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Applications of Codense and Compatible Ideals

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Abstract: *In this paper, we will discuss the further properties of some new types of set which have been defined with the help of ideal. We also characterize these sets, generalized continuities and $*$ - extremally disconnectedness in ideal topological spaces.*
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I. Introduction

The subject of ideals in topological spaces has been studied by Kuratowski [22] and Vaidyanathaswamy [35]. An ideal on a topological space (X,τ) as a non-empty collection I of subsets of X satisfying the following two conditions:
(1) If $A \in I$ and $B \subset A$, then $B \in I$;
(2) If $A \in I$ and $B \in I$, then $A \cup B \in I$.

An ideal topological space [7] is a topological space (X,τ) with an ideal I on X, and is denoted by (X,τ,I). An ideal in an ideal topological space (X,τ,I) is called a codense ideal [8] if $I \cap \tau = \{\emptyset\}$. Although Newcom [31], Jankovic and Hamlett [18] have used this as τ-boundary where as Dontchev [6] calls such spaces as Hayasi-Samuel spaces. In fact such ideals play very important role in the study of ideal topological spaces. Some remarkable results have been considered in [26, 27, 18, 15, 34, 25, 30, 29, 28, 13] using this ideal. Another important ideal is compatible ideal which had been introduced by Jankovic and Hamlett [19]. Mathematicians like Rose, Hamlett and Jankovic in [19, 34] and Modak and Bandyopadhyay in [26, 27, 5] have studied it extensively. An ideal I is said to be compatible with τ, denoted by $I\sim\tau$, for each $A \subset X$ and for each $a \in A$ there exists a neighbourhood U of a such that $U \cap A \in I$ then $A \in I$.

For a subset $A \subset X$, the set $A^*(I) = \{x \in X : U \cap A \notin I\}$ for every $U \in \tau$ with $x \in U$, is called the local function of A with respect to I and τ [22]. We simply write A^* instead of $A^*(I)$ in case there is no chance of confusion. It is well known that $Cl^*(A) = A \cup A^*$ defines a kuratowski closure operator for $\tau^*(I)$ and one of its base is $\{U - A : U \in \tau, A \in I\}$. It is interesting that $\tau^*(I) = \{U - A : U \in \tau, A \in I\}$ when $\tau \sim I$ [19]. In this ideal topological space a set A is called $* -$ dense [8] if $Cl^*(A) = X$, and the set A is called I - open if $A \subset Int\,(A^*)$Again in ideal topological space, Modak and Bandyopadhyay [26, 27] have proved two remarkable results: $Cl(O) = O^*$, for the open set O if and only if $I \cap \tau = \{\emptyset\}$; and for any nonempty $G \in \tau^*(I), Cl^*(G) = Cl(G)$, when I is codense and $I\sim \tau$.

The study of generalized sets in the topological space and in the ideal topological space are the important part in the study of topological spaces.
Let space (X,τ) be a topological space and $A \subset X$. A is said to be preopen [24] (resp. α - open [32], semi - open [23], semi - preopen [3], b - open [2]) if $A \subset Int(Cl(A))$ $\left(resp. A \subset Int\left(Cl(Int(A))\right), A \subset ClIntA,\ A\subset ClIntClA,\ A\subset IntClA\cup ClIntA,\right.$ where Cl and Int denote as the closure and interior operators respectively. The collection of all preopen (resp. α- open, semi - open, semi - preopen , b - open) sets in (X,τ) is denoted by $PO(X,\tau)$ $(resp.\ \alpha O(X,\tau),\ SO(X,\tau),\ SPO(X,\tau),\ BO(X,\tau)$).

A subset A of an ideal topological space (X,τ,I) is said to be pre - I-open [6] (resp. semi - I - open [17], α- I - open [17], β- I - open [17], b - I - open [14], strongly β - I - open [16], t - I - set [17], semi - I - regular [21], AB_I - set [21], weakly I - locally closed ([20] [21]) if $A \subset Int(Cl^*(A))$ $(resp.\ A \subset Cl^*(Int(A)),\ A \subset Int(Cl^*(Int(A))),\ A \subset Cl(Int(Cl^*(A))),$ $A \subset Int(Cl^*(A)) \cup Cl^*(Int(A)),\ A \subset Cl^*(Int(Cl^*(A))),\ Int(A) = Int(Cl^*(A))$, A is a t - set and semi - I - open, $A = U \cap V$ where U is open and V is semi - I - regular, $A = U \cap K$ where U is open and K is $*$ - closed). The collection of all pre - I - open (resp. semi - I - open,α - I - open, b - I - open, Weakly I - locally closed, strongly β - I - open) sets is denoted by $PIO(X)$ $(resp.\ SIO(X),\ \alpha\,IO(X),\ BIO(X),\ WILC(X),$ $s\,\beta IO(X))$.
A space (X,τ) is called extremally disconnected if $Cl(O)$ is open for every $O \in \tau$, and the space called submaximal space if every dense set is open.
An ideal topological space (X,τ,I) is said to be $*$ - extremally disconnected [10] if the $Cl^*(O)$ is open, for every open set O, and it is called I - submaximal [11] if every $*$ - dense subset of X is open.

II. $*$-Extremally Disconnected Spaces

Theorem 2.1. *For an ideal topological space (X,τ,I) following hold:*
(1) Every semi - open set is b - I - open, when I is codense.
(2) Every preopen set of $(X, \tau^*(I))$ is b - I - open, when I is codense.
(3) Every I - open set is b - I - open, when I is codense.
(4) Every $b - I -$ open set is semi - preopen (or β - open) set of $(X, \tau^*(I))$, when $I \sim \tau$ and I is condense.
Proof. The proof is straightforward from Proposition 1 of [14].

If I is a condense ideal in the ideal topological space (X,τ,I) , then the concept of $*$ - maximally disconnected and the extremally disconnected are coincident.

Let (X,τ,I) be the extremally disconnected space. Then for $V \in \tau$, $Cl(V)$ is open. From [26, 27], $V^* = Cl(V)$ is open. This implies that $V \cup V^*$ is open, and hence $Cl^*(V)$ is open. So the space (X,τ,I) is the $*$ - extremally disconnected. Reciprocally suppose that the space (X,τ,I) is $*$ - extremally disconnected. Then for $V \in \tau$, $Cl^*(V)$ is open. From [26, 27], $V^* = Cl(V)$, since I is codense. So $Cl^*(V) = V \cup V^* = Cl(V)$, and it is open.

Theorem 2.2. *For an ideal topological space (X,τ,I) where I is codense, then the following properties are equivalent:*
(1) X is extremally disconnected,
(2) $Cl^*(Int(V)) \subset Int(Cl^*(V))$ for every subset V of X.
Proof. Proof is obvious from [10].
Theorem 8 of [10] can be restated by the following, using extremally disconnectedness.

Theorem 2.3. *Let (X,τ,I) be an extremally disconnected ideal topological space, where I is codense. Then for $A \subset X$, the following properties are equivalent:*
(1) A is an open set,
(2) A is α - I - open and weakly I - local closed,
(3) A is pre - I - open and weakly I - local closed,
(4) A is semi - I - open and weakly I - local closed,
(5) A is b - I - open and weakly I - local closed.

Theorem 2.4. *Let (X,τ,I) be an extremally disconnected ideal space, where I is codense. Then for $A \subset X$, the following properties are equivalent:*
(1) A is an open set,
(2) A is α - I - open and a locally closed set,
(3) A is pre - I - open and a locally closed,
(4) A is semi - I - open and a locally closed set,
(5) A is b - I - open and a locally closed set.
Proof. The proof of this theorem is obvious from Theorem 9 of [10] and [27].

Now we shall discuss about decompositions of continuous functions. For this we define following:
Definition 2.5. A function $f : (X,\tau,I) \rightarrow (Y,\sigma)$ is α - I - continuous [17, 18] (resp. pre - I - continuous [7], semi - I - continuous [17], b - I - continuous [14], $W_I LC$ - continuous [20], LC - continuous [12]) if $f^{-1}(V)$ is α - I - open (resp. pre - I - open, semi - I - open, b - I - open, weakly I - locally closed, locally closed) for each open set V in Y .

If I is codense, then every semi-open set is a b - I - open set, and every preopen set of $\tau^*(I)$ is a b - I - open set. Therefore every semicontinuous function is a b - I - continuous function, and every precontinuous function on $(X,\tau^*(I))$ is always a b - I - continuous function.

We can replaced the term $*$ - extremally disconnected by extremally disconnected in Theorem 26 of [10].

Theorem 2.6. *For a function $f : (X,\tau,I) \rightarrow (Y,\sigma)$, where (X,τ,I) is an extremally disconnected ideal space and I is codense, the following properties are equivalent:*
(1) f is continuous,
(2) f is α - I - continuous and $W_I LC$ - continuous,
(3) f is pre - I - continuous and $W_I LC$ - continuous,
(4) f is semi - I - continuous and $W_I LC$ - continuous,
(5) f is b - I - continuous and $W_I LC$ - continuous.
According to the intimation of codense ideal we can further discuss of Theorem 27 of [10].

Theorem 2.7. *Let $f : (X,\tau,I) \rightarrow (Y,\sigma)$ be a function, where (X,τ,I) is an extremally disconnected and I is codense. Then following properties are equivalent:*
(1) f is continuous,
(2) f is α - I - continuous and LC - continuous,
(3) f is pre - I - continuous and LC - continuous,
(4) f is semi - I - continuous and LC - continuous,
(5) f is b - I - continuous and LC - continuous.
We rewrite the Lemma 15 of [11] using codense ideal.

Theorem 2.8. *For an ideal space (X,τ,I) where I is codense, the following properties are equivalent:*
(1) X is extremally disconnected,
(2) Every semi - I - open set is pre - I - open,
(3) The $*$ - closure of every strongly β - I - open subset of X is open,
(4) Every strongly β - I - open set is pre - I - open.
We are ending this section with the following Remark for importance of codense ideal.
Remark 2.9. *Let (X,τ,I) be an ideal topological space with $I \cap \tau = \{\emptyset\}$. Then X is $*$-extremally disconnected if and only if it is extremally disconnected.*
Proof. Proof is obvious from Theorem 20 of [10].

III. *I*-Submaximal Spaces

In previous section, we have interrelated the $*$ - extremally disconnected spaces and the extremally disconnected spaces. In this section we try to interrelate the submaximal spaces and the I - submaximal spaces in presence of restriction on ideal. Although Modak in [26, 27] have proved following:
Theorem 3.1. *If (X,τ) is extremally disconnected and submaximal then $(X,\tau^*(I))$ is extremally disconnected and submaximal, where $I \sim \tau$ and $I \cap \tau = \{\emptyset\}$.*

He has also given an example for encounter of submaximality of $\tau^*(I)$ implies submaximality of τ in the space (X,τ,I).
Here we further discuss the properties of I - submaximality.
Theorem 3.2. *For a subset A of an ideal space (X,τ,I) where I is codense, the following properties are equivalent:*
(1) $A \in PO\big(X,\tau^*(I)\big)$,
(2) $A = G \cap B$, where G is open and B is $*$ - dense.

Theorem 3.3. *Let (X,τ,I) be an ideal space, where I is codense and $A \subset X$. Then A is α - I - open if and only if it is semi-open and pre - I - open in $\big(X,\tau^*(I)\big)$.*
Theorem 3.4. *For an ideal topological space (X,τ,I) where I is codense, the following properties are equivalent:*
(1) X is I - submaximal,
(2) Every pre - open set in $\big(X,\tau^*(I)\big)$ is open,
(3) Every pre - open set in $\big(X,\tau^*(I)\big)$ is semi – open and every α – open set is open.
Proof. We know from Theorem 3 of [11]:
X is I - submaximal ⇒ Every pre - I- open set is open ⇒ Every pre - I- open set is semi - I -open and every α - I - open set is open.
Then X is I - submaximal ⇒ Every pre - open set in $\big(X,\tau^*(I)\big)$ is open ⇒ Every pre – open set in $\big(X,\tau^*(I)\big)$ is semi-open and every α- open set is open (since I is condense).
Proof of the reverse part is similar.
Theorem 3.5. *For a subset A of an I - submaximal ideal space (X,τ,I) where I is codense, the following are equivalent:*
(1) A is semi – open,
(2) A is strongly β - I - open.
Theorem 3.6. *For an ideal space (X,τ,I) where I is codense, the following are equivalent:*
(1) X is I - submaximal,
(2) Every pre – open set in $\big(X,\tau^*(I)\big)$ is an AB_I – set,
(3) Every $*$ - dense set is an AB_I - set.
Proof. We know from Theorem 10 of [11],
X is I – submaximal ⇔ every pre $- I -$ open set is an AB_I - set ⇔ Every $*$ - dense set is an AB_I - set.
Therefore we get,

X is I – submaximal $\Leftrightarrow$ every pre-open set in $(X, \tau^*(I))$ is an AB_I – set $\Leftrightarrow$ Every $*$ -dense set is an AB_I - set.
Theorem 3.7. *For an ideal space (X, τ, I) where I is codense, the following are equivalent:*
(1) X is I - submaximal and extremally disconnected,
(2) Any subset of X is strongly $\beta - I -$ open if and only if it is open.

Theorem 3.8. *For an ideal space (X, τ, I), where I is codense, if X is I -submaximal and extremally disconnected, the following are equivalent for a subset $A \subset X$:*
(1) A is strongly $\beta - I$ – open,
(2) A is semi $- I$ – open,
(3) A is pre $- I$ – open,
(4) A is $\alpha - I$ – open,
(5) A is open.
Proof. Proof is obvious from Corollary 17 of [11].
Theorem 3.9. *Every AB_I - set is semi-open in an ideal topological space (X, τ, I), where I is codense.*
Proof. Proof is obvious from Lemma 18 of [11].
Theorem 3.10. *For an ideal space (X, τ, I), where I is codense, if X is I - submaximal and extremally disconnected, the following properties are equivalent for a subset $A \subset X$:*
(1) A is semi – $I -$ open ,
(2) A is an AB_I - set .
Proof. Proof is similar with the Theorem 19 of [11].

IV. RI-Open Sets

At the starting of this section we shall discuss the following sets which are already in literature.

Let (X, τ, I) be an ideal space. A subset A of X is said to be a semipre* - I - closed [33] (resp. RI - open set [36]) if $Int(Cl^*(Int(A))) \subset A$ ($resp.$ $A = Int(Cl^*(A))$). We will denote the family of all RI - open sets by $RIO(X)$.

A subset A of an ideal space (X, τ, I) is said to be a $\mathcal{A}_{1I}$ [33] (resp. $\mathcal{B}_{1I}$ [4]) if $A = U \cap V$ where U is open (resp. α- I - open) and $Cl^*(Int(V)) = X$. We will denote the family of all $\mathcal{B}_{1I}$ -sets (resp. $\mathcal{A}_{1I}$ - sets) by $\mathcal{B}_{1I}(X)$ (resp. $\mathcal{A}_{1I}(X)$).

A subset A of an ideal space (X, τ, I) is said to be a $\mathcal{B}_I$ - set [17] (resp. $\mathcal{C}_I$ - set [17]) if $A = U \cap V$ where U is open and V is a $t - I -$ set (resp. semipre*$- I -$ closed set). We will denote the family of all $\mathcal{B}_I$ - set (resp. $\mathcal{C}_I$ - set) by $\mathcal{B}_I(X)$ (resp. $\mathcal{C}_I(X)$).

A subset A of an ideal space (X, τ, I) is said to be a $\mathcal{A}_{2I}$ - set [4] (resp. $\mathcal{B}_{2I}$ - set [4] ($\alpha_1 M_2$ - set [1]) if $A = U \cap V$ where U is open (resp. $\alpha - I$ – open) and $Cl^*(V) = X$. We will denote the family of all $\mathcal{A}_{2I}$ - sets (resp. $\mathcal{B}_{2I}$ - sets) by $\mathcal{A}_{2I}(X)$ (resp. $\mathcal{B}_{2I}(X)$). Clearly $\mathcal{A}_{2I}(X) \subset \mathcal{B}_{2I}(X)$.

A subset A of an ideal space (X, τ, I) is said to be an $\alpha_I N_5$ - set [1] if $A = U \cap V$ where U is $\alpha - I -$ open and V is $*$ - closed. We will denote the family of all $\alpha_I N_5$ - sets of an ideal space (X, τ, I) by $\alpha_I N_5(X)$.

A subset A of an ideal space (X, τ, I) is said to be IR - closed set [36] if $A = Cl^*(Int(A))$.

A subset A of an ideal space (X, τ, I) is said to be an $\alpha\mathcal{A}_I$ - set [4] (resp. $\mathcal{A}_{IR}$ - set [1]) if $A = U \cap V$ where U is an $\alpha - I -$ open (resp. open) set and V is an IR - closed. $\mathcal{A}_{IR}$ - sets are called as $\mathcal{A}_I$ - sets in [4].

A subset A of an ideal space (X, τ, I) is said to be a $\mathcal{A}_{3I}$ - set [33] if $A = U \cap V$ where U is open and $Cl^*(Int(V)) \subset V$. We will denote the family of all $\mathcal{A}_{3I}$ - sets by $\mathcal{A}_{3I}(X)$.

Now we shall discuss some characterizations of the above sets.
Theorem 4.1. *Let (X, τ, I) be an ideal space (X, τ, I) , where I is codense and $A \subset X$. Then*
$\mathcal{B}_{1I}(X) = \alpha IO(X) = \mathcal{A}_{1I} = \tau^\alpha$.
Proof. Proof is similar with the Theorem 2.8 of [33] and the condition $O^* = Cl(O)$ for open set O.
Theorem 4.2. *Let (X, τ, I) be an ideal space where I is codense. Then following are equivalent:*
(1) A is pre – $I -$ open,
(2) A is pre - open in $(X, \tau^*(I))$,
(3) There exists an RI- open set G such that $A \subset G$ and $Cl^*(G) = Cl^*(A)$,
(4) $A = G \cap D$ where G is RI – open and D is $*$ - dense,
(5) $A = G \cap D$ where G is open and D is $*$ - dense.
Proof. Proof is obvious from the Theorem 2.9 of [33] and [26, 27].
Theorem 4.3. *Let (X, τ, I) be an ideal space, where I is codense. Then $\mathcal{A}_{2I}(X) = PIO(X) = \mathcal{B}_{2I}(X) = PO(X, \tau^*(I))$.*
Proof. Proof is obvious from the [26, 27].

Theorem 4.4. *Let (X,τ,I) be an ideal space, where I is codense and $A \subset X$. Then A is $\alpha_I N_5$ - set if and only if $A = U \cap Cl^*(A)$ for some $U \in \tau^\alpha$.*

Proof. Proof is obvious from Theorem 2.13 of [33] and [26, 27].

Theorem 4.5. *Let (X,τ,I) be an ideal space where I is codense. Then $\alpha\mathcal{A}_I(X) = s\beta IO(X) \cap \alpha_I N_5(X) = SIO(X) = SO(X)$.*

Proof. Proof is obvious from the fact that $I \cap \tau = \{\emptyset\}$ if and only if $O^* = Cl(O)$ for any open set O.

Theorem 4.6. *Let (X,τ,I) be an ideal space and $A \subset X$, where I is codense. Then following are equivalent:*

(1) A is $\alpha -$ open.

(2) A is $\alpha - I -$ open.

(3) A is pre $- I -$ open and semi $- I -$ open.

(4) A is a $\mathcal{B}_{2I}$ – set and $\alpha\mathcal{A}_I -$ sets.

Proof. Proof is obvious from Proposition 1.1, Theorem 2.3 of [4], Corollary 2.15 of [33] and [26, 27].

Theorem 4.7. *Let (X,τ,I) be an ideal space, where I is codense and $A \subset X$. Then the following are equivalent:*

(1) A is open.

(2) A is $\alpha - I -$ open and $\mathcal{A}_{IR}$ – set.

(3) A is $\alpha -$ open and $\mathcal{A}_{IR}$ – set.

(4) A is pre – open set in $(X,\tau^*(I))$ and $\mathcal{A}_{IR}$ – set.

(5) A is $\alpha - I -$ open and weakly I - locally closed.

(6) A is $\alpha -$ open and $\mathcal{B}_I$ – set.

(7) A is $\alpha - I -$ open and $\mathcal{C}_I$ – set.

Proof. Proof is obvious from Corollary 2.16 of [33] and [26, 27].

Theorem 4.8. *Let (X,τ,I) be an ideal space, where I is codense and $A \subset X$. Then the following are equivalent.*

(1) $A \in \mathcal{A}_{IR}(X)$.

(2) $A \in SO(X) \cap \mathcal{A}_{3I}(X)$.

(3) $A \in \alpha\mathcal{A}_I(X) \cap \mathcal{A}_{3I}(X)$.

(4) $A \in s\beta IO(X) \cap \alpha_I N_5(X) \cap \mathcal{A}_{3I}(X)$.

(5) $A \in s\beta IO(X) \cap WILC(X)$.

Proof. Proof is obvious from Corollary 2.20 of [33] and [26, 27].

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