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On D α -Closed Sets in Supra Topological Spaces

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Abstract: *In this paper, a new class of sets called D α-closed sets in supra topological spaces are introduced and studied. Also we introduce supra D α-continuous function, totally supra D α -continuous and supra contra D α-continuous function in supra topology spaces and some of its properties are studied.*

I. Introduction and Preliminaries

In 1983 , A.S.Mashhour [5]introduced the supra topological spaces. In 1970 N.Levine [3] introduced and studied a class of generalized open sets in topological space called α-open sets. In 1985 I.L.Reilly [9] introduced α-continuity in topological spaces.

In 2008, R.Devi[2] introduced and studied a class of sets and a class of maps called supra α-open sets and supra α-continuous functions, respectively. Now we introduced the concepts of supra $D\alpha$-closed sets and totally $D\alpha$-continuous functions, and contra supra $D\alpha$-continuous functions and investigated some of its properties.

Definition 1.1. *Let (X, τ) be a topological space [7], and $A \subseteq X$. Then*
(i) A is α-open if $A \subseteq Int(Cl(Int(A)))$ and α-closed if $Cl(Int(Cl(A))) \subseteq A$
(ii) A is generalized closed (briefly g-closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(iii) A is generalized open (briefly g-open) if X-A is g-closed

Definition 1.2. *A subset A of a space X is called $D\alpha$-closed [11], if $Cl^*(Int(Cl^*(A))) \subseteq A$.*

Definition 1.3. *A subset A of a space X is called an $D\alpha$-open [11], if $A \subseteq Int^*(Cl(Int^*(A)))$*

Definition 1.4. *A subfamily τ^* of X is said to a supra topology on X [5], if,*
(i) $X, \emptyset \in \tau^*$
(ii) if $A_i \in \tau^*$ for all $i \in j$ then $\cup A_i \in \tau^*$

(X, τ^*) is called a supra topological space. The elements of τ^* are called supra open sets in (X, τ^*) and complement of a supra open set is called a supra closed set.

Definition 1.5. *Let (X, τ) be topological and τ^* be a supra topology on X [5]. We call τ^* a supra topology associated with τ if $\tau \subset \tau^*$*

II. On supra D -closed sets

Definition 2.1. *Let (X, τ) be a supra topological space. A set A is called supra $D\alpha$-closed set if $Cl_*^\mu(Int^\mu(Cl_*^\mu(A))) \subseteq A$. The complement of a supra $D\alpha$-closed set is a supra $D\alpha$-open set.*

The intersection of all supra g-closed sets containing A is called the supra g-closure of A and denoted by $Cl_*^\mu(A)$, and the supra g-interior of A is the union of all supra g-open sets contained in A and is denoted by $Int_*^\mu(A)$

Lemma 2.1. *If there exists an supra g-closed set F such that $Cl_*^\mu(Int(F)) \subseteq A \subseteq F$, then A is supra $D\alpha$-closed*

Proof. Since F is supra g-closed, $Cl_*^\mu(F) = F$. Therefore, $Cl_*^\mu(Int(Cl_*^\mu(A))) \subseteq Cl_*^\mu(Int(Cl_*^\mu(F))) = Cl_*^\mu(Int(F)) \subseteq A$. Hence A is supra $D\alpha$-closed. □

Theorem 2.1. *Let (X, τ) be a supra topological space. Then*
(i) Every supra α-closed subset of (X, τ) is supra $D\alpha$-closed.
(ii) Every supra g-closed subset of (X, τ) is supra $D\alpha$-closed.
Proof. (i) Since supra closed set is supra g-closed, $Cl_*^\mu(A) \subseteq Cl^\mu(A)$. Now suppose A is supra α-closed in X, then $Cl^\mu(Int(Cl^\mu(A))) \subseteq A$. Therefore, $Cl_*^\mu(Int(Cl_*^\mu(A))) \subseteq Cl^\mu(Int(Cl^\mu(A))) \subseteq A$. Hence A is supra $D\alpha$-closed in X.
(ii) Suppose A is supra g-closed. Then $Cl_*^\mu(A) = A$. Therefore $Int(Cl_*^\mu(A)) \subseteq Cl_*^\mu(A)$. Then $Cl_*^\mu(Int(Cl_*^\mu(A))) \subseteq Cl_*^\mu(Cl_*^\mu(A)) \subseteq Cl_*^\mu(A) = A$. Hence A is supra $D\alpha$-closed. □

Theorem 2.2. *Finite unoin of supra $D\alpha$-open sets is always a supra $D\alpha$-open set and Finite intersection of supra $D\alpha$-closed sets is always a supra $D\alpha$-closed set. Finite intersection of supra $D\alpha$-open sets may fail to be a supra $D\alpha$-open set and Finite union of supra $D\alpha$-closed sets may fail to be a supra $D\alpha$-closed set.*

Example 2.1. *Let (X, τ) associated be a supra topological space, where $X = \{a, b, c\}$ and*
$\tau = \{\emptyset, \{a\}, \{a, b\}, \{b, c\}, X\}, \tau^*$ *be associated supra topology with* τ *and*
$\tau^* = \{\emptyset, \{b, c\}, \{c\}, \{a\}, X\}$,
$\alpha C(X) = \{\emptyset, \{a\}, \{c\}, \{b, c\}, X\}$,
$\alpha O(X) = \{\emptyset, \{b, c\}, \{a, b\}, \{a\}, X\}, GC(X) = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}, X\}$,
$GO(X) = \{\emptyset, \{a\}, \{b\}, \{b, c\}, \{a, b\}, X\}$,
$D\alpha O(X) = \{\{b, c\}, \{a, c\}, \{a, b\}, \{a\}\}, D\alpha C(X) = \{\{a\}, \{b\}, \{c\}, \{b, c\}\}$ *Therefore*
$\{b\} \in D\alpha C(X)$ *but* $\{b\} \notin \alpha C(X)$ *and* $\{b\} \notin GC(X)$.

Remark 1. *The union of two supra $D\alpha$-closed sets need not to be supra $D\alpha$-closed as shown in the example, Let (X,τ) associated be a supra topological space, where $X = \{a,b,c\}$ and*
$\tau = \{\emptyset,\{a\},\{a,b\},\{b,c\},X\}, \tau^*$ *be associated supra topology with τ and*
$\tau^* = \{\emptyset,\{b,c\},\{c\},\{a\},X\}$,
$\alpha C(X) = \{\emptyset,\{a\},\{c\},\{b,c\},X\}$,
$\alpha O(X) = \{\emptyset,\{b,c\},\{a,b\},\{a\},X\}, GC(X) = \{\emptyset,\{a\},\{c\},\{a,c\},\{b,c\},X\}$,
$GO(X) = \{\emptyset,\{a\},\{b\},\{b,c\},\{a,b\},X\}$,
$D\alpha O(X) = \{\{b,c\},\{a,c\},\{a,b\},\{a\}\}, D\alpha C(X) = \{\{a\},\{b\},\{c\},\{b,c\}\}$ *Therefore*
$\{b\} \in D\alpha C(X)$ *but* $\{b\} \notin \alpha C(X)$ *and* $\{b\} \notin GC(X)$. *where both* $\{a\}$ *and* $\{b\}$ *are supra $D\alpha$-closed stes but* $\{a\} \cup \{b\} = \{a,b\}$ *is not $D\alpha$-closed.*

Lemma 2.2. *Let $A \subseteq X$, then*
(i) $X - Cl^{\mu}_{*}(X - A) = Int^{\mu}_{*}(A)$.
(ii) $X - Int^{\mu}_{*}(X - A) = Cl^{\mu}_{*}$.

Proof. Obvious. □

Theorem 2.3. *A subset A of a space X is supra $D\alpha$-open if and only if if $A \subseteq Int^{\mu}_{*}(Cl(Int^{\mu}_{*}(A)))$.*

Proof. Let A be supra $D\alpha$-open set. Then X-A is supra $D\alpha$-closed and $Cl^{\mu}_{*}(Int(Cl^{\mu}_{*}(X - A))) \subseteq X - A$.By lemma 1.2 $A \subseteq Int^{\mu}_{*}(Cl(Int^{\mu}_{*}(A)))$. Conversely, suppose $A \subseteq Int^{\mu}_{*}(Cl(Int^{\mu}_{*}(A)))$. Then $X - Int^{\mu}_{*}(Cl(Int^{\mu}_{*}(A))) \subseteq X - A$. Hence $Int^{\mu}_{*}(Cl(Int(X - A))) \subseteq X - A$. This shows that X-A is supra $D\alpha$-closed.Thus A is supra $D\alpha$-open. □

Theorem 2.4. *Let(X,τ) be a supra topological space. Then*
(i) Every supra α-open subset of (X,τ) is supra $D\alpha$-open.
(ii) Every supra α-open subset of (X,τ) is supra $D\alpha$-open.

Proof. (i) Since open set is supra g-open, $Int^{\mu}_{*}(A) \subseteq Int(A)$. Now suppose A is supra α-open in X. Then $A \subseteq Int^{/}mu_{*}(Cl(Int^{\mu}_{*}(A)))$. Therefore $A \subseteq Int^{\mu}_{*}(Cl(Int^{\mu}_{*}(A))) \subseteq A \subseteq Int(Cl(Int(A)))$. Hence A is supra $D\alpha$-open in X.
(ii) Suppose A is supra g-open. Then $Int^{\mu}_{*}(A) = A$. Therefore $Cl(Int(A)) \subseteq Int^{\mu}_{*}(A)$. Then $Int^{\mu}_{*}(Cl(Int^{\mu}_{*}(A))) \subseteq Int^{\mu}_{*}(Int^{\mu}_{*}(A)) \subseteq Int^{\mu}_{*}(A) = A$. Hence A is supra $D\alpha$-open. □

Theorem 2.5. *Arbitrary union of supra $D\alpha$-open set is supra $D\alpha$-open.*

Proof. Let $\{F_i : i \in \Lambda\}$ be a collection of supra $D\alpha$-open sets in X.
Then $F_i \subseteq Int^{\mu}_{*}(Cl(Int^{\mu}_{*}(F_i)))$ for each i.
Since $\cup F_i \subseteq F_i$ for each i, $Int^{\mu}_{*}(\cup F_i) \subseteq Int^{\mu}_{*}(F_i)$ for each i.
Hence $Int^{\mu}_{*}(\cup F_i) \subseteq \cup Int^{\mu}_{*}(F_i), i \in \Lambda$.
Therefore $Int^{\mu}_{*}(Cl(Int^{\mu}_{*}\cup(F_i))) \subseteq Int^{\mu}_{*}(Cl(\cup Int^{\mu}_{*}(F_i))) \subseteq Int^{\mu}_{*}(\cup Cl(Int^{\mu}_{*}(F_i))) \subseteq \cup Int^{\mu}_{*}(Cl(Int^{\mu}_{*}(F_i))) \subseteq \cup F_i$. . Hence $\cup F_i$ is supra $D\alpha$-open. □

Remark 2. *The intersection of two supra Dα-open sets need not be supra Dα-open as seen example ,Let (X, τ) associated be a supra topological space, where $X = \{a, b, c\}$ and*
$\tau = \{\emptyset, \{a\}, \{a, b\}, \{b, c\}, X\}, \tau^*$ *be associated supra topology with* τ *and*
$\tau^* = \{\emptyset, \{b, c\}, \{c\}, \{a\}, X\}$,
$\alpha C(X) = \{\emptyset, \{a\}, \{c\}, \{b, c\}, X\}$,
$\alpha O(X) = \{\emptyset, \{b, c\}, \{a, b\}, \{a\}, X\}, GC(X) = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c\}, X\}$,
$GO(X) = \{\emptyset, \{a\}, \{b\}, \{b, c\}, \{a, b\}, X\}$,
$D\alpha O(X) = \{\{b, c\}, \{a, c\}, \{a, b\}, \{a\}\}, D\alpha C(X) = \{\{a\}, \{b\}, \{c\}, \{b, c\}\}$ *Therefore*
$\{b\} \in D\alpha C(X)$ *but* $\{b\} \notin \alpha C(X)$ *and* $\{b\} \notin GC(X)$. *where both {b,c} and {a,c} are supra Dα-open sets but* $\{b, c\} \cap \{a, c\} = \{c\}$ *is not supra Dα-open.*

III. Totally supra D -continuous functions

In this section, the notion of totally $D\alpha$-continuous function is introduced. If A is both supra $D\alpha$-open and supra $D\alpha$-closed, then it is said to be supra $D\alpha$-clopen.

Definition 3.1. *Let (X, τ) and (Y, σ) be two toplogical spaces and τ^* be associated supra topology with τ we define a function $f : (X, \tau) \to (Y, \sigma)$ is called a supra Dα-continuous function if the inverse image of each open set Y is supra Dα-open in X.*

Definition 3.2. *Let (X, τ) and (Y, σ) be two toplogical spaces and τ^* be associated supra topology with τ we define a function $f : (X, \tau) \to (Y, \sigma)$ is called a totally supra Dα-continuous function if the inverse image of each open set Y is supra Dα-clopen in X.*

Theorem 3.1. *Every totally supra Dα-continuous function is supra Dα-continuous*

Proof. Suppose that $f : (X, \tau) \to (Y, \sigma)$ is totally supra $D\alpha$-continuous function and A is any open set in Y, since f is totally supra $d\alpha$-continuous function, $f^{-1}(A)$ is supra $D\alpha$-open in X. Therefore f is supra $D\alpha$-continuous.

The converse of the above theorem need not be a true as seen from the following example.

Example 3.1. $X = \{a, b, c\}$ *associated with the topology* $\tau = \{\emptyset, \{a, b\}, X\}$,$Y = \{a, b, c\}$ *associated with the topology* $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}, \tau^*$ *be associated supra topology with* τ *and* $\tau^* = \{\emptyset, \{a\}, \{a, b\}, \{b, c\}X\}$.
Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined follows $f(a) = a, f(b) = b, f(c) = c$, the inverse image of the open set $\{a, b\}$ is $\{a, b\}$ which is supra Dα-open but it is not supra Dα-clopen in X, then f is supra Dα-continuous but it is not totally supra Dα-continuous

Theorem 3.2. *Let (X, τ) and (Y, σ) be two toplogical spaces and τ^* be associated supra topology with τ, the following statements are equivalent.*
(i) f is totally supra $D\alpha$-continuous.
(ii) For each $x \in X$ and each open set V in Y with $f(x) \in V$, there is a supra $D\alpha$-clopen set U in X such that $x \in V$ and $f(U) \subset V$

Proof. $(i) \to (ii)$ Suppose that $f : (X, \tau) \to (Y, \sigma)$ is totally supra $D\alpha$-continuous function and V be any open set in Y containing f(x) so that $x \in f^{-1}(V)$, since f is totally supra $D\alpha$-continuous function, $f^{-1}(V)$ is supra $D\alpha$-clopen in X. Let $U = f^{-1}(V)$ is supra $D\alpha$-clopen set in X and $x \in U$. Also $f(U) = f(f^{-1}(V)) \subset V$ this implies $f(U) \subset V$.
$(ii) \to (i)$ Let V be open in Y and $x \in f^{-1}(V)$ be any arbitrary point. This implies $f(x) \in V$, therefore by (ii) there is a supra $D\alpha$-clopen set U in X containing x such that $f(U) \subset V$. This implies $U \subset f^{-1}(V)$.Hence $f^{-1}(V)$ is supra $D\alpha$-clopen of X. Hence it is supra $D\alpha$-clopen set in X. Therefore f is totally supra $D\alpha$-continuous function. □

Theorem 3.3. *If $f : (X, \tau) \to (Y, \sigma)$ is totally supra $D\alpha$-continuous function and $g : (y, \sigma) \to (Z, \nu)$ continuous function, then gof is totally supra $D\alpha$-continuous function.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ is totally supra $D\alpha$-continuous function and $g : (Y, \sigma) \to (Z, \nu)$ continuous. Let V be open in Z, since g is continuous $f^{-1}(g)$ is open in Y. Now since f is totally supra $D\alpha$-continuous function, then $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is supra $D\alpha$-clopen in X. Hence gof : $(X, \tau) \to (Z, \nu)$ is totally supra $D\alpha$-continuous function. □

Definition 3.3. *A supra topological space X is said to be*
(i) Supra $D\alpha - T_1$ if for each pair of distinct points x and y of X, there exists supra $D\alpha$-open sets U and V, respectively such that $x \in U$ and $y \notin U$ and $y \in V, x \notin V$
(ii) Supra $D\alpha - T_2$ if for each pair of distinct points x and y of X, there exist disjoint supra $D\alpha$-open sets U and V in X such that $y \in U$ and $x \in V$ id disjoint.

Theorem 3.4. *Let (X, τ) and (Y, σ) be two topological spaces and τ^* be associated supra topology with τ. Let $f : (X, \tau) \to (Y, \sigma)$ be a totally supra $D\alpha$-continuous injection if Y is T_1 then (X, τ) is supra $D\alpha - T_1$.*

Proof. Let x and y be any two distinct point in X. Since f is injective we have f(x) and $f(y) \in Y$ such that $f(x) \neq f(y)$. Since Y is T_1 there exist open sets U and V in Y such that $f(x) \in U, f(y) \notin U, f(y) \in V, f(x) \notin V$, therefore $f^{-1}(U)$ and $f^{-1}(V)$ are supra $D\alpha$-clopen subsets of X because f is totally supra $D\alpha$-continuous, thus (X, τ) is supra $D\alpha - T_1$ □

Theorem 3.5. *Let (X, τ) and (Y, σ) be two topological spaces and τ^* be associated supra topology with τ. Let $f : (X, \tau) \to (Y, \sigma)$ be a totally supra $D\alpha$-continuous injection if Y is T_0 then (X, τ^*) is supra $\alpha - T_2$*

Proof. Let $x, y \in X$ with $x \neq y$, since f is injection $f(x) \neq f(y)$ since Y is T_0, there exist an open subset V of Y containing f(x) but not f(y), or containing f(y) but not f(x). Thus we have, $x \in f^{-1}(V)$ and $y \notin f^{-1}(V)$. Since f is totally supra $D\alpha$-continuous and V is an open subset of Y, $f^{-1}(V)$ and $X - f^{-1}(V)$ are disjoint supra $D\alpha$-clopen subsets of X containing x and y. Thus X is supra $D\alpha - T_2$ □

Theorem 3.6. *Let (X, τ) and (Y, σ) be two topological spaces and τ^* be associated supra topology with τ, let $f : (X, \tau) \to (Y, \sigma)$ be a totally supra $D\alpha$-continuous injection if Y is T_2 then (X, τ) is supra $D\alpha - T_2$*

Proof. Let $x, y \in X$ with $x \neq y$. Since f is injection $f(x) \neq f(y)$. By hypothesis there exist U and V open sets in Y such that $f(x) \in U, f(y) \in V$ and $U \cap V = \emptyset$. This implies $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$, since f is totally supra $D\alpha$-continuous $f^{-1}(U)$ and $f^{-1}(V)$ are supra $D\alpha$-clopen subsets of X such that $f^{-1}(U) \cap f^{-1}(V) = \emptyset$, D$\alpha$-clopen sets, therefore X is supra $D\alpha - T_2$. □

Definition 3.4. *Let (X, τ) and (Y, σ) be two topological spaces and τ^* be associated supra topology with τ we define a function $f : (X, \tau) \to (Y, \sigma)$ is called a strongly supra $D\alpha$-continuous function if the inverse of every subset of Y is supra $D\alpha$-clopen subset of X*

Theorem 3.7. *Every strongly supra $D\alpha$-continuous function is totally supra $D\alpha$-continuous*

Proof. Suppose that $f : (X, \tau) \to (Y, \sigma)$ is strongly supra $D\alpha$-continuous function and A is any open set in Y. By definition $f^{-1}(A)$ is supra $D\alpha$-clopen in X. Therefore f is totally supra $D\alpha$-continuous. □

The converse of the above theorem need not be true as seen from the following example

Example 3.2. *$X = \{a, b, c\}$ associated with the topology $\tau = \{\emptyset, \{a\}, X\}, Y = \{a, b, c\}$ associated with the topology $\sigma = \{\emptyset, \{a, b\}, Y\}, \tau^*$ be associated supra topology with τ and $\tau^* = \{\emptyset, \{a\}, \{a, b\}, \{b, c\}, X\}$.*
Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function, then f is totally supra $D\alpha$-continuous but not strongly supra $D\alpha$-continuous

Theorem 3.8. *Every totally supra $D\alpha$-continuous function into a finite T_1 spaces is strongly supra $D\alpha$-continuous*

Proof. Suppose $f : (X, \tau) \to (Y, \sigma)$ is totally supra $D\alpha$-continuous function (Y, σ) be a finite T_1 space and $B \subset Y$. Since Y is finite T_1, then Y must be a discrete space, therefore B is an open set in Y, since f is totally supra $D\alpha$-continuous $f^{-1}(B)$ is supra $D\alpha$-clopen in X. Therefore f is strongly supra $D\alpha$-continuous. □

IV. Contra supra D -continuous functions

In this section , we introduce the concept of contra supra $D\alpha$-continuous function and investigate some of the basic properties for this class of function.

Definition 4.1. *Let (X, τ) and (Y, σ) be two topological spaces and τ^* be associated supra topology with τ we define a function $f : (X, \tau) \to (Y, \sigma)$ is called a contra supra continuous function if the inverse image of each open set Y is τ^* supra closed set of X*

Definition 4.2. *Let (X, τ) and (Y, σ) be two topological spaces and τ^* be associated supra topology with τ we define a function $f : (X, \tau) \to (Y, \sigma)$ is called a contra supra $D\alpha$-continuous function if the inverse image of each open set Y is supra $D\alpha$-closed set of X*

Theorem 4.1. *Every contra continuous function is contra supra $D\alpha$-continuous.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be contra continuous function and A be closed in Y. Then $f^{-1}(A)$ is closed set in X. But τ^* is associated with τ, that is $\tau \subset \tau^*$. This implies $f^{-1}(A)$ is supra closed in X. Since every supra closed is supra $D\alpha$-closed, $f^{-1}(A)$ is supra $D\alpha$-closed in X. Hence f is a contra supra $D\alpha$-closed in X. Hence f is a contra supra $D\alpha$-continuous. □

Theorem 4.2. *Every contra supra continuous function is contra supra $D\alpha$-continuous.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a contra supra continuous function and A is any open set in Y. Since f is contra supra continuous, then $f^{-1}(A)$ is supra closed set in X, since every supra closed is supra $D\alpha$-closed, $f^{-1}(A)$ is supra $D\alpha$-closed in X. Hence f is a contra supra $D\alpha$-continuous □

The converse of the above theorems need not be true as seen from the following example.

Example 4.1. *Let $X = \{a, b, c\}$ associated with the topology $\tau = \{\emptyset, \{a, b\}, X\}$, $Y = \{a, b, c\}$ associated with the topology $\sigma = \{\emptyset, \{a\}, \{a, c\}, Y\}$, τ^* associated supra topology with τ and $\tau^* = \{\emptyset, \{a\}, \{a, b\}, \{b, c\}, X\}$, let $f : (X, \tau) \to (Y, \sigma)$ be a function defined follows $f(a) = a, f(b) = b, f(c) = c$, since inverse image of $\{a, c\}$ is supra $D\alpha$-closed in X, then f is contra supra $D\alpha$-continuous function but the inverse image of $\{a, c\}$ is not a supra closed set so f is not contra supra continuous*

Theorem 4.3. *Let (X, τ) and (Y, σ) be two topological spaces. Let f be a function from X into Y. into Y. Let τ^* be associated supra topology with τ, the following statements are equivalent.*
(i) f is contra supra $D\alpha$-continuous.
(ii) The inverse image of closed set in Y is supra $D\alpha$-open set in X.

Proof. $(i) \to (ii)$ Let A be a closed set in Y, then Y-A is open in Y, since f is contra supra $D\alpha$-continuous, then $f^{-1}(X - A) = X - f^{-1}(A)$ is supra $D\alpha$-closed in X. It follows that $f^{-1}(A)$ is supra $D\alpha$-open set of X.
$(ii) \to (i)$ Let V is open in Y. Then Y-V is closed in Y. By hypothesis $f^{-1}(Y - V) = X - f^{-1}(V)$ is supra $D\alpha$-open in X, which implies $f^{-1}(V)$ is supra $D\alpha$-closed. Therefore f is a contra supra $D\alpha$-continuous □

Theorem 4.4. *If $f : (X, \tau) \to (Y, \sigma)$ is contra supra $D\alpha$-continuous function and $g : (Y, \sigma) \to (Z, \nu)$ be continuous, then gof is contra supra $D\alpha$-continuous function.*

Proof. Let V be open in Z, since g is continuous $g^{-1}(V)$ is open in Y. Now since f is contra supra $D\alpha$-continuous function then $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is supra $D\alpha$-closed in X. Hence $gof = (X, \tau) \to (Z, \nu)$ is contra supra $D\alpha$-continuous function. Then gof is contra supra $D\alpha$-continuous function. □

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