i \leq n\}$

$= \{f(u_1u_1^1), f(u_2u_2^1),..., f(u_nu_n^1), f(v_1v_1^1), f(v_2v_2^1), ...,f(v_nv_n^1), f(u_0w_2), f(w_{m-1}v_0), f(w_2w_3), f(w_3w_4),..., f(w_{m-2}w_{m-1})\} \cup \{f(u_0u_1), f(u_0u_2), ..., f(u_0u_n), f(v_0v_1), f(v_0v_2), ..., f(v_0v_n)\}$

$= \{\left|\frac{f(u_1)-f(u_1^1)}{2}\right|, \left|\frac{f(u_2)-f(u_2^1)}{2}\right|,..., \left|\frac{f(u_n)-f(u_n^1)}{2}\right|, \left|\frac{f(v_1)-f(v_1^1)}{2}\right|, \left|\frac{f(v_2)-f(v_2^1)}{2}\right|,..., \left|\frac{f(v_n)-f(v_n^1)}{2}\right|, \left|\frac{f(u_0)-f(w_2)}{2}\right|, \left|\frac{f(w_{m-1})-f(v_0)}{2}\right|, \left|\frac{f(w_2)-f(w_3)}{2}\right|, \left|\frac{f(w_3)-f(w_4)}{2}\right|,..., \left|\frac{f(w_{m-2})-f(w_{m-1})}{2}\right|\} \cup \{\left|\frac{f(u_0)-f(u_1)}{2}\right|, \left|\frac{f(u_0)-f(u_2)}{2}\right|,..., \left|\frac{f(u_0)-f(u_n)}{2}\right|, \left|\frac{f(v_0)-f(v_1)}{2}\right|, \left|\frac{f(v_0)-f(v_2)}{2}\right|,..., \left|\frac{f(v_0)-f(v_n)}{2}\right|\}$

$= \{\left|\frac{f(u_1)-2F_{n+1}-f(u_1)}{2}\right|, \left|\frac{f(u_2)-2F_{n+2}-f(u_2)}{2}\right|,..., \left|\frac{f(u_n)-2F_{2n}-f(u_n)}{2}\right|, \left|\frac{f(v_1)-2F_{3n+m}-f(v_1)}{2}\right|, \left|\frac{f(v_2)-2F_{3n+m+1}-f(v_2)}{2}\right|,..., \left|\frac{f(v_n)-2F_{4n+m-1}-f(v_n)}{2}\right|, \left|\frac{1-2F_{2n+1}-1)}{2}\right|, \left|\frac{f(w_{m-1})-2F_{2n+m-1}-f(w_{m-1})}{2}\right|, \left|\frac{f(w_2)-2F_{2n+2}-f(w_2)}{2}\right|, \left|\frac{f(w_3)-2F_{2n+3}-f(w_3)}{2}\right|,..., \left|\frac{f(w_{m-2})-2F_{2n+m-2}-f(w_{m-2})}{2}\right|\} \cup \{\left|\frac{1-2F_1-1}{2}\right|, \left|\frac{1-2F_2-1}{2}\right|, ..., \left|\frac{1-2F_n-1}{2}\right|, \left|\frac{f(v_0)-2F_{2n+m}-f(v_0)}{2}\right|, \left|\frac{f(v_0)-2F_{2n+m+1}-f(v_0)}{2}\right|,..., \left|\frac{f(v_0)-2F_{3n+m-1}-f(v_0)}{2}\right|\}$

$= \{F_{n+1}, F_{n+2}, ..., F_{2n}, F_{3n+m}, F_{3n+m+1},..., F_{4n+m-1}, F_{2n+1}, F_{2n+m-1}, F_{2n+2}, F_{2n+3}, ..., F_{2n+m-2}\} \cup \{F_1, F_2,..., F_n, F_{2n+m}, F_{2n+m+1},..., F_{3n+m-1}\}$

$= \{F_1, F_2,..., F_n, F_{n+1}, F_{n+2},..., F_{2n}, F_{2n+1}, F_{2n+2}, F_{2n+3}, ..., F_{2n+m-2}, F_{2n+m-1}, F_{2n+m}, F_{2n+m+1},..., F_{3n+m-1}, F_{3n+m}, F_{3n+m+1},..., F_{4n+m-1}\}$

$= \{F_1, F_2, ..., F_{4n+m-1}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2, ..., F_{4n+m-1}$.
Hence, the graph $P_m \ominus S_{2,n}$ is Skolem difference Fibonacci mean graph for all m, $n \geq 2$.

Example 3.4:
The Skolem difference Fibonacci mean labelling of the graph $P_4 \ominus S_{2,3}$ is

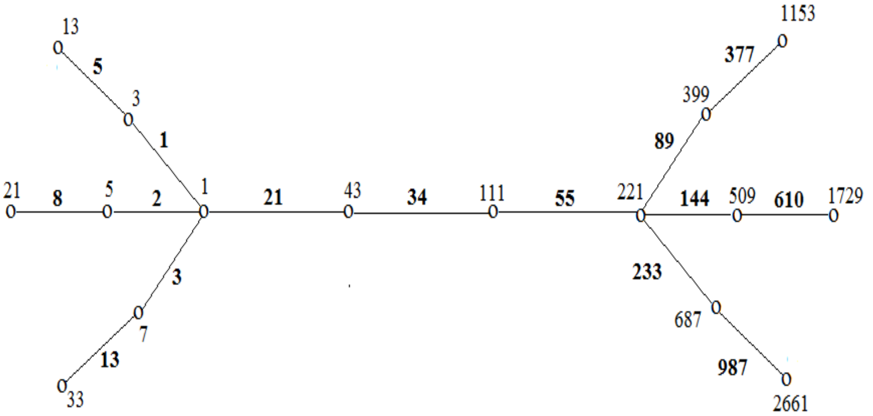

Figure 2

Theorem 3.5

The graph $P_{n_{(m)}}$ is skolem difference Fibonacci mean graph for all n, m ≥ 2.

Proof:

Let G be the graph $P_{n_{(m)}}$.

Let V (G) = $\{v_{ij} / 1 \le i \le m$ and $1 \le j \le n\}$

E (G) = $\{v_{ij}v_{i(j+1)} / 1 \le i \le m$ and $1 \le j \le n-1\} \cup \{v_{i2}v_{(i+1)2} / 1 \le i \le m\}$

Then $|V(G)|$ = mn and $|E(G)|$ = mn-1

Let f: V (G) → $\{1, 2,..., F_{2mn-1}\}$ be defined as follows

$f(v_{11}) = 4$

$f(v_{1j}) = 2F_{j-1} + f(v_{1(j-1)}),\ 2 \le j \le n$

$f(v_{i2}) = 2F_{n(i-1)} + f(v_{(i-1)2}),\ 2 \le i \le m$

$f(v_{i1}) = 2F_{n(i-1)+1} + f(v_{i2}),\ 2 \le i \le m$

$f(v_{ij}) = 2F_{n(i-1)+(j-1)} + f(v_{i(j-1)}),\ 2 \le i \le m$ and $2 \le j \le n$

$f^{+}(E) = \{f(v_{ij}v_{i(j+1)}) / 1 \le i \le m$ and $1 \le j \le n-1\} \cup \{f(v_{i2}v_{(i+1)2}) / 1 \le i \le m-1\}$

$= \{f(v_{11}v_{12}), f(v_{12}v_{13}),..., f(v_{1(n-1)}v_{1n}), f(v_{21}v_{22}), f(v_{22}v_{23}),..., f(v_{2(n-1)}v_{2n}),..., f(v_{m1}v_{m2}), f(v_{m2}v_{m3}),..., f(v_{m(n-1)}v_{mn})\} \cup \{f(v_{12}v_{22}), f(v_{22}v_{32}),..., f(v_{(m-1)2}v_{m2})\}$

$= \{\left|\frac{f(v_{11})-f(v_{12})}{2}\right|, \left|\frac{f(v_{12})-f(v_{13})}{2}\right|,..., \left|\frac{f(v_{1(n-1)})-f(v_{1n})}{2}\right|, \left|\frac{f(v_{21})-f(v_{22})}{2}\right|, \left|\frac{f(v_{22})-f(v_{23})}{2}\right|,..., \left|\frac{f(v_{2(n-1)})-f(v_{2n})}{2}\right|,..., \left|\frac{f(v_{m1})-f(v_{m2})}{2}\right|, \left|\frac{f(v_{m2})-f(v_{m3})}{2}\right|,..., \left|\frac{f(v_{m(n-1)})-f(v_{mn})}{2}\right|\} \cup \{\left|\frac{f(v_{12})-f(v_{22})}{2}\right|, \left|\frac{f(v_{22})-f(v_{32})}{2}\right|,..., \left|\frac{f(v_{(m-1)2})-f(v_{m2})}{2}\right|\}$

$= \{\left|\frac{f(v_{11})-2F_1-f(v_{11})}{2}\right|, \left|\frac{f(v_{12})-2F_2-f(v_{12})}{2}\right|,..., \left|\frac{f(v_{1(n-1)})-2F_{n-1}-f(v_{1(n-1)})}{2}\right|, \left|\frac{2F_{n+1}+f(v_{22})-f(v_{22})}{2}\right|, \left|\frac{f(v_{22})-2F_{n+2}-f(v_{22})}{2}\right|,..., \left|\frac{f(v_{2(n-1)})-2F_{2n-1}-f(v_{2(n-1)})}{2}\right|,..., \left|\frac{2F_{n(m-1)+1}+f(v_{m2})-f(v_{m2})}{2}\right|, \left|\frac{f(v_{m2})-2F_{n(m-1)+2}-f(v_{m2})}{2}\right|,..., \left|\frac{f(v_{m(n-1)})-2F_{n(m-1)+n-1}-f(v_{m(n-1)})}{2}\right|\} \cup \{\left|\frac{f(v_{12})-2F_{n}-f(v_{12})}{2}\right|, \left|\frac{f(v_{22})-2F_{2n}-f(v_{22})}{2}\right|,..., \left|\frac{f(v_{(m-1)2})-2F_{n(m-1)}-f(v_{(m-1)2})}{2}\right|\}$

$= \{F_1, F_2,..., F_{n-1}, F_{n+1}, F_{n+2},..., F_{2n-1},..., F_{n(m-1)+1}, F_{n(m-1)+2},..., F_{n(m-1)+(n-1)}, F_n, F_{2n},..., F_{n(m-1)}\}$

$= \{F_1, F_2,..., F_{n-1}, F_n, F_{n+1}, F_{n+2},..., F_{2n-1}, F_{2n},..., F_{n(m-1)}, F_{n(m-1)+1}, F_{n(m-1)+2},..., F_{mn-1}\}$

$= \{F_1, F_2, ..., F_{mn-1}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2, ..., F_{mn-1}$.

Hence, the graph $P_{n_{(m)}}$ is skolem difference Fibonacci mean graph for all n, m ≥ 2.

Example 3.6:

The Skolem difference Fibonacci mean labelling of the graph $P_{5_{(4)}}$ is

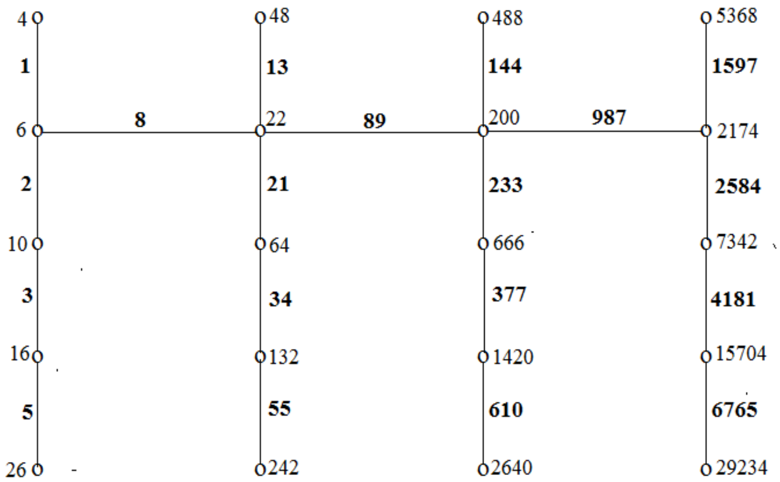

Figure 3

Theorem 3.7

The graph ($P_n : C_3$) is Skolem difference Fibonacci mean graph for all n >1.

Proof:

Let G be the graph ($P_n : C_3$).

Let V (G) = $\{v_i, v_{ij} / 1 \le i \le n$ and j = 1, 2, 3$\}$

E (G) = $\{v_iv_{i+1} / 1 \le i \le n-1\} \cup \{v_iv_{i1} / 1 \le i \le n\} \cup \{v_{i1}v_{i2}, v_{i2}v_{i3}, v_{i3}v_{i1} / 1 \le i \le n\}$

Then $|V(G)| = 4n$ and $|E(G)| = 5n-1$

Let $f: V(G) \to \{1, 2, \ldots, F_{9n-1}\}$ be defined as follows

$f(v_i) = 2F_{i+1}, 1 \le i \le n$

$f(v_{i1}) = 2F_{n+4(i-1)} + f(v_i), 1 \le i \le n$

$f(v_{i2}) = 2F_{n+1+4(i-1)} + f(v_{i1}), 1 \le i \le n$

$f(v_{i3}) = 2F_{n+2+4(i-1)} + f(v_{i2}) = 2F_{n+2+4(i-1)} + 2F_{n+1+4(i-1)} + f(v_{i1}), 1 \le i \le n$

$f^+(E) = \{f(v_iv_{i+1})/1 \le i \le n-1\} \cup \{f(v_iv_{i1})/1 \le i \le n\} \cup \{f(v_{i1}v_{i2}), f(v_{i2}v_{i3}), f(v_{i3}v_{i1})/1 \le i \le n\}$

$= \{f(v_1v_2), f(v_2v_3), \ldots, f(v_{n-1}v_n)\} \cup \{f(v_1v_{11}), f(v_2v_{21}), \ldots, f(v_nv_{n1})\} \cup \{f(v_{11}v_{12}), f(v_{21}v_{22}), \ldots, f(v_{n1}v_{n2}), f(v_{12}v_{13}), f(v_{22}v_{23}), \ldots, f(v_{n2}v_{n3}), f(v_{13}v_{11}), f(v_{23}v_{21}), \ldots, f(v_{n3}v_{n1})\}$

$$= \left\{\left|\frac{f(v_1)-f(v_2)}{2}\right|, \left|\frac{f(v_2)-f(v_3)}{2}\right|, \ldots, \left|\frac{f(v_{n-1})-f(v_n)}{2}\right|\right\} \cup \left\{\left|\frac{f(v_1)-f(v_{11})}{2}\right|, \left|\frac{f(v_2)-f(v_{21})}{2}\right|, \ldots, \left|\frac{f(v_n)-f(v_{n1})}{2}\right|\right\} \cup \left\{\left|\frac{f(v_{11})-f(v_{12})}{2}\right|, \left|\frac{f(v_{21})-f(v_{22})}{2}\right|, \ldots, \left|\frac{f(v_{n1})-f(v_{n2})}{2}\right|, \left|\frac{f(v_{12})-f(v_{13})}{2}\right|, \left|\frac{f(v_{22})-f(v_{23})}{2}\right|, \ldots, \left|\frac{f(v_{n2})-f(v_{n3})}{2}\right|, \left|\frac{f(v_{13})-f(v_{11})}{2}\right|, \left|\frac{f(v_{23})-f(v_{21})}{2}\right|, \ldots, \left|\frac{f(v_{n3})-f(v_{n1})}{2}\right|\right\}$$

$$= \left\{\left|\frac{2F_2-2F_3}{2}\right|, \left|\frac{2F_3-2F_4}{2}\right|, \ldots, \left|\frac{2F_n-2F_{n+1}}{2}\right|\right\} \cup \left\{\left|\frac{f(v_1)-2F_n-f(v_1)}{2}\right|, \left|\frac{f(v_2)-2F_{n+4}-f(v_2)}{2}\right|, \ldots, \left|\frac{f(v_n)-2F_{n+4(n-1)}-f(v_n)}{2}\right|\right\} \cup \left\{\left|\frac{f(v_{11})-2F_{n+1}-f(v_{11})}{2}\right|, \left|\frac{f(v_{21})-2F_{n+5}-f(v_{21})}{2}\right|, \ldots, \left|\frac{f(v_{n1})-2F_{n+1+4(n-1)}-f(v_{n1})}{2}\right|, \left|\frac{f(v_{12})-2F_{n+2}-f(v_{12})}{2}\right|, \left|\frac{f(v_{22})-2F_{n+6}-f(v_{22})}{2}\right|, \ldots, \left|\frac{f(v_{n2})-2F_{n+2+4(n-1)}-f(v_{n2})}{2}\right|, \left|\frac{2F_{n+2}+2F_{n+1}+f(v_{11})-f(v_{11})}{2}\right|, \left|\frac{2F_{n+6}+2F_{n+5}+f(v_{21})-f(v_{21})}{2}\right|, \ldots, \left|\frac{2F_{n+2+4(n-1)}+2F_{n+1+4(n-1)}+f(v_{n1})-f(v_{n1})}{2}\right|\right\}$$

$= \{F_1, F_2, \ldots, F_{n-1}\} \cup \{F_n, F_{n+4}, \ldots, F_{n+4(n-1)}\} \cup \{F_{n+1}, F_{n+5}, \ldots, F_{n+1+4(n-1)}, F_{n+2}, F_{n+6}, \ldots, F_{n+2+4(n-1)}, F_{n+3}, F_{n+7}, \ldots, F_{5n-1}\}$

$= \{F_1, F_2, \ldots, F_{n-1}\} \cup \{F_n, F_{n+4}, \ldots, F_{5n-4}\} \cup \{F_{n+1}, F_{n+5}, \ldots, F_{5n-3}, F_{n+2}, F_{n+6}, \ldots, F_{5n-2}, F_{n+3}, F_{n+7}, \ldots, F_{5n-1}\}$

$= \{F_1, F_2, \ldots, F_{n-1}, F_n, F_{n+1}, F_{n+2}, F_{n+3}, F_{n+4}, F_{n+5}, F_{n+6}, F_{n+7}, \ldots, F_{5n-4}, F_{5n-3}, F_{5n-2}, F_{5n-1}\}$

$= \{F_1, F_2, \ldots, F_{5n-1}\}$

Thus, the induced edge labels are distinct and are $F_1, F_2, \ldots, F_{5n-1}$.

Hence, $(P_n : C_3)$ is a Skolem difference Fibonacci mean graph for all $n > 1$.

Example 3.8:

Skolem difference Fibonacci mean labelling of the graph $(P_4: C_3)$ is

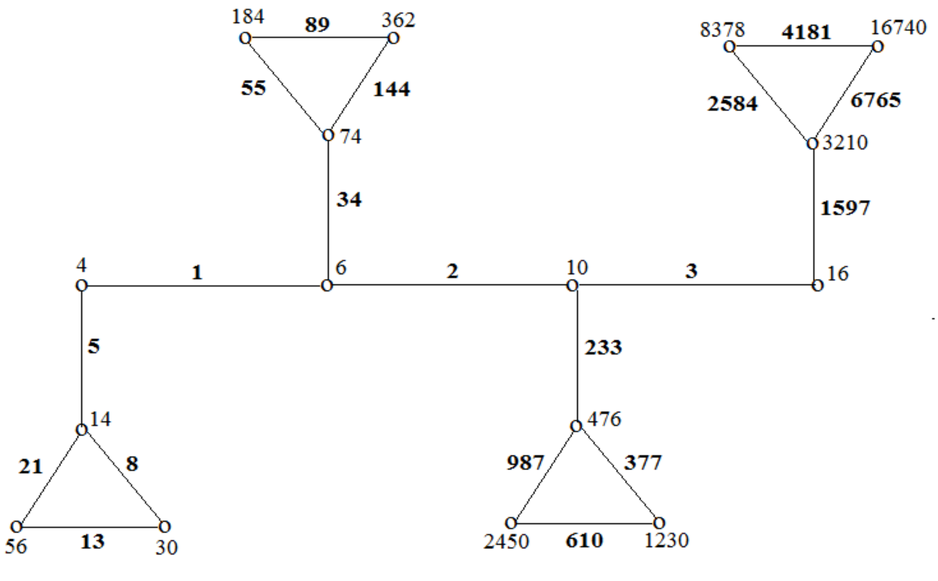

Figure 4

Theorem 3.9

$(P_n \otimes S_m)^k$ is a Skolem difference Fibonacci mean graph for all $n, m, k \ge 2$.

Proof:

Let G be $(P_n \otimes S_m)^k$

Let $V(G) = \{u, v_{ij}, x_{i\ell} / 1 \le i \le k, 1 \le j \le n-1 \text{ and } 1 \le \ell \le m\}$

Let $E(G) = \{uv_{i1} / 1 \le i \le k\} \cup \{v_{ij} v_{i(j+1)} / 1 \le i \le k \text{ and } 1 \le j \le n-2\} \cup \{v_{i(n-1)} x_{i\ell} / 1 \le i \le k \text{ and } 1 \le \ell \le m\}$

Then $|V(G)| = (n+m-1)k+1$ and $|E(G)| = (n+m-1)k$

Let $f: V(G) \rightarrow \{1, 2, ..., F_{2(n+m-1)k+1}\}$ be defined as follows
$f(u) = 1$
$f(v_{i1}) = 2F_i+1,\ 1 \le i \le k$
$f(v_{ij}) = 2F_{(j-1)k+i} + f(v_{i(j-1)}),\ 1 \le i \le k$ and $2 \le j \le n-1$
$f(x_{i\ell}) = f(v_{i(n-1)}) + 2F_{(n-1)k+m(i-1)+\ell},\ 1 \le i \le k$ and $1 \le \ell \le m$
$f^+(E) = \{f(uv_{i1}) / 1 \le i \le k\} \cup \{f(v_{ij}v_{i(j+1)}) / 1 \le i \le k$ and $1 \le j \le n-2\} \cup \{f(v_{i(n-1)}x_{i\ell}) / 1 \le i \le k$ and $1 \le \ell \le m\}$

$$= \{f(uv_{11}), f(uv_{21}),..., f(uv_{k1})\} \cup \{f(v_{11}v_{12}), f(v_{12}v_{13}),..., f(v_{1(n-2)}v_{1(n-1)}), f(v_{21}v_{22}), f(v_{22}v_{23}),..., f(v_{2(n-2)}v_{2(n-1)}),..., f(v_{k1}v_{k2}), f(v_{k2}v_{k3}),..., f(v_{k(n-2)}v_{k(n-1)})\} \cup \{f(v_{1(n-1)}x_{11}), f(v_{1(n-1)}x_{12}),..., f(v_{1(n-1)}x_{1m}), f(v_{2(n-1)}x_{21}), f(v_{2(n-1)}x_{22}),..., f(v_{2(n-1)}x_{2m}),..., f(v_{k(n-1)}x_{k1}), f(v_{k(n-1)}x_{k2}),..., f(v_{k(n-1)}x_{km})\}$$

$$= \left\{\left|\frac{f(u)-f(v_{11})}{2}\right|, \left|\frac{f(u)-f(v_{21})}{2}\right|,..., \left|\frac{f(u)-f(v_{k1})}{2}\right|\right\} \cup \left\{\left|\frac{f(v_{11})-f(v_{12})}{2}\right|, \left|\frac{f(v_{12})-f(v_{13})}{2}\right|,..., \left|\frac{f(v_{1(n-2)})-f(v_{1(n-1)})}{2}\right|, \left|\frac{f(v_{21})-f(v_{22})}{2}\right|, \left|\frac{f(v_{22})-f(v_{23})}{2}\right|,..., \left|\frac{f(v_{2(n-2)})-f(v_{2(n-1)})}{2}\right|, \left|\frac{f(v_{k1})-f(v_{k2})}{2}\right|, \left|\frac{f(v_{k2})-f(v_{k3})}{2}\right|,..., \left|\frac{f(v_{k(n-2)})-f(v_{k(n-1)})}{2}\right|\right\} \cup \left\{\left|\frac{f(v_{1(n-1)})-f(x_{11})}{2}\right|, \left|\frac{f(v_{1(n-1)})-f(x_{12})}{2}\right|,..., \left|\frac{f(v_{1(n-1)})-f(x_{1m})}{2}\right|, \left|\frac{f(v_{2(n-1)})-f(x_{21})}{2}\right|, \left|\frac{f(v_{2(n-1)})-f(x_{22})}{2}\right|,..., \left|\frac{f(v_{2(n-1)})-f(x_{2m})}{2}\right|,..., \left|\frac{f(v_{k(n-1)})-f(x_{k1})}{2}\right|, \left|\frac{f(v_{k(n-1)})-f(x_{k2})}{2}\right|,..., \left|\frac{f(v_{k(n-1)})-f(x_{km})}{2}\right|\right\}$$

$$= \left\{\left|\frac{1-2F_1-1}{2}\right|, \left|\frac{1-2F_2-1}{2}\right|,..., \left|\frac{1-2F_k-1}{2}\right|\right\} \cup \left\{\left|\frac{f(v_{11})-2F_{k+1}-f(v_{11})}{2}\right|, \left|\frac{f(v_{12})-2F_{2k+1}-f(v_{12})}{2}\right|,..., \left|\frac{f(v_{1(n-2)})-2F_{(n-2)k+1}-f(v_{1(n-2)})}{2}\right|, \left|\frac{f(v_{21})-2F_{k+2}-f(v_{21})}{2}\right|, \left|\frac{f(v_{22})-2F_{2k+2}-f(v_{22})}{2}\right|,..., \left|\frac{f(v_{2(n-2)})-2F_{(n-2)k+2}-f(v_{2(n-2)})}{2}\right|,..., \left|\frac{f(v_{k1})-2F_{2k}-f(v_{k1})}{2}\right|, \left|\frac{f(v_{k2})-2F_{3k}-f(v_{k2})}{2}\right|,..., \left|\frac{f(v_{k(n-2)})-2F_{(n-2)k+k}-f(v_{k(n-2)})}{2}\right|\right\} \cup \left\{\left|\frac{f(v_{1(n-1)})-f(v_{1(n-1)})-2F_{(n-1)k+1}}{2}\right|, \left|\frac{f(v_{1(n-1)})-f(v_{1(n-1)})-2F_{(n-1)k+2}}{2}\right|,..., \left|\frac{f(v_{1(n-1)})-f(v_{1(n-1)})-2F_{(n-1)k+m}}{2}\right|, \left|\frac{f(v_{2(n-1)})-f(v_{2(n-1)})-2F_{(n-1)k+m+1}}{2}\right|, \left|\frac{f(v_{2(n-1)})-f(v_{2(n-1)})-2F_{(n-1)k+m+2}}{2}\right|,..., \left|\frac{f(v_{2(n-1)})-f(v_{2(n-1)})-2F_{(n-1)k+m+m}}{2}\right|,..., \left|\frac{f(v_{k(n-1)})-f(v_{k(n-1)})-2F_{(n-1)k+m(k-1)+1}}{2}\right|, \left|\frac{f(v_{k(n-1)})-f(v_{k(n-1)})-2F_{(n-1)k+m(k-1)+2}}{2}\right|,..., \left|\frac{f(v_{k(n-1)})-f(v_{k(n-1)})-2F_{(n-1)k+m(k-1)+m}}{2}\right|\right\}$$

$= \{F_1, F_2,...,F_k\} \cup \{F_{k+1}, F_{2k+1},..., F_{(n-2)k+1}, F_{k+2}, F_{2k+2},..., F_{(n-2)k+2},..., F_{2k}, F_{3k},..., F_{(n-2)k+k}\} \cup \{F_{(n-1)k+1}, F_{(n-1)k+2},..., F_{(n-1)k+m}, F_{(n-1)k+m+1}, F_{(n-1)k+m+2},..., F_{(n-1)k+2m},..., F_{(n-1)k+m(k-1)+1}, F_{(n-1)k+m(k-1)+2},..., F_{(n-1)k+m(k-1)+m}\}$
$= \{F_1, F_2,...,F_k, F_{k+1}, F_{k+2},..., F_{2k}, F_{2k+1}, F_{2k+2},..., F_{3k},..., F_{(n-2)k+1}, F_{(n-2)k+2},..., F_{(n-2)k+k}, F_{(n-1)k+1}, F_{(n-1)k+2}, ..., F_{(n-1)k+m}, F_{(n-1)k+m+1}, F_{(n-1)k+m+2}, ..., F_{(n-1)k+2m},..., F_{(n-1)k+m(k-1)+1}, F_{(n-1)k+m(k-1)+2}, ..., F_{(n-1)k+m(k-1)+m}\}$
$= \{F_1, F_2, ..., F_{(n-1)k+m(k-1)+m}\}$
$= \{F_1, F_2, ..., F_{(n-1)k+m(k-1+1)}\}$
$= \{F_1, F_2, ..., F_{(n-1)k+mk}\}$
$= \{F_1, F_2, ..., F_{(n+m-1)k}\}$
Thus, the induced edge labels are distinct and are $F_1, F_2, ..., F_{(n+m-1)k}$.
Hence, $(P_n \otimes S_m)^k$ is a Skolem difference Fibonacci mean graph for all $n, m, k \ge 2$.

Example 3.10:
Skolem difference Fibonacci mean labelling of the graph $(P_5 \otimes S_4)^2$ is

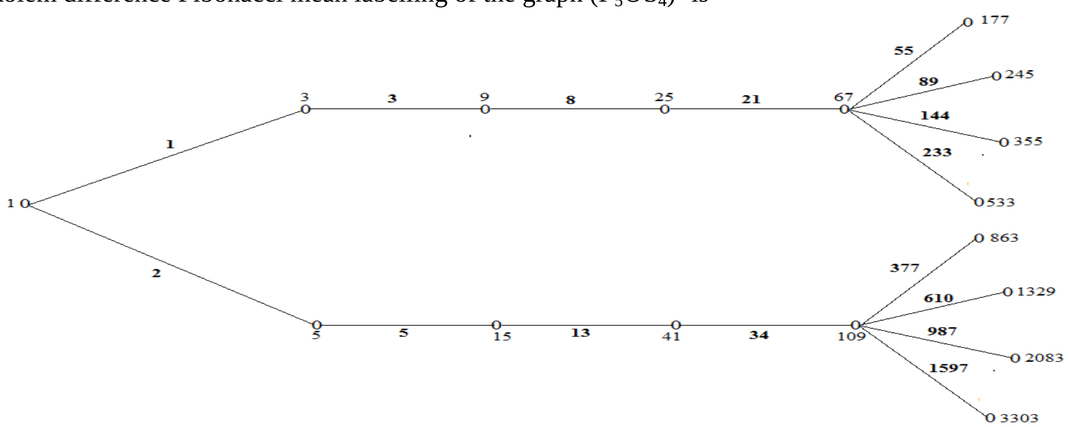

Figure 5

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