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Existence of Solutions for a Three-Order *P*-Laplacian BVP on Time Scales

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Abstract: *This paper is concerned with the existence of solution to p-Laplacian dynamic equation*

$$\left(\phi_p\left(u^{\Delta\nabla}(t)\right)\right)^{\nabla}+\lambda f\left(t,u(t),u^{\Delta}(t)\right)=0,\quad t\in[0,T]_{\mathrm{T}},$$

subject to boundary conditions

$$u(0)=\beta(\xi),u^{\Delta}(t)=0,\phi_p\left(u^{\Delta\nabla}(0)\right)=\sigma\phi_p\left(u^{\Delta\nabla}(\xi)\right),$$

where $\phi_p(u)=|u|^{p-2}u$ *with* $p>1$*. Depending on the relevant theory and properties on time scales, we get the solution expression. We establish a proper Banuch space and the cone for this equation and define the corresponding operator. By Leray-Schauder nonlinear alternative theorem, we establish the sufficient condition for the existence of at leust one solution.*

Keyword: *time scales, p-Laplacian operator, Leray-Schauder nonlinear alternative theorem.*

I. Introduction

Recently, some authors have obtained many results on the existence of positive solutions to boundary value problems on time scales, for details, see [1-6] and the references therein. However, there is very few reported work considered the existence of solutions to boundary value problems with nonlinear terms involving with the derivative explicitly.

In[7], Wei Han studied the following m-point *p*-Laplacian eigenvalue problems

$$\begin{cases}\left(\phi_p\left(u^{\Delta\nabla}(t)\right)\right)^{\nabla}+\lambda f\left(t,u(t),u^{\Delta}(t)\right)=0, & t\in(0,T)_{\mathrm{T}},\quad \lambda>0,\\ \alpha u(0)-\beta u^{\Delta}(0)=0,\quad u(T)=\sum_{i=0}^{m-2}\alpha_i u(\xi_i), & u^{\Delta\nabla}(0)=0.\end{cases}$$

The author showed the existence and uniqueness of a nontrivial solution by way of the leray-schauder nonlinear alternative.

In[8], You-Hui Su concerned the following *p*-Laplacian dynamic equation

$$\begin{cases}\left(q_p\left(u^{\Delta}(t)\right)\right)^{\nabla}+h(t)f\left(t,u(t),u^{\Delta}(t)\right)=0, & t\in[0,T]_{\mathrm{T}},\\ u(0)-B_0\left(\sum_{i=0}^{m-2}\alpha_i u^{\Delta}(\xi_i)\right)=0, & u^{\Delta}(T)=0.\end{cases}$$

The author obtained that the boundary value problem has at Least triple or arbitrary positive solutions by using a generalization of Leggett-williams fixed point theorem. Similarly, authors of [9] considered the boundary value problem

$$\begin{cases}\left(\phi_p\left(u^{\Delta\nabla}(t)\right)\right)^{\nabla}+h(t)f\left(t,u(t),u^{\Delta}(t)\right)=0, t\in[0,T]_{\mathrm{T}},\\ u(0)-B_0\left(\sum_{i=0}^{m-2}\alpha_i u^{\Delta}(\xi_i)\right)=0,\\ u^{\Delta\nabla}(0)=u^{\Delta}(T)=0.\end{cases}$$

Motivated by the above mentioned works, in this paper, we study the boundary value problem

$$\begin{cases}\left(\phi_p\left(u^{\Delta\nabla}(t)\right)\right)^{\nabla}+\lambda f\left(t,u(t),u^{\Delta}(t)\right)=0, t\in[0,T]_{\mathrm{T}},\\ u(0)=\beta(\xi), u^{\Delta}(T)=0,\\ \phi_p\left(u^{\Delta\nabla}(0)\right)=\sigma\phi_p\left(u^{\Delta\nabla}(\xi)\right),\end{cases} \tag{1}$$

where T is a time scale,

$0,T\in\mathrm{T}$, $[0,T]_{\mathrm{T}}=[0,T]\cap\mathrm{T}$. $\phi_p(u)=|u|^{p-2}u$, $p>1$, $(\phi_p)^{-1}=\phi_q$, $\frac{1}{p}+\frac{1}{q}=1$.

By Leray-Schauder nonlinear alternative theorem we establish sufficient condition for the existence of at least one solution.

We note that by a solution u of the problem (1) we mean that $u:\mathrm{T}\to\mathrm{R}$, which is a delta differential, u^{Δ} and $\left(\phi_p\left(u^{\Delta\nabla}(t)\right)\right)^{\nabla}$ are both continuous on $\mathrm{T}^k\cap\mathrm{T}_k$, and u satisfies (1).

The interrelated definitions on time scales can be found in [10]. Throughout this paper it is assumed that

(H_1) $0<\beta$, $\sigma<1$, $\xi\in(0,T)_{\mathrm{T}}$;

(H_2) $f:[0,T]_{\mathrm{T}}\times\mathrm{R}\times\mathrm{R}\to\mathrm{R}$

is C_{ld} continuous and does not vanish identically on any closed subinterval of $[0,T]_{\mathrm{T}}$, where R^+ denotes the nonnegative real numbers.

Let $X=C_{ld}[0,T]_{\mathrm{T}}$ be the Banach space with norm $\|u\|=\max_{t\in[0,T]_{\mathrm{T}}}u(t)$ and order relation $x\le y$ if $x(t)\le y(t)$, $t\in[0,T]_{\mathrm{T}}$.

Let $Y=C^1_{ld}[0,T]_{\mathrm{T}}$ with norm $\|u\|_1=\|u\|+\|u^{\Delta}\|=\max_{t\in[0,T]_T}|u(t)|+\max_{t\in[0,T]_T}|u^{\Delta}(t)|$ Then $(Y,\|u\|_1)$ is a Banach space.

For convenient, we denote

$$D=-A=\frac{\sigma}{1-\sigma}\int_0^{\xi}f\left(\tau,u,u^{\Delta}\right)\nabla\tau,$$

$$\varphi(s)=\int_0^s\left(p(\tau)+q(\tau)\right)\nabla\tau,$$

$$\psi(s)=\int_0^s r(\tau)\nabla\tau+\frac{D}{\lambda},$$

$$M_{\varphi}=\int_0^T s\left(\varphi(s)\right)^{\frac{1}{p-1}}\nabla s+\frac{\beta}{1-\beta}\int_0^{\xi}(\xi-s)\left(\varphi(s)\right)^{\frac{1}{p-1}}\nabla s+\frac{1+\beta(\xi-1)}{1-\beta}\int_0^T\left(\varphi(s)\right)^{\frac{1}{p-1}}\nabla s.$$

Lemma1.1 ([10])

$$\phi_q(s+t)\le\begin{cases}\frac{1}{2^{p-1}}\left(\phi_q(s)+\phi_q(t)\right), & p\ge 2, \quad s,t>0,\\ \phi_q(s)+\phi_q(t), & 1<p<2, \quad s,t>0.\end{cases}$$

.

II. Main Results

Lemma2.1. The solution expression of the boundary value problem (1) is

$$u(t)=-\int_0^t (t-s)\phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s+t\int_0^T \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s$$
$$+\frac{\beta}{1-\beta}\left[-\int_0^\xi (\xi-s)\phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s+\xi\int_0^T \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s\right]. \quad (2)$$

Proof. By(1), we have

$$u(t)=-\int_0^t (t-s)\phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau-A\right)\nabla s+Bt+C, \quad (3)$$

and

$$u^\Delta(t)=-\int_0^t \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau-A\right)\nabla s+B, \quad (4)$$

$$u^{\Delta\nabla}(t)=-\phi_q\left(\int_0^t \lambda f(\tau,u,u^\Delta)\nabla\tau-A\right). \quad (5)$$

Then $A=-\frac{\sigma}{1-\sigma}\int_0^\xi \lambda f(\tau,u,u^\Delta)\nabla\tau$ since

$$\phi_p\left(u^{\Delta\nabla}(0)\right)=A=\sigma\left(-\int_0^\xi \lambda f(\tau,u,u^\Delta)\nabla\tau+A\right).$$

On the other hand, using

$$u^\Delta(T)=-\int_0^T \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau-A\right)\nabla s+B=0$$

we can get

$$B=\int_0^T \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s.$$

Furthermore, by

$$u(0)=C=\beta\left[-\int_0^\xi \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s+\xi\int_0^T \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s+C\right].$$

We see that

$$C=\frac{\beta}{1-\beta}\left[-\int_0^\xi \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s+\xi\int_0^T \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s\right]. \quad (6)$$

Substituting A, B and C into(3), we may see that (2) holds.

Next we will show

$$u(t)\ge 0,\quad u^\Delta(t)\ge 0,\quad u^{\Delta\nabla}(t)\le 0. \quad (7)$$

From(5), $u^{\Delta\nabla}(t)\le 0,\quad t\in[0,T]_{\mathbb{T}}$, and $u^\Delta(T)=0$ then $u^\Delta(t)$ is decreasing.

Thus $u(t)\ge u(0)$ By (6), and $u^\Delta(T)=0$ we have $u(t)\ge 0,\quad u^\Delta(t)\ge 0.$

We define the operator $Q:\ \mathrm{Y}\to\mathrm{Y}$ as follows:

$$(Qu)(t)=-\int_0^t (t-s)\phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s+t\int_0^T \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s$$
$$+\frac{\beta}{1-\beta}\left[-\int_0^\xi (\xi-s)\phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s+\xi\int_0^T \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau+D\right)\nabla s\right].$$

Lemma2.2. $Q: \ Y \to Y$ is complete continuous.

Proof. Let $C > 0$, and $u \in \bar{Y}_C = \{x \in Y : \|x\| < C\}$. By lemma1.1, for $1 < p < 2, \quad s,t > 0,$ we have

$$\|Qu\| = \max_{t\in[0,T]_{\mathrm{T}}} |Qu(t)| = Qu(T) \le \int_0^T T\phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau + D\right)\nabla s$$

$$+\frac{\beta}{1-\beta}\left[\int_0^\xi T\phi_q\left(\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau\right)+\phi_q(D)\right)\nabla s + \xi\int_0^T \phi_q\left(\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau\right)+\phi_q(D)\right)\nabla s\right] < +\infty.$$

Similarly, we may obtain when $\|Qu\| < +\infty$ when $p \ge 2$.

There fore, $Q\bar{Y}_C$ is bounded uniformly.

On the other have, for $t_1 < t_2$,

$$|(Qu)(t_2) - (Qu)(t_1)|$$

$$= \left| -\int_0^{t_1}(t_1 - s)\phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau + D\right)\nabla s + t_1\int_0^T \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta) + D\right)\nabla s\right.$$

$$\left. +\int_0^{t_2}(t_2 - s)\phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau + D\right)\nabla s - t_2\int_0^T \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta) + D\right)\nabla s\right|$$

$$\le (t_2 - t_1)\left|2\int_0^T \phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau + D\right)\nabla s\right|$$

$$+(t_2 - t_1)\left|\phi_q\left(\lambda\int_0^s f(\tau,u,u^\Delta)\nabla\tau + D\right)\right| \to 0\,(t_2 \to t_1)$$

Arzela-Ascoli theorem and continuity of f show that $Q: \ \bar{Y}_C \to \mathrm{R}$ is a completely continuous operator.

Theorem 2.1. Suppose $(H_1)(H_2)$ hold. There exists nonnegative functions $p(t), q(t), r(t) \in L^1$ satisfy

$$|f(t,u,v)| \le p(t)|u|^{p-1} + q(t)|u|^{p-1} + r(t), \quad (t,u,v) \in [0,T]_{\mathrm{T}} \times \mathrm{R} \times \mathrm{R}. \qquad (8)$$

Where $p(t), q(t)$ do not vanish identically.

Then there exists a constant number $\lambda^* > 0$, for $\forall\lambda \in (0,\lambda^*)$, the problem (1) has at least one solution $u^* \in C_{ld}^1([0,T]_{\mathrm{T}}, \mathrm{R})$.

Proof. First, from $p(t_0) \ne 0$ or $q(t_0) \ne 0$, we have

$$\int_0^T \psi(s)\nabla s > 0.$$

Let $m = \dfrac{M_\psi}{M_\varphi}, \quad \Omega = \{u \in C_{ld}^1[0,T]_{\mathrm{T}} : \|u\|_1 < m\}$.

Assume $u \in \partial\Omega,\quad Qu = \mu u,\quad \mu > 1,$ then

$$\mu m = \mu\|u\|_1 = \|Qu\|_1 = \|Qu\| + \left\|(Qu)^\Delta\right\|.$$

Since

$$\|Qu\| = \max_{t\in[0,T]_{\mathbb{T}}} \left|(Qu)(t)\right| = \left|Qu(T)\right|$$

$$= \Bigg|\int_0^T s\phi_q\left(\lambda\int_0^s f\left(\tau,u,u^\Delta\right)\nabla\tau + D\right)\nabla s$$
$$+\frac{\beta}{1-\beta}\left[-\int_0^\xi(\xi-s)\phi_q\left(\lambda\int_0^s f\left(\tau,u,u^\Delta\right)+D\right)\nabla s + \xi\int_0^T\phi_q\left(\lambda\int_0^s f\left(\tau,u,u^\Delta\right)\nabla\tau + D\right)\nabla s\right]\Bigg|$$
$$\le \int_0^T s\phi_q\left(\lambda\int_0^s\left(p(\tau)|u|^{p-1} + q(\tau)\left|u^\Delta\right|^{p-1} + r(\tau)\right)\nabla\tau + D\right)\nabla s$$
$$+\frac{\beta}{1-\beta}\left[\int_0^\xi(\xi-s)\phi_q\left(\lambda\int_0^s\left(p(\tau)|u|^{p-1} + q(\tau)\left|u^\Delta\right|^{p-1} + r(\tau)\right)\nabla\tau + D\right)\nabla s\right.$$
$$\left.+\xi\int_0^T\phi_q\left(\lambda\int_0^s\left(p(\tau)|u|^{p-1} + q(\tau)\left|u^\Delta\right|^{p-1} + r(\tau)\right)\nabla\tau + D\right)\nabla s\right]$$
$$\le \int_0^T s\phi_q\left(\lambda\|u\|_1^{p-1}\int_0^s\left(p(\tau)+q(\tau)\right)\nabla\tau + \lambda\int_0^S r(\tau)\nabla\tau + D\right)\nabla s$$
$$+\frac{\beta}{1-\beta}\left[\int_0^\xi(\xi-s)\phi_q\left(\lambda\|u\|_1^{p-1}\int_0^s\left(p(\tau)+q(\tau)\right)\nabla\tau + \lambda\int_0^S r(\tau)\nabla\tau + D\right)\nabla s\right.$$
$$\left.+\xi\int_0^T\phi_q\left(\lambda\|u\|_1^{p-1}\int_0^s\left(p(\tau)+q(\tau)\right)\nabla\tau + \lambda\int_0^S r(\tau)\nabla\tau + D\right)\nabla s\right]$$
$$= \int_0^T s\phi_q(\lambda)\phi_q\left(\|u\|_1^{p-1}\varphi(s)+\psi(s)\right)\nabla s \qquad + \frac{\beta}{1-\beta}\left[\int_0^T(\xi-s)\phi_q(\lambda)\phi_q\left(\|u\|_1^{p-1}\varphi(s)+\psi(s)\right)\nabla s\right.$$
$$\left.+\xi\int_0^T\phi_q(\lambda)\phi_q\left(\|u\|_1^{p-1}\varphi(s)+\psi(s)\right)\nabla s\right]$$

Next, we consider two cases

(I) If $p \ge 2$, then by using inequality $x^{p-1} + y^{p-1} \le (x+y)^{p-1}$, $x, y \in \mathrm{R}^+$ we have

$$\phi_q\left(\|u\|_1^{p-1}\varphi(s)+\psi(s)\right) = \phi_q\left(\phi_p\left(\|u\|_1(\varphi(s))^{\frac{1}{p-1}}\right) + \phi_p\left((\psi(s))^{\frac{1}{p-1}}\right)\right)$$
$$\le \|u\|_1(\varphi(s))^{\frac{1}{p-1}} + (\psi(s))^{\frac{1}{p-1}}.$$

Thus

$$\|Qu\| \le \phi_q(\lambda)\|u\|_1\left(\int_0^T s(\varphi(s))^{\frac{1}{p-1}}\nabla s + \frac{\beta}{1-\beta}\int_0^\xi(\xi-s)(\varphi(s))^{\frac{1}{p-1}}\nabla s + \frac{\beta\xi}{1-\beta}\int_0^T(\varphi(s))^{\frac{1}{p-1}}\nabla s\right)$$
$$+\phi_q(\lambda)\|u\|_1\left(\int_0^T s(\psi(s))^{\frac{1}{p-1}}\nabla s + \frac{\beta}{1-\beta}\int_0^\xi(\xi-s)(\psi(s))^{\frac{1}{p-1}}\nabla s + \frac{\beta\xi}{1-\beta}\int_0^T(\psi(s))^{\frac{1}{p-1}}\nabla s\right),$$

and

$$\left\|(Qu)^\Delta\right\| = \max_{t\in[0,T]_{\mathbb{T}}}\left|(Qu)^\Delta\right| = \left|(Qu)^\Delta(0)\right|$$
$$= \left|\int_0^T\phi_q\left(\lambda\int_0^s f\left(\tau,u,u^\Delta\right)\nabla\tau + D\right)\nabla s\right| \quad \le \int_0^T\phi_q\left(\lambda\int_0^s\left(p(\tau)|u|^{p-1} + q(\tau)\left|u^\Delta\right|^{p-1} + r(\tau)\right)\nabla\tau + D\right)\nabla s$$

$$\le \int_0^T \phi_q(\lambda)\phi_q\left(\|u\|_1^{p-1}\varphi(s)+\psi(s)\right)\nabla s$$

$$\le \int_0^T \phi_q(\lambda)\left(\|u\|_1(\varphi(s))^{\frac{1}{p-1}}+(\psi(s))^{\frac{1}{p-1}}\right)\nabla s$$

$$=\phi_q(\lambda)\|u\|_1\int_0^T(\varphi(s))^{\frac{1}{p-1}}\nabla s+\phi_q(\lambda)\int_0^T(\psi(s))^{\frac{1}{p-1}}\nabla s.$$

Then

$$\|Qu\|_1\le\phi_q(\lambda)\|u\|_1\left(\int_0^T s(\varphi(s))^{\frac{1}{p-1}}\nabla s+\frac{\beta}{1-\beta}\int_0^\xi(\xi-s)(\varphi(s))^{\frac{1}{p-1}}\nabla s+\frac{\beta(\xi-1)}{1-\beta}\int_0^T(\varphi(s))^{\frac{1}{p-1}}\nabla s\right)$$

$$+\phi_q(\lambda)\left(\int_0^T s(\psi(s))^{\frac{1}{p-1}}\nabla s+\frac{\beta}{1-\beta}\int_0^\xi(\xi-s)(\psi(s))^{\frac{1}{p-1}}\nabla s+\frac{1+\beta(\xi-1)}{1-\beta}\int_0^T(\psi(s))^{\frac{1}{p-1}}\nabla s\right)$$

$$=\phi_q(\lambda)\|u\|_1M_\varphi+\phi_q(\lambda)M_\psi.$$

(II) For $1<p<2$

$$\|Qu\|\le\int_0^T s\phi_q(\lambda)\left[\phi_q\left(\|u\|_1^{p-1}\varphi(s)\right)+\phi_q(\psi(s))\right]\nabla s$$

$$+\frac{\beta}{1-\beta}\phi_q(\lambda)\int_0^\xi(\xi-s)\left[\phi_q\left(\|u\|_1^{p-1}\varphi(s)\right)+\phi_q(\psi(s))\right]\nabla s$$

$$+\phi_q(\lambda)\frac{\beta\xi}{1-\beta}\int_0^T\left[\phi_q\left(\|u\|_1^{p-1}\varphi(s)\right)+\phi_q(\psi(s))\right]\nabla s\quad\le\int_0^T s\phi_q(\lambda)\left(\|u\|_1(\varphi(s))^{\frac{1}{p-1}}+(\psi(s))^{\frac{1}{p-1}}\right)\nabla s$$

$$+\frac{\beta}{1-\beta}\phi_q(\lambda)\int_0^\xi(\xi-s)\left(\|u\|_1(\varphi(s))^{\frac{1}{p-1}}+(\psi(s))^{\frac{1}{p-1}}\right)\nabla s$$

$$+\phi_q(\lambda)\frac{\beta\xi}{1-\beta}\int_0^T\left(\|u\|_1(\varphi(s))^{\frac{1}{p-1}}+(\psi(s))^{\frac{1}{p-1}}\right)\nabla s.$$

And

$$\left\|(Qu)^\Delta\right\|=\left|(Qu)^\Delta(0)\right|$$

$$\le\int_0^T\phi_q(\lambda)\left[\phi_q\left(\phi_p\left(\|u\|_1(\varphi(s))^{\frac{1}{p-1}}\right)\right)+\phi_q\left(\phi_p(\psi(s))^{\frac{1}{p-1}}\right)\right]\nabla s$$

$$=\phi_q(\lambda)\|u\|_1\int_0^T(\varphi(s))^{\frac{1}{p-1}}\nabla s+\phi_q(\lambda)\int_0^T(\psi(s))^{\frac{1}{p-1}}\nabla s.$$

There fore,

$$\|Qu\|_1\le\phi_q(\lambda)\|u\|_1M_\varphi+\phi_q(\lambda)M_\psi. \tag{9}$$

Choose $\lambda^*=\left(\frac{1}{2M_\varphi}\right)^{p-1}$, when for $0<\lambda\le\lambda^*$, we can get

$$\mu=\frac{\mu\|u\|_1}{m}\le\frac{1}{2M_\varphi}\frac{\|u\|_1}{m}+\frac{M_\psi}{2M_\varphi m}\le 1.$$

Which is contract with $\mu>1$. Thus Q has a fixpoint $u^*\in\overline{\Omega}$. Since $f(t,0,0)$ does not vanish identically, (1) has a non-trivial Solution in $C^1_{ld}([0,T]_T,R)$.

Theorem2.2. Assume that $(H_1)(H_2)$ hold and

$$0\le L=\lim_{|u|+|v|\to\infty}\max_{t\in[0,T]_T}\frac{f(t,u,v)}{|u|^{p-1}+|v|^{p-1}}<\infty \tag{10}$$

holds, then there exists a constant $\lambda^*>0$ such that the problem(1) has at least one solution $u^*\in C^1_{ld}([0,T]_T,R)$

(10)

when $\lambda \in (0, \lambda^*]$.

证明： $\forall \varepsilon > 0$, satisfies $L + 1 - \varepsilon > 0$, by(10), there is $H > 0$ such that

$|f(t,u,v)| \le (L+1-\varepsilon)\left(|u|^{p-1} + |v|^{p-1}\right)$, $|u| + |v| \ge H$, $0 \le t \le T$.

Let $K = \max_{t \in [0,T]_{\mathrm{T}}, |u|+|v| \ge H} |f(t,u,v)|$, then for all $(t,u,v) \in [0,T]_{\mathrm{T}} \times \mathrm{R} \times \mathrm{R}$,

$|f(t,u,v)| \le (L+1-\varepsilon)\left(|u|^{p-1} + |v|^{p-1}\right) + K$

holds. In the view of the Theorem2.1, (1)has at least or solution $u^* \in C^1_{ld}([0,T]_{\mathrm{T}}, \mathrm{R})$.

Corollary 2.1 Assume that $(H_1)(H_2)$ hold and the inequality

$$0 \le L = \lim_{|u|+|v| \to \infty} \max_{t \in [0,T]_{\mathrm{T}}} \frac{f(t,u,v)}{|u|^{p-1}} < \infty,$$

$$0 \le L = \lim_{|u|+|v| \to \infty} \max_{t \in [0,T]_{\mathrm{T}}} \frac{f(t,u,v)}{|v|^{p-1}} < \infty$$

holds, then there exist a constant $\lambda^* > 0$ such the problem(1) has at least one solution $u^* \in C^1_{ld}([0,T]_{\mathrm{T}}, \mathrm{R})$ when $\lambda \in (0, \lambda^*]$.

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