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Optimal Inventory Policy for Deteriorating Items with Seasonal Demand under the Effect of Price Discounting on Lost Sales

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Abstract: *In this paper an inventory model is developed with seasonal quadratic demand and time proportional deterioration. Shortages are considered and are partially backlogged. At the start of shortage period price discount is declared on unit selling price for backordered quantity so as to enhance the demand and to reduce the lost sales which in turn maximizes the total profit per unit time. Numerical illustration and sensitivity analysis of the optimal solution with respect to various parameters is carried out to describe the model.*

Kew Words: *Deterioration, Partial backlogging, Seasonal demand, Price discounting*.

I. Introduction

Inventory models with constant demand rate have been developed by many researchers in the past. But the demand rate of many commodities may be in dynamic state. This work started with Silver and Meal (1969), who developed EOQ model with time-varying demand rate. Since then many researchers developed inventory models with time varying demand. Khanna and Chaudhari (2003),Manna and Chaudhari (2006), Panda et al (2008) developed an inventory models for perishable products with time varying demand. Skouri et al (2009) presented inventory model with ramp type demand rate, partial backlogging &Weibull deterioration rate.

Vaish and Garg (2011) formulated an inventory model for non-instantaneous deteriorating items with stock dependent and time decreasing demand. Garg, Vaish and Gupta (2012) developed an inventory model with variable production and linearly increasing demand. Vaish and Agarwal (2012) developed a production inventory model with weibull distribution deterioration, quadratic demand and price discounting during deterioration. Karmarkar and Chaudhary (2014) developed inventory model with ramp type demand and partial backlogging. In any inventory control problem a major factor is the maintenance of inventory of deteriorating items. Many product decay or deteriorate over time. Product such as fruits, vegetables, food stuffs are subject to direct spoilage while kept in store. Further it is common experience that some products deteriorate as soon as they are received in stock. Most of researchers have taken interest in developing inventory models with constant and time proportional deterioration. Ghare and Schrader (1963) were the first to develop an inventory model with an exponential deterioration rate. Then Covert and Philip(1973) generalized Ghare and Schrader's constant exponential deterioration rate to a two-parameter Weibull distribution. Dave and Patel (1981), Philip(1974), Mishra(1975b), Chakarbarty et al (1998), Mukhopadhyayet al (2005), Shah and Acharya (2008), Bhuniaet al (2009), Skouri et al (2009) and many others developed inventory models for time dependent deteriorating items. Mishra (2013) developed an inventory model with instantaneous deterioration, linear time dependent demand and partial backlogging. It has been seen many times that stock ends before the arrival of next replenishment and some customers do not want wait up to the next replenishment. Cheng and Dye (1999) formulated an inventory model with time-varying demand and partial backlogging. Dye et al. (2007) developed an inventory model to find optimal selling price and lot size with variable demand and deterioration rate and exponential partial backlogging. They assumed that backlogging increases exponentially as the waiting time for the next replenishment decreases. Mishra, Singh and Kumar (2013) developed an inventory model with instantaneously deteriorated items with linearly increasing demand and partial backlogging. Karmakar and Chaudhary (2014) developed an inventory model with time proportional deterioration, ramp type demand and partial backlogging. Price discount on unit selling price of goods attracts the customers to buy more and more. The main reasons for discounts include: Firstly, suppliers offer discounts to clear their old stocks and free-up space for the new stock. Secondly, a supplier wants to sell more to make more profits, thirdly, customers are attracted by discounts and they buy more than usual. Further discount offers also reduces the loss due to deterioration for suppliers. Thus price discount is one of the key factors which enhance the demand which in turn increases the total profit per unit time. Ardalan (1994) developed an inventory policy where temporary price discount resulted in increase in demand. Papachristos and Skouri (2003) presented an inventory model for deteriorating item where demand rate is a decreasing function of the selling price. Sana and Chaudhari (2008) presented an EOQ model with price discount offers. Hsu and Yu (2009) proposed an EOQ model under a one-time discount. Panda et al. (2009) developed an EOQ model for deteriorating products with discounted selling price and stock dependent demand. Cardanas-Barron et al (2010) presented an inventory model for determining the optimal ordering policies with advantage of a one-time discount offer and back orders. Vaish, Garg and

Gupta (2012) presented an inventory model for perishable products with ramp type demand and price discounting. Vaish, B and Agarwal, D (2012) formulated a production lot size model for non-instantaneous deterioration with price discounting and quadratic demand.

The present paper is based on of **Mishra, Singh and Kumar** (2013) and **Garima and Vaish** (2011) papers. In Garima and Vaish paper price discount is given on unit selling price before deterioration and after deterioration. The modal is developed for no shortage case while Mishra's paper is developed for linear trend in demand. Shortages were allowed and a constant fraction of the demand was backlogged. Price discount is not given in the model. In the present paper an economic ordered quantity model is developed considering seasonal quadratic type of demand which starts with zero reaches its maximum and ends with zero. Deterioration is instantaneous and time dependent. Shortages are allowed. A fraction of demand is backordered which depends on waiting time up to the next replenishment and a price discount is given on the backordered quantity. Price discount on the backordered quantity reduces the lost sales. The modal is developed for the fixed length of the inventory cycle. Special case of no discount is discussed. Profit maximization technique is used to solve the modal. Numerical illustration, tables and graphs are presented to describe the model. In addition sensitivity analysis of the optimal solution with respect to various parameters involved in inventory problem is carried out.

II. Assumptions

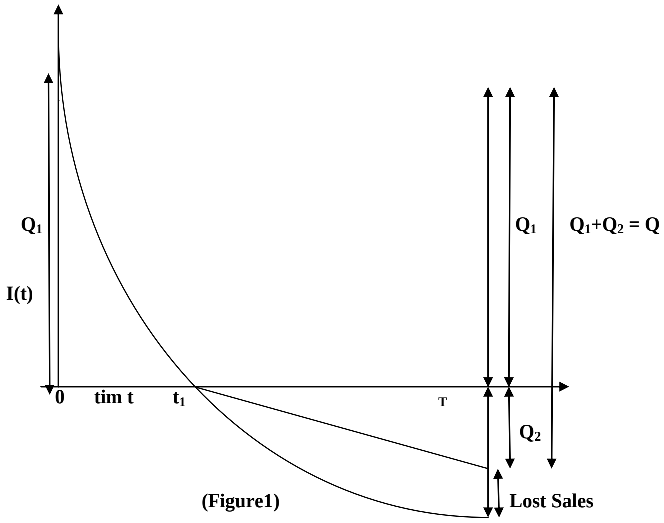
1. Demand is quadratic in nature which starts from zero and ends with zero and it follows the pattern D (t) = at (T-t).Where a is demand rate and a > 0.
2. Shortages are allowed and are partial backlogged. The backlogged rate is described as decreasing function of the waiting time $\frac{1}{1+\delta(T-t)}$ where ($\delta > 0$).Thus a fraction of the demand is backlogged
3. d ($0 \leq d \leq 1$)is the percentage offer on unit selling price on backordered quantity declared at the start of the stock out period. $\alpha = (1-d)^{-n}$ ($n \in R$, the set of real numbers and $n \geq 1$) is the positive effect of discounted selling price on demand during stock out period. When $d \to 0, \alpha \to 1$ i.e. the demand during stocked period will not be increased.
5. The deterioration is instantaneous and is linearly time dependent which *is* θ (t)= θt.There is no repair or replacement of deteriorating items during the period under consideration.
6. Delivery lead time is zero and cycle length of the inventory model is finite.

III. Notations

C	Purchasing cost per unit
c	Cost of each deteriorated unit
p	Selling price per unit
θ	Deterioration coefficient, θ<<<1
T	Cycle length
t_1	The time at which inventory level becomes zero
Q_1	Initial inventory level at the beginning of each cycle and
Q_2	Backordered quantity
Q	Ordered Quantity (Q_1+Q_2)
DQ	Deteriorated quantity
h	Holding cost per unit
s	Shortage cost per unit
l	Lost sale cost per unit
A	Ordering cost per order
δ	Rate of backlogging
R	Sales revenue per replenishment cycle
I(t)	The inventory level at time t.
F (t_1, d)	Profit per unit time

t_1^*, d^*, $F^*(t_1,d)$,Q^*, DQ^* represents the optimal values of t_1,d, F(t_1,d),Q,DQ respectively.

The behavior of the inventory level during cycle T is depicted in figure1.



(Figure1)

The differential equations showing the fluctuation of inventory with time t are shown as below:

$$\frac{dI(t)}{dt} = -\theta t I_1(t) - at(T-t) \qquad 0 \le t \le t_1 \qquad \ldots (1)$$

$$\frac{dI(t)}{dt} = -\frac{at\alpha(T-t)}{1+\delta(T-t)} \qquad t_1 \le t \le T \qquad \ldots (2)$$

With boundary condition $I(t_1) = 0$ the solutions of these equations are given by:

$$I(t) = a\{(\frac{Tt_1^2}{2} - \frac{t_1^3}{3} + \frac{\theta T t_1^4}{8} - \frac{\theta T t_1^5}{10}) + (-\frac{T}{2} - \frac{\theta T t_1^4}{4} + \frac{\theta t_1^6}{6})t^2 + \frac{\theta T}{8}t^4 - \frac{\theta}{15}t^5\} \quad 0 \le t \le t_1 \qquad \ldots (3)$$

$$I(t) = a(1-d)^{-n}(\frac{(t_1^2 - t^2)}{2\delta} + \frac{(t_1 - t)}{\delta^2} + \left(\frac{1+\delta T}{\delta^3}\right)\{\log[1+\delta(T-t_1)] - \log[1+\delta(T-t)]\})$$

$$t_1 \le t \le T \qquad \ldots (4)$$

Now if $F(t_1,d)$) is the unit time profit function then:

$F(t_1,d) = \frac{1}{T}$ [Sales revenue – purchasing cost – deterioration cost – shortage cost -Lost sale cost – holding cost – ordering cost]

Sales revenue R= $pQ_1 + p(1-d)Q_2$ where

$$Q_1 = p\int_0^{t_1} at(T-t)dt$$

$$Q_1 = \frac{apTt_1^2}{2} - \frac{apt_1^3}{3} \qquad \ldots (5)$$

$$Q_2 = -I(T)$$

$$Q_2 = a(1-d)^{-n}(1-d)\left(\frac{\left(T^2-t_1{}^2\right)}{2\delta}+\frac{\left(T-t_1\right)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta\left(T-t_1\right)\right]\right) \quad \ldots(6)$$

$$R = ap\{\frac{t_1{}^2T}{2}-\frac{t_1{}^3}{3}+\left(1-d\right)^{-n}\left(1-d\right)\left(\frac{\left(T^2-t_1{}^2\right)}{2\delta}+\frac{\left(T-t_1\right)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta\left(T-t_1\right)\right]\right)\} \quad \ldots(7)$$

Purchasing cost PC = (Q_1+Q_2) C =C $(I_1(0) + (-I_2(T))$

$$= aC\left(\frac{t_1{}^2T}{2}-\frac{t_1{}^3}{3}+\frac{\theta T t_1{}^4}{8}-\frac{\theta t_1{}^5}{10}+\left(1-d\right)^{-n}\left(\frac{\left(T^2-t_1{}^2\right)}{2\delta}+\frac{\left(T-t_1\right)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta\left(T-t_1\right)\right]\right)\right) \quad \ldots(8)$$

Deterioration Cost DC $= ac\{\frac{\theta T t_1{}^4}{8}-\frac{\theta t_1{}^5}{10}\}$ … (9)

Holding Cost HC $= h\int_0^{t_1} I(t)dt = ah\left(\frac{Tt_1{}^3}{3}-\frac{t_1{}^4}{4}-\frac{\theta T t_1{}^5}{15}-\frac{\theta t_1{}^6}{18}\right)$ … (10)

Shortage cost SC $= as\left(\frac{T^3}{6}-\frac{t_1{}^2T}{2}+\frac{t_1{}^3}{3}\right)$ … (11)

Lost Sale Cost LSC $= al\int_{t_1}^{T}(t(T-t)-\frac{\alpha t(T-t)}{1+\delta(T-t)})dt$

$$= al\left(1-d\right)^{-n}\{\left(\frac{t_1}{\delta^2}+\frac{t_1{}^2}{2\delta}+\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta\left(T-t_1\right)\right]-\frac{T^2}{2\delta}-\frac{T}{\delta^2}\}+l(\frac{T^3}{6}-\frac{t_1{}^2T}{2}+\frac{t_1{}^3}{3}\right) \quad \ldots(12)$$

Ordering Cost OC = A … (13)

Total Profit per Unit Time

$$F\left(t_1,d\right)=\frac{a}{T}\left\{p\{\frac{t_1{}^2T}{2}-\frac{t_1{}^3}{3}+\left(1-d\right)^{-n}\left(1-d\right)\left(\frac{\left(T^2-t_1{}^2\right)}{2\delta}+\frac{\left(T-t_1\right)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta\left(T-t_1\right)\right]\right)\}\right.$$

$$-C\left(\frac{t_1{}^2T}{2}-\frac{t_1{}^3}{3}+\frac{\theta T t_1{}^4}{8}-\frac{\theta t_1{}^5}{10}+\left(1-d\right)^{-n}\left(\frac{\left(T^2-t_1{}^2\right)}{2\delta}+\frac{\left(T-t_1\right)}{\delta^2}-\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta\left(T-t_1\right)\right]\right)\right)$$

$$-c\{\frac{\theta T t_1{}^4}{8}-\frac{\theta t_1{}^5}{10}\}-h\left(\frac{Tt_1{}^3}{3}-\frac{t_1{}^4}{4}-\frac{\theta T t_1{}^5}{15}-\frac{\theta t_1{}^6}{18}\right)-s\left(\frac{T^3}{6}-\frac{t_1{}^2T}{2}+\frac{t_1{}^3}{3}\right)$$

$$\left.-l\left(1-d\right)^{-n}\{\left(\frac{t_1}{\delta^2}+\frac{t_1{}^2}{2\delta}+\left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta\left(T-t_1\right)\right]-\frac{T^2}{2\delta}-\frac{T}{\delta^2}\}+l(\frac{T^3}{6}-\frac{t_1{}^2T}{2}+\frac{t_1{}^3}{3}\right)-A\right\}$$

… (14)

Order Quantity

$$\mathbf{Q} = a\left(\frac{t_1^{\,2}T}{2} - \frac{t_1^{\,3}}{3} + \frac{\theta T t_1^{\,4}}{8} - \frac{\theta t_1^{\,5}}{10} + (1-d)^{-n}\left(\frac{(T^2 - t_1^{\,2})}{2\delta} + \frac{(T-t_1)}{\delta^2} - \left(\frac{1+\delta T}{\delta^3}\right)\log\left[1+\delta(T-t_1)\right]\right)\right)$$

… (15)

SOLUTION PROCEDURE

In the model unit time profit is a function of two variables t_1 and d. To find out the optimal solution

$$\frac{\partial F(t_1,d)}{\partial t_1} = 0 \quad \frac{\partial F(t_1,d)}{\partial d} = 0$$

… (16)

The optimal values of t_1 and d are obtained by solving these equations simultaneously provided

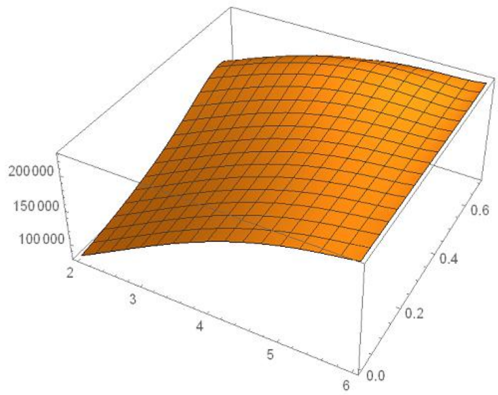
$$\frac{\partial^2 F(t_1,d)}{\partial t_1^2}\cdot\frac{\partial^2 F(t_1,d)}{\partial d^2} - \left(\frac{\partial^2 F(t_1,d)}{\partial t_1 \partial d}\right)^2 > 0$$

... (17)

NUMERICAL ILLUSTRATION

T=6 months, θ=0.009, δ =2 units, p=100, s=0.9rs/unit, l=1.2 rs/unit, a=600, A = 200rs/ order h=3.2 rs/unit, n=3, C=26, c=5.

Applying the solution procedure described above the optimal values obtained is as follows:
t_1*= 4.6750, d^* =0.628, $F^*(t_1,d)$=227996rs, Q^* =40186.6 units, DQ^*=728.68 units



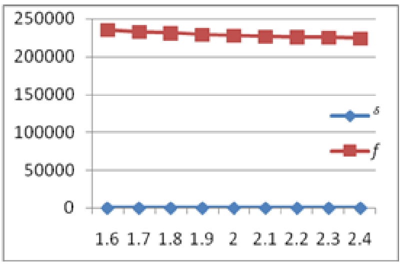
3D Graph shows the concavity of the unit time profit function $F(t_1,d)$

Effects of parameters, a & p) **on Total Profit per Unit Time**

From the numerical Illustration given above, the effects of changing different parameters like h, a & p on time t_1^* and discount on unit selling price d^* total profit per unit time $F^*(t_1,d)$ are studied.

Effects of parameter" δ "on Total Profit per Unit Time

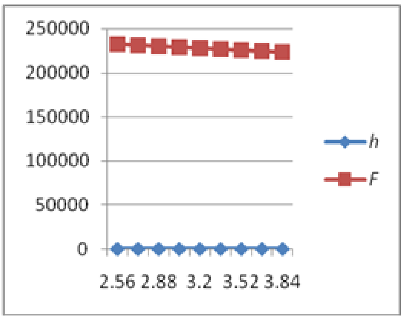
% variation in δ	δ	t_1	d	$F(t_1,d)$
-20%	1.6	4.2857	0.628	234912
-15%	1.7	4.395599	0.628	232716
-10%	1.8	4.496241	0.628	230872
-5%	1.9	4.589017	0.628	229315
0%	2	4.6750	0.628	227996
5%	2.1	4.75514	0.628	226874
10%	2.2	4.83019	0.628	225917
15%	2.3	4.900824	0.628	225099
20%	2.4	4.9676436	0.628	224400



(Table1)Variation in F (t_1, d) with the variation in δ

Effects of parameter "h" on Total Profit per Unit Time

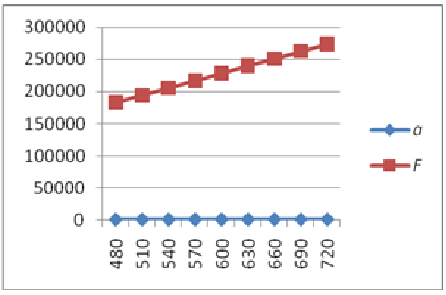
% variation in h	h	t_1	d	$F(t_1,d)$
-20%	2.56	4.72034	0.628	232603
-15%	2.72	4.70965	0.628	231447
-10%	2.88	4.698534	0.628	230294
-5%	3.04	4.686999	0.628	229144
0%	3.2	4.6750	0.628	227996
5%	3.36	4.66256	0.628	226850
10%	3.52	4.64964	0.628	225708
15%	3.68	4.6362	0.628	224568
20%	3.84	4.62232	0.628	223432



(Table 2) Variation in F (t_1, d) with the variation in h

Effects of "a" on Total Profit per Unit Time

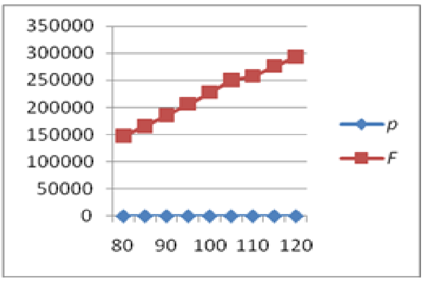
% variation in a	a	$F(t_1,d)$
-20%	480	182397
-15%	510	193796
-10%	540	205196
-5%	570	216596
0%	600	227996
5%	630	239395
10%	660	250795
15%	690	262195
20%	720	273595



(Table 3) Variation in F (t_1, d) with the variation in a

Effects of parameter "p" on Total Profit per Unit Time

% change p	p	t_1	d	$F(t_1,d)$
-20	80	5.40314	0.529944	147410
-15	85	5.41125	0.56317	166329
-10	90	5.05992	0.586667	185864
-5	95	4.8519	0.608421	206421
0	100	4.675	0.628	227996
5	105	4.50939	0.645714	250725
10	110	5.88216	0.661818	258042
15	115	5.90164	0.676522	276162
20	120	5.91672	0.69	294259



(Table 4) Variation in F (t_1, d) with the variation in p

Sensitivity Analysis

parameter	% change	%change t_1	%change d	%change $F(t_1,d)$
a	**-20**	**0**	**0**	**-0.1999**
	-10	**0**	**0**	**-0.1**
	10	**0**	**0**	**0.0999**
	20	**0**	**0**	**0.1999**
h	**-20**	**0.00969**	**0**	**0.0202**
	-10	**0.00503**	**0**	**0.1007**
	10	**-0.0054**	**0**	**-0.010035**
	20	**-0.01128**	**0**	**-0.02001**
δ	**-20**	**-0.08327**	**0**	**0.0303**
	-10	**-0.03824**	**0**	**0.01261**
	10	**0.03317**	**0**	**-0.009118**
	20	**0.0625**	**0**	**-0.01577**
p	**-20**	**0.1557**	**-0.15614**	**-0.3534**
	-10	**0.08233**	**-0.06581**	**-0.18479**
	10	**0.25821**	**0.05385**	**0.13178**
	20	**0.2656**	**0.09872**	**0.290632**

(Table 5)

OBSERVATIONS

1. From table (1) it is observed that as the rate of backlogging (δ) decreases, the unit time profit of The system increases.
2. From table (2) it is observed that as holding cost decreases the unit time profit of the system Increases
3. From table (3) it has been noticed that an increment in demand coefficient (a) left unchanged d & $t_1$and increases the unit time profit of the system.
4. Table (4) reveals that as the selling price (p) increases, the unit time profit of the system also
5. From sensitivity table (5) it has been seent_1 is not sensitive to (a) but it shows small sensitivity to(δ),(h) & (p).d is not sensitive to (a),(δ)&(h)but it is sensitive to selling price (p). F (t_1, d) is Small sensitive (a),(δ)& (h) and shows more sensitivity to selling price (p).

SPECIAL CASE OF THE MODAL (NO DISCOUNT d=0)

In this case for no discount(d=0), the profit function F(t_1,d) becomes a function of single variable t_1 that is F(t_1).The solution procedure to find out the optimal values is given by $\frac{dF(t_1)}{dt_1} = 0$ provided $\frac{d^2F(t_1)}{dt_1^2} < 0$ .This gives optimal values as follows: $t_1^* = 5.7864, F^*(t_1) = 219980, Q^* = 22620.1 \& DQ^* = 1037.38$

IV. Conclusion

In the present paper an inventory model is developed with realistic features of seasonal quadratic demand which starts with zero and end with zero and end with zero, time dependent deterioration, partial backlogging. The most important feature of the modal is the declaration of price discount at the start of shortage period so that demand is boosted in this period and more customers will be willing to wait for the next replenishment. Thus in turn the discount on unit selling price increases optimum total profit per unit time. A numerical illustration is given to describe the model. Special case for no discount case for the model is discussed and the numerical illustration with no discount shows that given optimal price discount at the start of shortage period is profitable. Effect of some parameters involved in the problem on optimal discount and total profit per unit has been discussed through tables and graphs. Sensitivity analysis with respect to above parameters has been carried out. Results noticed in tables and graphs are suitable to real situations. The model could be useful in retail business of seasonal products where partial backlogging occurs. Declaration of discount on unit selling price is the key factor to enhance the demand and to increases the total back ordered quantity. The present study can be further extended for some other factor useful for inventory problems.

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