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Common Fixed Point Theorems Using E.A. Property in Intuitionistic Fuzzy Metric Spaces

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Abstract: *In this paper, we prove common fixed point theorems for six mappings by using property satisfying contractive condition in intuitionistic fuzzy metric spaces.*
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I. Introduction

Atanassov [10] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park [9] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al. [2] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Mich´alek [8]. In 2006, T¨urkoˇglu et al. [4] proved Jungck's common fixed point theorem [7] in the setting of intuitionistic fuzzy metric spaces for commuting mappings. For more detail, one can refer to papers ([3], [12], [5], [6]).

In this paper, we prove a common fixed point theorem for six mappings by using E.A Property satisfying contractive condition in intuitionistic fuzzy metric spaces.

II. Preliminaries

Definition 2.1: [13] A binary operation $\ast\ [0,1] \times [0,1] \rightarrow [0,1]$ is continuous t-norm if $\ast$ satisfies the following condition:
(i) $\ast$ is commutative and associative;
(ii) $\ast$ is continuous;
(iii) $a \ast 1 = a$ for all $a\epsilon[0,1]$;
(iv) $a \ast b \le c \ast d$ whenever $a \le c$ and $b \le d$ for all a, b, c, d $\in[0,1]$

Definition 2.2: [13] A binary operation $\diamond\ [0,1] \times [0,1] \rightarrow [0,1]$ is continuous t-norm if $\ast$ satisfies the following condition:
(i) $\diamond$ is commutative and associative;
(ii) $\diamond$ is continuous;
(iii) $a \diamond 0 = a$ for all $a\epsilon[0,1]$;
(iv) $a \diamond b \le c \diamond d$ whenever $a \le c$ and $b \le d$ for all a, b, c, d $\in[0,1]$

Definition 2.3: [13] A 5-tuple $(X,M,N, \ast ,\diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $\ast$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0,\infty)$ satisfying

(i) $M(x, y, t)+N(x, y, t)\le1$ for all $x, y \in X$ and $t>0$;
(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;
(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t>0$ if and only if $x = y$;
(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t>0$;
(v) $M(x, y, t) \ast M(y, z, s) \le M(x, z, t+s)$ for all $x, y, z \in X$ and $s, t>0$;
(vi) for all $x, y \in X$, $M(x, y, .) : [0, \infty)\rightarrow[0,1]$ is left continuous ;
(vii) $\lim_{t\rightarrow\infty} M(x,y,t) = 1$ for all $x, y \in X$ and $t>0$;
(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;
(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t>0$ if and only if $x = y$;
(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t>0$;
(xi) $N(x, y, t) \diamond (y, z, s) \ge N(x, z, t+s)$ for all $x, y, z \in X$ and $s, t>0$;
(xii) for all $x, y \in X$, $N(x, y, .) : [0, \infty)\rightarrow[0,1]$ is right continuous;

(xiii) $\lim_{t\to\infty} N(x,y,t) = 1$ for all x, y ∈ X and t>0.

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of nonnearness between x and y with respect to t, respectively.

Remark : Every fuzzy metric space (X,M, ∗) is an intuitionistic fuzzy metric space of the form (X, M, 1 − M, ∗,⋄) such that t-norm ∗ and t-conorm ⋄ are associated as x ⋄ y = 1 − ((1 − x) ∗ (1 − y)) for all x, y ∈ X.

Remark : In an intuitionistic fuzzy metric space (X, M, N, ∗, ⋄), M(x, y, .) is non-decreasing and N(x, y, .) is non-increasing for all x, y ∈ X.

Definition 2.4: [13] Let (X,M,N, ∗, ⋄) be an intuitionistic fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be

(i) convergent to a point x ∈ X if $\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 1$ for all t>0.

(ii) Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n\to\infty} N(x_{n+1}, x_n, t) = 1$ for all t>0 and p>0.

Definition 2.5: [13] An intuitionistic fuzzy metric space (X,M,N, ∗,⋄) is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 2.6: [13] Let A and S be self-mappings of an intuitionistic fuzzy metric space (X,M,N, ∗,⋄). Then a pair (A, S) is said to be commuting if M(ASx, SAx, t) = 1 and N(ASx, SAx, t) = 0 for all x ∈ X and t > 0.

Definition 2.7: [11] A pair of self mappings (f, g) of an intuitionistic fuzzy metric space (X, M, N, ∗,⋄) is said to satisfy the property E.A if there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} M(fx_n, gx_n, t) = 1$ and $\lim_{n\to\infty} M(fx_n, gx_n, t) = 0$.

Lemma 2.8: ([2], [4]) Let (X,M,N, ∗,⋄) be an intuitionistic fuzzy metric space. If there exists a constant k∈(0, 1) such that M(x, y, kt) ≥ M(x, y, t) and N(x, y, kt) ≤ N(x, y, t) for all x, y ∈ X and t > 0, then x = y.

III. Main Results

Theorem 3.1: Let (X, M, N, ∗, ⋄) be a complete intuitionistic fuzzy metric space with t ∗ t ≥ t and (1 − t) ⋄ (1 − t) ≤ (1 − t) for all t ϵ [0, 1]. Further, let A, B, P, Q, S and T be six self mappings of X satisfying

(1) A(X) ⊂ PT(X) and B(X) ⊂ QS(X),

(2) there exists a constant k ϵ (0, 1) such that

$$[1 + aM(QSx, PTy, kt)] * M(Ax, By, kt) \geq a[M(Ax, QSx, kt) * M(By, PTy, kt) * M(By, QSx, kt)] + M(PTy, QSx, t) * M(Ax, QSx, t) * M(By, PTy, t) * M(By, QSx, \propto t) * M(Ax, PTy, (2-\propto)t)$$

and

$$[1 + aN(QSx, PTy, kt)] \diamond N(Ax, By, kt) \leq a[N(Ax, QSx, kt) \diamond N(By, PTy, kt) \diamond N(By, QSx, kt)] + N(PTy, QSx, t) \diamond N(Ax, QSx, t) \diamond N(By, PTy, t) \diamond N(By, QSx, \propto t) \diamond N(Ax, PTy, (2-\propto)t)$$

for all $x, y \epsilon X, a \geq 0, \propto \epsilon (0,2)$ and $t > 0$.
(3) (A, QS) and (B, PT) satisfies the property (E.A.)
(4) One of A(X), B(X), QS(X) and PT(X) is a complete subspace of X.
Then the pair (A,QS) and (B, PT) have a unique common fixed point provided the pairs (A, QS) and (B, PT) commute pair wise (i. e. AQ=QA, BS=SB, QS=SQ, BP=PB, BT=TB, AS=SA and PT=TP.)
Proof: If the pair (B, PT) satisfies the property (E. A.), then there exists a sequence $\{y_n\}$ in X such that $By_n \to z$ and $PTy_n \to z$, for some $z \epsilon X$ an $n \to \infty$. Since B(X)⊂QS(X). there exists a sequence $\{x_n\}$ in X such that $By_n = QSx_n$. Hence $QSx_n \to z$ as $n \to \infty$. Now we show that $Ax_n \to z$ as $n \to \infty$. By putting $x = x_n$ and $y = y_n$ in (2), we have

$$\Rightarrow \quad [1 + aM(QSx_n, PTy_n, kt)] * M(Ax_n, By_n, kt) \geq a[M(Ax_n, QSx_n, kt) * M(By_n, PTy_n, kt) * M(By_n, QSx_n, kt)] + M(PTy_n, QSx_n, t) * M(Ax_n, QSx_n, t) * M(By_n, PTy_n, t) * M(By_n, QSx_n, \propto t) * M(Ax_n, PTy_n, (2-\propto)t)$$

and

$$[1 + aN(QSx_n, PTy_n, kt)] \diamond N(Ax_n, By_n, kt) \leq a[N(Ax_n, QSx_n, kt) \diamond N(By_n, PTy_n, kt) \diamond N(By_n, QSx_n, kt)] + N(PTy_n, QSx_n, t) \diamond N(Ax_n, QSx_n, t) \diamond N(By_n, PTy_n, t) \diamond N(By_n, QSx_n, \propto t) \diamond N(Ax_n, PTy_n, (2-\propto)t)$$

Let $Ax_n \to l(\neq z)$ for $t > 0$ as $n \to \infty$. Then letting $n \to \infty$, with $\propto = 1$, we have

$$\Rightarrow \quad [1 + aM(z, z, kt)] * M(l, z, kt) \geq a[M(l, z, kt) * M(z, z, kt) * M(z, z, kt)] + M(z, z, t) * M(l, z, t) * M(z, z, t) * M(z, z, t) * M(l, z, t)$$

and

$$[1 + aN(z, z, kt)] \diamond N(l, z, kt) \leq a[N(l, z, kt) \diamond N(z, z, kt) \diamond N(z, z, kt)] + N(z, z, t) \diamond N(l, z, t) \diamond N(z, z, t) \diamond N(z, z, t) \diamond N(l, z, t)$$

$$\Rightarrow \quad [1 + a] * M(l, z, kt) \geq a[M(l, z, kt)] + 1 * M(l, z, t) * M(l, z, t)$$

and

$$[1 + a] \diamond N(l, z, kt) \leq a[N(l, z, kt)] + 1 \diamond N(l, z, t) \diamond N(l, z, t)$$

Which implies that

$M(l, z, kt) \geq M(l, z, t)$ and $N(l, z, kt) \leq N(l, z, t)$

Therefore by lemma 2.8 , we have, $l = z$. It follows that $Ax_n \to z$ as $n \to \infty$. Suppose that QS(X) is complete subspace of X. then $z = QSu$ for some $u \in X$. By putting $x = u$ and $y = y_n$ with $\propto = 1$ in (2), we have

$$\Rightarrow \quad [1 + aM(QSu, PTy_n, kt)] * M(Au, By_n, kt) \geq a[M(Au, QSu, kt) * M(By_n, PTy_n, kt) M(By_n, QSu, kt)] + M(PTy_n, QSu, t) * M(Au, QSu, t) * M(By_n, PTy_n, t) * M(By_n, QSu, t) * M(Au, PTy_n, t)$$

and

$$[1 + aN(QSu, PTy_n, kt)] \diamond N(Au, By_n, kt) \leq a[N(Au, QSu, kt) \diamond N(By_n, PTy_n, kt) \diamond N(By_n, QSu, kt)] + N(PTy_n, QSu, t) \diamond N(Au, QSu, t) \diamond N(By_n, PTy_n, t) \diamond N(By_n, QSx_n, t) \diamond N(Au, PTy_n, t)$$

$$\Rightarrow \quad [1 + aM(z, z, kt)] * M(Au, z, kt) \geq a[M(Au, z, kt) * M(z, z, kt) * M(z, z, kt)] + M(z, z, t) * M(Au, z, t) * M(z, z, t) * M(z, z, t) * M(Au, z, t)$$

and

$$[1 + aN(z, z, kt)] \diamond N(Au, z, kt) \leq a[N(Au, z, kt) \diamond N(z, z, kt) \diamond N(z, z, kt)] + N(z, z, t) \diamond N(Au, z, t) \diamond N(z, z, t) \diamond N(z, z, t) \diamond N(Au, z, t)$$

$$\Rightarrow \quad [1 + a] * M(Au, z, kt) \geq a[M(Au, z, kt)] + 1 * M(Au, z, t) * M(Au, z, t)$$

and

$$[1 + a] \diamond N(Au, z, kt) \leq a[N(Au, z, kt)] + 1 \diamond N(Au, z, t) \diamond N(Au, z, t)$$

Which implies that

$M(Au, z, kt) \geq M(Au, z, t)$
and
$N(Au, z, kt) \leq N(Au, z, t)$

Therefore by lemma 2.8, we have hence Au = z. Thus Au = QSu = z which shows that the pair (A, QS) has a point of coincidence on the other hand since, A(X) ⊂ PT(X) and Au = z, there exists a point v ϵ X such that PTv = z. now we show that PTv = Bv. Putting x = u and y = v with α = 1 in (2), we have

$$\Rightarrow \quad [1 + aM(QSu, PTv, kt)] * M(Au, Bv, kt) \geq a[M(Au, QSu, kt) * M(Bv, PTv, kt) * M(Bv, QSu, kt)] + M(PTv, QSu, t) * M(Au, QSu, t) * M(Bv, PTv, t) * M(Bv, QSu, t) * M(Au, PTv, t)$$

and

$$[1 + aN(QSu, PTv, kt)] \diamond N(Au, Bv, kt) \leq a[N(Au, QSu, kt) \diamond N(Bv, PTv, kt) \diamond N(Bv, QSu, kt)] + N(PTv, QSu, t) \diamond N(Au, QSu, t) \diamond N(Bv, PTv, t) \diamond N(Bv, QSu, t) \diamond N(Au, PTv, t)$$

$$\Rightarrow \quad [1 + aM(z, z, kt)] * M(z, Bv, kt) \geq a[M(z, z, kt) * M(Bv, z, kt) * M(Bv, z, kt)] + M(z, z, t) * M(z, z, t) * M(Bv, z, t) * M(Bv, z, t) * M(z, z, t)$$

and

$$[1 + aN(z, z, kt)] \diamond N(z, Bv, kt) \leq a[N(z, z, kt) \diamond N(Bv, z, kt) \diamond N(Bv, z, kt)] + N(z, z, t) \diamond N(z, z, t) \diamond N(Bv, z, t) \diamond N(Bv, z, t) \diamond N(z, z, t)$$

Which implies that

$$M(z, Bv, kt) \geq M(Bv, z, t)$$

and

$$N(z, Bv, kt) \leq N(Bv, z, t)$$

Therefore by lemma 2.8 , we have hence Bv = z. Thus Bv = PTv = z. Which shows that the pair (B, PT) has a point of coincidence since the pairs (A, QS) and (B, PT) are commuting pair wise i.e. AQ = QA, AS = SA, QS = SQ, BP = PB, BT = TB and PT = TP. It implies that both the pairs (A, QS) and (B. PT) are weakly compatible at u and v respectively, i.e. z = Au = QSu = Bv = PTv, therefore Az = A(QSu) = QS(Au) = QSz and Bz = B(PTu) = PT(Bu) = PTz. Now we assert that z is a fixed point of the self-map A, Q and S. putting x = z and y = v with $\alpha = 1$, in (2), we have

$$\Rightarrow \quad [1 + aM(QSz, PTv, kt)] * M(Az, Bv, kt) \geq a[M(Az, QSz, kt) * M(Bv, PTv, kt) * M(Bv, QSz, kt)] + M(PTv, QSz, t) * M(Az, QSz, t) * M(Bv, PTv, t) * M(Bv, QSz, t) * M(Az, PTv, t)$$

and

$$[1 + aN(QSz, PTv, kt)] \diamond N(Az, Bv, kt) \leq a[N(Az, QSz, kt) \diamond N(Bv, PTv, kt) \diamond N(Bv, QSz, kt)] + N(PTv, QSz, t) \diamond N(Az, QSz, t) \diamond N(Bv, PTv, t) \diamond N(Bv, QSz, t) \diamond N(Az, PTv, t)$$

$$\Rightarrow \quad [1 + aM(Az, z, kt)] * M(Az, z, kt) \geq a[M(Az, Az, kt) * M(z, z, kt) * M(z, Az, kt)] + M(z, Az, t) * M(Az, Az, t) * M(z, z, t) * M(z, Az, t) * M(Az, z, t)$$

and

$$[1 + aN(Az, z, kt)] \diamond N(Az, z, kt) \leq a[N(Az, Az, kt) \diamond N(z, z, kt) \diamond N(z, Az, kt)] + N(z, Az, t) \diamond N(Az, Az, t) \diamond N(z, z, t) \diamond N(z, Az, t) \diamond N(Az, z, t)$$

Which implies that

$$M(Az, z, kt) \geq M(Az, z, t)$$

And

$$N(Az, z, kt) \leq N(Az, z, t)$$

Therefore by lemma 2.8 , we have hence Az = z. Since Az = QSz which implies QSz = z. now we assert that z is a fixed point of the pair (B, PT), x = y = z with α = 1, in (2), we have

$$\Rightarrow \quad [1 + aM(QSz, PTz, kt)] * M(Az, Bz, kt) \geq a[M(Az, QSz, kt) * M(Bz, PTz, kt) * M(Bz, QSz, kt)] + M(PTz, QSz, t) * M(Az, QSz, t) * M(Bz, PTz, t) * M(Bz, QSz, t) * M(Az, PTz, t)$$

and

$$[1 + aN(QSz, PTz, kt)] \diamond N(Az, Bz, kt) \leq a[N(Az, QSz, kt) \diamond N(Bz, PTz, kt) \diamond N(Bz, QSz, kt)] + N(PTz, QSz, t) \diamond N(Az, QSz, t) \diamond N(Bz, PTz, t) \diamond N(Bz, QSz, t) \diamond N(Az, PTz, t)$$

$$\Rightarrow \quad [1 + aM(z, Bz, kt)] * M(z, Bz, kt) \geq a[M(z, z, kt) * M(Bz, Bz, kt) * M(Bz, z, kt)] + M(Bz, z, t) * M(z, z, t) * M(Bz, Bz, t) * M(Bz, z, t) * M(z, Bz, t)$$

and

$$[1 + aN(z, Bz, kt)] \diamond N(z, Bz, kt) \leq a[N(z, z, kt) \diamond N(Bz, Bz, kt) \diamond N(Bz, z, kt)] + N(Bz, z, t) \diamond N(z, z, t) \diamond N(Bz, Bz, t) \diamond N(Bz, z, t) \diamond N(z, Bz, t)$$

Which implies that

$M(z, Bz, kt) \geq M(Bz, z, t)$
and
$N(z, Bz, kt) \leq N(Bz, z, t)$

Therefore by lemma 2.8 , we have hence $Bz = z$. Since Bz = PTz. Which implies PTz = z. Now to prove that Sz = z. putting x = Sz and y = z, with α = 1 in (2), then we have

$$\Rightarrow \quad [1 + aM(QS(Sz), PTz, kt)] * M(A(Sz), Bz, kt) \geq a[M(A(Sz), QS(Sz), kt) * M(Bz, PTz, kt) * M(Bz, QS(Sz), kt)] + M(PTz, QS(Sz), t) * M(A(Sz), QS(Sz), t) * M(Bz, PTz, t) * M(Bz, QS(Sz), t) * M(A(Sz), PTz, t)$$

and

$$[1 + aN(QS(Sz), PTz, kt)] \diamond N(A(Sz), Bz, kt) \leq a[N(A(Sz), QS(Sz), kt) \diamond N(Bz, PTz, kt) \diamond N(Bz, QS(Sz), kt)] + N(PTz, QS(Sz), t) \diamond N(A(Sz), QS(Sz), t) \diamond N(Bz, PTz, t) \diamond N(Bz, QS(Sz), t) \diamond N(A(Sz), PTz, t)$$

$$\Rightarrow \quad [1 + aM(Sz, z, kt)] * M(Sz, z, kt) \geq a[M(Sz, Sz, kt) * M(z, z, kt) * M(z, Sz, kt)] + M(z, Sz, t) * M(Sz, Sz, t) * M(z, z, t) * M(z, Sz, t) * M(Sz, z, t)$$

and

$$[1 + aN(Sz, z, kt)] \diamond N(Sz, z, kt) \leq a[N(Sz, Sz, kt) \diamond N(z, z, kt) \diamond N(z, Sz, kt)] + N(z, Sz, t) \diamond N(Sz, Sz, t) \diamond N(z, z, t) \diamond N(z, Sz, t) \diamond N(Sz, z, t)$$

Which implies that

$M(Sz, z, kt) \geq M(Sz, z, t)$
and
$N(Sz, z, kt) \leq N(Sz, z, t)$

Therefore by lemma 2.8 , we have hence $Sz = z$. Since z = QSz. Which implies that Qz = z. Now, we prove that Tz = z. Putting x = z and y = Tz with α = 1 in (2), then we have

$$\Rightarrow \quad [1 + aM(QSz, PT(Tz), kt)] * M(Az, B(Tz), kt) \geq a[M(Az, QSz, kt) * M(B(Tz), PT(Tz), kt) * M(B(Tz), QSz, kt)] + M(PT(Tz), QSz, t) * M(Az, QSz, t) * M(B(Tz), PT(Tz), t) * M(B(Tz), QSz, t) * M(Az, PT(Tz), t)$$

and

$$[1 + aN(QSz, PT(Tz), kt)] \diamond N(Az, B(Tz), kt)$$

$$\leq a[N(Az, QSz, kt) \diamond N(B(Tz), PT(Tz), kt) \diamond N(B(Tz), QSz, kt)] + N(PT(Tz), QSz, t) \diamond N(Az, QSz, t) \diamond N(B(Tz), PT(Tz), t) \diamond N(B(Tz), QSz, t) \diamond N(Az, PT(Tz), t)$$

$$\Rightarrow \quad [1 + aM(z, Tz, kt)] * M(z, Tz, kt) \geq a[M(z, z, kt) * M(Tz, Tz, kt) * M(Tz, z, kt)] + M(Tz, z, t) * M(z, z, t) * M(Tz, Tz, t) * M(Tz, z, t) * M(z, Tz, t)$$

and

$$[1 + aN(z, Tz, kt)] \diamond N(z, Tz, kt) \leq a[N(z, z, kt) \diamond N(Tz, Tz, kt) \diamond N(Tz, z, kt)] + N(Tz, z, t) \diamond N(z, z, t) \diamond N(Tz, Tz, t) \diamond N(Tz, z, t) \diamond N(z, Tz, t)$$

Which implies that

$M(z, Tz, kt) \geq M(Tz, z, t)$
and
$N(z, Tz, kt) \leq N(Tz, z, t)$

Therefore by lemma 2.8 , we have hence Tz = z. Since z = PTz. Which implies Pz = z. then combining all the above result we have Az = Bz = Pz = z = Qz = Sz = Tz. Hence z is a common fixed point of A, B, P, Q, S and T.

Uniqueness: let w be an another common fixed point of A, B, P, Q, S and T. then putting x = z and y = w, in (2) with α = 1, we have

$$\Rightarrow [1 + aM(QSz, PTw, kt)] * M(Az, Bw, kt) \geq a[M(Az, QSz, kt) * M(Bw, PTw, kt) * M(Bw, QSz, kt)] + M(PTw, QSz, t) * M(Az, QSz, t) * M(Bw, PTw, t) * M(Bw, QSz, t) * M(Az, PTw, t)$$

and

$$\Rightarrow \quad [1 + aN(QSz, PTw, kt)] \diamond N(Az, Bw, kt) \leq a[N(Az, QSz, kt) \diamond N(Bw, PTw, kt) \diamond N(Bw, QSz, kt)] + N(PTw, QSz, t) \diamond N(Az, QSz, t) \diamond N(Bw, PTw, t) \diamond N(Bw, QSz, t) \diamond N(Az, PTw, t)$$

$$\Rightarrow \quad [1 + aM(z, w, kt)] * M(z, w, kt) \geq a[M(z, z, kt) * M(w, w, kt) * M(w, z, kt)] + M(w, z, t) * M(z, z, t) * M(w, w, t) * M(w, z, t) * M(z, w, t)$$

and

$$[1 + aN(z, w, kt)] \diamond N(z, w, kt) \leq a[N(z, z, kt) \diamond N(w, w, kt) \diamond N(w, z, kt)] + N(w, z, t) \diamond N(z, z, t) \diamond N(w, w, t) \diamond N(w, z, t) \diamond N(z, w, t)$$

Which implies that

$M(z, w, kt) \geq M(z, w, t)$
and
$N(z, w, kt) \leq N(z, w, t)$

Therefore by lemma 2.8 , we have hence $z = w$. thus , z is a unique common fixed point of A, B, P, Q, S and T.

Corollary: Let (X, M, N, $*,\diamond$) be a complete intuitionistic fuzzy metric space with $t * t \geq t$ and $(1 - t) \diamond (1 - t) \leq (1 - t)$ for all t ϵ [0, 1]. Further, let A, B, S and T be six self mappings of X satisfying

(1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$,

(2) there exists a constant k ϵ (0, 1) such that

$$[1 + aM(Sx, Ty, kt)] * M(Ax, By, kt) \geq a[M(Ax, Sx, kt) * M(By, Ty, kt) * M(By, Sx, kt)] + M(Ty, Sx, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, \propto t) * M(Ax, Ty, (2 - \propto)t)$$

and

$$[1 + aN(Sx, Ty, kt)] \diamond N(Ax, By, kt) \leq a[N(Ax, Sx, kt) \diamond N(By, Ty, kt) \diamond N(By, Sx, kt)] + N(Ty, Sx, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(By, Sx, \propto t) \diamond N(Ax, Ty, (2 - \propto)t)$$

for all $x, y \epsilon X, a \geq 0, \propto \epsilon (0, 2)$ and $t > 0$.

(3) (A, S) and (B, T) satisfies the property (E.A.).
(4) One of A(X), B(X), S(X) and T(X) is a complete subspace of X.
Then the pair (A, S) and (B, T) have a unique common fixed point provided the pairs (A, S) and (B,T) commute pair wise.

IV. Conclusion

In this paper, we have presented common fixed point theorems in intuitionistic Fuzzy metric spaces through concept of property (E.A).

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