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Abstract. In this paper an inventory model is formulated and analyzed for random deteriorating items with the assumption of life period of time. Demand rate is taken to be a linear function of the on hand inventory. Replenishment is assumed to be instantaneous. The model is developed for infinite time horizon. Shortages are allowed and they are partially backlogged. The backlogging rate is taken to be inversely proportional to the waiting time for the next replenishment. Two special cases also have been discussed. d and they are partially backlogging rate is taken to be inversely proportional to the next replenishment. Two special cases also have been discussed. The waiting time for the next replenishment. Two special cases also have been discussed.

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I. Introduction

The effect of deterioration is very important in many inventory systems. Deterioration is defined as decay or damage such that the item can not be used for its original purpose. Food items, pharmaceuticals and radioactive substances are example of items in which sufficient deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the system. Since deterioration of an item depends upon the fluctuation of humidity, temperature etc. It would be more reasonable and realistic if we assume the deterioration function θ to depend upon a parameter " γ " in addition to t which ranges over a space "[" and in which probability density function $p(\gamma)$ is defined. In many inventory models demand rate are either constant or time dependent but independent of the stock level. However for certain types of commodities particularly consumer goods, the demand rate of may be depend on the on hand inventory. For this type of commodity - the sale would increase as the amount of inventory increase. Gupta and Vrat (1986) assumed the demand rate to be function of order quality. Baker and Urban (1988), Mandal and Phaujdar (1989) have assumed the demand rate to be the function of the on hand inventory. In all these papers the deterioration of items and the possibility of shortages have not been taken into account. Some researchers developed models for deteriorating items with initial stock level dependent demand and instantaneous stock level dependent demand. Dutta and Pal (1990) formulated inventory model for deteriorating items with demand rate to be a linear function of a hand inventory. Goyal and Giri (2001) presented an EOQ model for deteriorating items with time varing demand and partial backlogging. Perumal (2002) presented inventory model with two rates of production and backorders. Teng el al (2003) developed inventory model for deteriorating items with time varying demand anpartial backlogging. In the present paper an inventory model is presented considering random rate of deteriorating with concept of life period of an item, instantaneous stock level dependent demand rate, and shortages with partial backlogging. The replenishment rate is taken to be instantaneous and time horizon is considered to be infinite.

II. Notations

- 1. C: Set up cost for each replenishment.
- 2. C_1 : Holding cost per unit per unit time.
- 3. C_2 : Shortage cost per unit per unit time.
- 4. C_3 : Cost of each unit.
- 5. I: Opportunity cost due to last sales per unit.
- 6. T: Planning horizon.
- 7. Q: Total amount of inventory produced or purchased at the beginning of each production cycle.
- 8. S: Initial inventory after fulfilling back orders.
- 9. q (t) : Inventory level at time t.
- 10. t_1 : Time at which shortages starts.
- 11. μ : Life period of the item.
- 12. $TC(t_1,T)$: The total average cost per unit time for each replenishment cycle.

III. Assumptions

1. The replenishment occurs instantaneously at an infinite rate.

2. There is no repair or replacement of deteriorated items.

3. The system involves only a single item and the lead time in zero.

4. The demand rate function (D(t)) is deterministic and it is a known function of instantaneous stock level (q(t)). Demand rate function D(t) is given by:-

$$\mathbf{D}(t) = \begin{bmatrix} \alpha + \beta \mathbf{q}(t) & , 0 \le t \le t_1 \\ \alpha & , 0 \le t \le t_1 \end{bmatrix}$$

where $\alpha > 0$ and $0 < \beta < 1$

5. Shortages are allowed and the backlogging rate is $\frac{\alpha}{1 + \lambda(T - t)}$ when inventory is in shortage. The backlogging parameter λ is a positive constant and $t_1 \le t \le T$.

6. A variable fraction $Q(t, \gamma)$ of the on hand inventory deteriorates per unit time only after the expiry of the life period μ of the item.

The deterioration function θ is taken as follows:-

$$\theta = \theta_0(\gamma, t) H(t-\mu) , t, \mu > 0$$

Where $H(t - \mu)$ is Heaviside unit function defined as follows :-

$$\begin{aligned} H(t-\mu) = 1 &, t \ge \mu \\ = 0 &, t \ge \mu \end{aligned}$$

and

$$\theta_0(\gamma, t) = \theta_0(\gamma) t \quad , 0 < \theta_0(\gamma) << 1 \quad , t > 0$$

is a special form of two parameter Weibull function considered by Covert and Philip. The function is some function of the random variable γ which ranges over a space Γ and in which a probability density function $p(\alpha)$ is defined such that—

$$\int_{\Gamma} \mathbf{p}(\boldsymbol{\gamma}) \, d\boldsymbol{\gamma} = 1$$

IV. Mathematical Formulation And Analysis Of The System:

Let Q be the total amount of inventory produced or purchased at the beginning of each period, and after fulfilling let us assume we get an amount S(>0) as initial inventory. During the period (0, μ). The inventory level gradually decreases due to the market demand only. Thereafter during the period (μ , t_1) the inventory level further decreases due to the market demand and deterioration till it reaches to zero at t = t_1 . The period (t_1 , T) is the period of shortages which are fully backlogged. The differential equations governing the inventory level q(t) at any time t during the production cycle [0, T] are given by:-

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} + \theta_0(\gamma)t \quad q(t) = -(\alpha + \beta q(t)), 0 \le t \le t_1 \qquad \dots \dots \dots (2)$$

The boundary conditions are given by-

(i) q(t) = S $\mathbf{t} = \mathbf{O}$ \mathbf{at} (ii) q(t) $= q(\mu)$ at $t = \mu$

(iii) q(t) = 0 at $t = t_1$ solution of equation (1) is given by:-

$$q(t)e^{\beta t} = -\int \alpha e^{\beta t} dt + A \qquad 0 \le t \le \mu$$

When A is some constant of integration:

Using boundary condition (i) at t = 0, q(t) = S one can get –

$$\mathbf{q}(t) = \left(s + \frac{\alpha}{\beta}\right) e^{-\beta t} - \frac{\alpha}{\beta}, \quad 0 \le t \le \mu \qquad \dots (4)$$

Now apply boundary condition (ii) at $t=\mu,\,q(t)=q(\mu)$

$$q(\mu) = \left(S + \frac{\alpha}{\beta}\right) e^{-\beta\mu} - \frac{\alpha}{\beta}$$
Solution of equation (2) is given by

Solution of equation (2) is given by-

$$q(t)e^{\beta t} \cdot e^{\theta_0(\gamma)\frac{t^2}{2}} = -\int_{\mu}^{t} \alpha e^{\left(\beta t + \theta_0(\gamma)\frac{t^2}{2}\right)} dt + B$$

where B is some constant of integration.

Using boundary condition (ii) at $t = \mu$, $q(t) = q(\mu)$ one can get-

$$\mathbf{q(t)}\mathbf{e}^{\beta t + \theta_0(\gamma)\frac{t^2}{2}} = -\int_{\mu}^{t} \alpha \mathbf{e}^{\left(\beta t + \theta_0(\gamma)\frac{t^2}{2}\right)} dt$$
$$+ \left[\left(\mathbf{S} + \frac{\alpha}{\beta}\right) \mathbf{e}^{-\beta\mu} - \frac{\alpha}{\beta} \right] \mathbf{e}^{\beta\mu} + \theta_0(\gamma)\frac{\mu^2}{2}$$

Since $\theta_0(\gamma)(0 < \theta_0(\gamma) << 1)$ and $\beta(0 < \beta << 1)$ are small so one can neglect second and higher powers of β and $\theta_0(\gamma)$ and their products.

Therefore-

$$\mathbf{q}(\mathbf{t})\mathbf{e}^{\left(\beta\mathbf{t}+\theta_{0}(\gamma)\frac{\mathbf{t}^{2}}{2}\right)} = -\int_{\mu}^{\mathbf{t}} \alpha \left(1+\beta\mathbf{t}+\theta_{0}(\gamma)\frac{\mathbf{t}^{2}}{2}\right) d\mathbf{t}$$
$$+\left[\left(\mathbf{S}+\frac{\alpha}{\beta}\right)\mathbf{e}^{-\beta\mu}-\frac{\alpha}{\beta}\right]\mathbf{e}^{\beta\mu}+\theta_{0}(\gamma)\frac{\mu^{2}}{2}$$
$$\int_{\Omega}^{\Omega} \mathbf{r} \left(\mathbf{t}+\frac{\beta\mathbf{t}^{2}}{2}+\frac{\theta_{0}(\gamma)\mathbf{t}^{3}}{2}-\mu-\frac{\beta\mu^{2}}{2}-\frac{\theta_{0}(\gamma)\mu^{3}}{2}\right)\left(1-\beta\mathbf{t}+\frac{\theta_{0}(\gamma)\mathbf{t}^{2}}{2}\right)$$

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$$q(t) = -\alpha \left[t + \frac{\beta t^2}{2} + \frac{\theta_0(\gamma)t^3}{6} - \mu - \frac{\beta \mu^2}{2} - \frac{\theta_0(\gamma)\mu^3}{6} \right] \left(1 - \beta t + \frac{\theta_0(\gamma)t^2}{2} \right)$$

$$+\left[\left(\mathbf{S}+\frac{\alpha}{\beta}\right)-\frac{\alpha}{\beta}\,\mathbf{e}^{\beta\mu}\,\right]\mathbf{e}^{-\left[\beta\mathbf{t}+\frac{\theta_{0}(\gamma)}{2}\left(\mathbf{t}^{2}-\mu^{2}\right)\right]}\qquad\dots(6)$$
$$=-\alpha\left[\mathbf{t}+\frac{\beta\mathbf{t}^{2}}{2}+\frac{\theta_{0}(\gamma)\mathbf{t}^{3}}{3}-\mu-\frac{\beta\mu^{2}}{2}-\frac{\theta_{0}(\gamma)\mu^{3}}{6}+\beta\mu\mathbf{t}+\theta_{0}(\gamma)\mu\frac{\mathbf{t}^{2}}{2}\right]$$
$$+\left[\left(\mathbf{S}+\frac{\alpha}{\beta}\right)-\frac{\alpha}{\beta}\left(\mathbf{1}+\beta\mu\right)\right]\left(\mathbf{1}-\beta\mathbf{t}-\frac{\theta_{0}(\gamma)}{2}\left(\mathbf{t}^{2}-\mu^{2}\right)\right)\qquad\dots(7)$$

By applying boundary conditions at $t = t_1$, $q(t_1) = 0$ from equation (6) one can obtain the value of S as-

$$S = \alpha \left[t_1 - \frac{\beta t_1^2}{2} - \frac{\theta_0(\gamma) t_1^3}{3} - \mu - \frac{\beta \mu^2}{2} - \frac{\theta_0(\gamma) \mu^3}{6} + \beta \mu t_1 + \frac{\theta_0(\gamma) \mu t_1^2}{2} \right]$$

$$\left(1 + \beta t_1 + \frac{\theta_0(\gamma) (t_1^2 - \mu^2)}{2} \right) + \alpha \mu$$

$$= \alpha \left[\left(1 - \frac{\theta_0(\gamma) \mu^2}{2} \right) t_1 + \frac{\beta t_1^2}{2} + \frac{\theta_0(\gamma) t_1^3}{6} - \frac{\beta \mu^2}{2} + \frac{\theta_0(\gamma) \mu^3}{3} \right]$$
......(8)

Using value of S from equation (8) in equation (7) one can get the solution equation (2) as-

Solution of equation (3) is given by-

$$q(t) = \int \frac{\alpha}{1 + \lambda(T - t)} dt + E$$

Where E is some constant of integration. By applying boundary condition at $t = t_1$, q(t) = 0 the solution is given by-

During period (0, t₁) total number of units holding can be obtained as-

$$q_{H} = \int_{0}^{\mu} q(t) dt + \int_{\mu}^{t_{1}} q(t) dt$$
$$= \int_{0}^{\mu} \left[\left(S + \frac{\alpha}{\beta} \right) e^{-\beta t} - \frac{\alpha}{\beta} \right] dt + \alpha \int_{\mu}^{t_{1}} \left[\frac{\theta_{0}(\gamma) t^{3}}{3} - (\beta - \theta_{0}(\gamma) t_{1}) \frac{t^{2}}{2} \right]$$

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67 | Page

Total amount of deteriorated units (q_{D}) during the period ($\ 0, t_{1}$) is given by-

Total amount of deteriorated units (q_D) during the period ($0, t_1$) is given by-

$$\begin{aligned} \mathbf{q}_{\mathrm{D}} &= \mathbf{q}_{\mu} - \int_{\mu}^{t_{1}} \left[\alpha + \beta \mathbf{q}(\mathbf{t}) \right] d\mathbf{t} \\ &= \left[\left(\mathbf{S} + \frac{\alpha}{\beta} \right) \mathbf{e}^{-\beta\mu} - \frac{\alpha}{\beta} \right] - \int_{\mu}^{t_{1}} \left[\alpha + \beta \alpha \left(\frac{\theta_{0}(\gamma) \mathbf{t}^{3}}{3} \right. \\ &+ \left(\beta - \theta_{0}(\gamma) \mathbf{t}_{1} \right) \frac{\mathbf{t}^{2}}{2} - \left(1 + \beta \mathbf{t}_{1} \right) \mathbf{t} + \frac{\theta_{0}(\gamma) \mathbf{t}_{1}^{3}}{6} + \frac{\beta \mathbf{t}_{1}^{2}}{2} + \mathbf{t}_{1} \right) \right] d\mathbf{t} \end{aligned}$$

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Total amount of shortage units (q_s) during the period (t_1, T) is given by-

Total amount of lost sales (q_L) during the period (t_1, T) is given by-

Total average cost of the system per unit time $TC(t_1, T)$ is given by-

To minimize total average cost per unit time $TC(t_1, T)$ the optimal values of t_1 and T (say t_1^* and T^*) can be obtained by solving the following equations simultaneously.

and

Provided they satisfy the following sufficient conditions

$$\frac{\partial^2 \operatorname{TC}(t_1, T)}{\partial t_1^2} \rangle 0 \quad , \quad \frac{\partial^2 \operatorname{TC}(t_1, T)}{\partial t^2} \rangle 0$$

and

$$\left(\frac{\partial^2 \operatorname{TC}(t_1, T)}{\partial t_1^2}\right) \left(\frac{\partial^2 \operatorname{TC}(t_1, T)}{\partial T^2}\right) - \left(\frac{\partial^2 \operatorname{TC}(t_1, T)}{\partial t_1, \partial T}\right)^2 \rangle 0$$
Equation (17) and (18) are equivalent to

Equation (17) and (18) are equivalent to

$$\frac{1}{T} \left[C_1 \alpha \left(\frac{\theta_0(\gamma) t_1^3}{3} + \frac{\beta t_1^2}{2} + t_1 - \frac{\theta_0(\gamma) \mu^3}{3} \right) + C_3 \alpha \right]$$
$$\left(\frac{\theta_0(\gamma) t_1^2}{2} - \frac{\theta_0(\gamma) \mu^2}{2} \right) - \frac{(C_2 + I\lambda) \alpha (T - t_1)}{1 + \lambda (T - t_1)} = 0$$

and

$$\begin{split} &-\frac{1}{T} \Bigg[C + C_1 \alpha \Bigg(\frac{\theta_0(\gamma) t_1^4}{12} + \frac{\beta t_1^3}{6} + \frac{t_1^2}{2} - \frac{\theta_0(\gamma) \mu^3 t_1}{3} + \frac{\theta_0(\gamma) \mu^4}{4} - \frac{2}{3} \beta \mu^3 \Bigg) + \\ &C_3 \alpha \Bigg(\frac{\theta_0(\gamma) t_1^3}{6} - \frac{\theta_0(\gamma) \mu^2 t_1}{2} + \frac{\theta_0(\gamma) \mu^3}{3} \Bigg) + \frac{(C_2 + I\lambda) \alpha}{\lambda^2} \\ &(\lambda (T - t_1) - \log(1 + \lambda (T - t_1))) \Bigg] + \frac{\alpha (C_2 + I\lambda) (T - t_1)}{T (1 + \lambda (T - t_1))} = 0 \qquad \dots (20) \end{split}$$

Equation (19) and (20) can be solved by some suitable computational numerical method with the help of computer to obtain the optional values (t_1 * and T*). With the use of these optimal values equation (16) provides minimum total average cost per unit time TC(t_1 *, T*) of the system in consideration.

V. Special Cases

CASE 1- If the planning horizon is finite, then the cost function TC (t_1,T) in the model presented will convert into TC (t_1) and condition for minimization of the total average cost per unit time TC (t_1) becomes-

It is equivalent to -

$$C_{1}\alpha \left[\frac{\theta_{0}(\gamma)t_{1}^{3}}{3} + \beta \frac{t_{1}^{2}}{2} + t_{1} - \frac{\theta_{0}(\gamma)\mu^{3}}{3}\right] + C_{3}\alpha$$

$$\left[\frac{\theta_{0}(\gamma)t_{1}^{2}}{2} - \frac{\theta_{0}(\gamma)\mu^{2}}{2}\right] - \frac{(C_{2} + I\lambda)\alpha(T - t_{1})}{1 + \lambda(T - t_{1})} = 0$$

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.....(19)

.....(22)

The value of t_1 obtained by equation (21) is the optimal value of $t = t_1^*$ provided it satisfy the sufficient condition-

It is equivalent to-

$$\frac{C_{1}\alpha \left[\Theta_{0}(\gamma)t_{1}^{2}+\beta t_{1}+1\right]+C_{3}\left[\Theta_{0}(\gamma)t_{1}\right]+}{\left(C_{2}+I\lambda\right)\alpha(1+\lambda)(T-t_{1})-\left(C_{2}+I\lambda\right)\alpha(T-t_{1})\lambda}{\left(1+\lambda(T-t_{1})\right)^{2}}\rangle0$$

.....(24)

The optimal value of $t = t_1^*$ obtained from equation (21) will give the minimum total average cost per unit time. *CASE 2-* If $Q_0(\gamma)t = Qt$ then the system reduces to the variable rate of deterioration model with partial backlogging.

VI. Conclusion

In this paper can inventory model is formulated and analyzed for random deteriorating items with the assumption of life period of time. Demand rate is taken to be a linear function of the on hand inventory. Replenishment is assumed to be instantaneous. The model is developed for infinite time horizon. Shortages are allowed and they are partially backlogged. The backlogging rate is taken to be inversely proportional to the waiting time for the next replenishment. Two special cases also have been discussed. This model can further be extended for other demand rates, different deterioration rates, without life time situations and with the condition of permissible delay in payments. The effect of inflation and time value of money can also be investigated for the discussed problem.

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