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Common Fixed Point Theorem under Sub Compatibility and Sub Sequentially Continuous Mappings in Menger Spaces by Using Implicit Relation

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Abstract: *In this paper we establish Common fixed point theorem for six mappings in Menger spaces by using implicit relation under the notion of sub compatibility and sub sequentially continuity.*
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I. Introduction

In 1942, K. Menger [16] introduced the notion of probabilistic metric space (briefly, PM-space) as a generalization of metric space. Such a probabilistic generalization of metric spaces appears to be well adapted for the investigation of physical quantities and physiological thresholds. It is also of fundamental importance in probabilistic functional analysis. The development of fixed point theory in PM-spaces was due to Schweizer and Sklar[23, 24].

In 1972, V. M. Sehgal and A. T. Bharucha-Reid [25] initiated the study of contraction mappings on probabilistic metric (briefly, PM) spaces. Since then there has been a massive growth of fixed point theorems using certain conditions on the mappings or on the space itself. Sesa [26] introduced weakly commuting maps in metric space. In 1986, Jungck [13] introduced the notion of compatible mappings in metric spaces, And this condition has further been weakened by introducing the notion of weakly compatible mappings by Jungck and Rhoades [14].

Pant [21] noticed these criteria for fixed points of contraction mappings and introduced a new continuity condition, known as reciprocal continuity and obtain a common fixed point theorem by using the compatibility in metric spaces. He also showed that in the setting of common fixed point theorems for compatible mappings satisfying contraction conditions, the notion of reciprocal continuity is weaker than the continuity of one of the mappings. Later on, Jungck and Rhoades [14] termed a pair of self maps to be coincidently commuting or equivalently weakly compatible if they commute at their coincidence points.

Recently, Bouhadjera and Godet-Thobie [4] introduced two new notions namely sub sequential continuity and sub compatibility which are weaker than reciprocal continuity and compatibility respectively (see also [3, 5]). Further, Imdad et al. [12] improved the results of Bouhadjera and Godet-Thobie [4] and showed that these results can easily recovered by replacing sub compatibility with compatibility or sub sequential continuity with reciprocally continuity. Several interesting and elegant results have been obtained by various authors in different settings [e.g. 5, 7, 8, 11, 12, 15, 19, 27]. Many authors [1, 2, 10] proved several fixed point theorems in Menger spaces and showed the applications of corresponding results in metric spaces. Most recently Pant and Chauhan[9]established a common fixed point theorem in Menger space using the notion of compatibility and sub sequentially continuous mapping of a pair of self maps. In this paper we generalize and extend the result of Pant and Chauhan[9] for six mappings in Menger space using the concept of sub compatibility and sub sequentially continuity by using implicit relation.

II. Preliminaries

Definition 2.1. [23] A probabilistic metric space (PM-space) is an ordered pair (X,F) consisting of a non empty set X and a function $F: X \times X \rightarrow L$, where L is the collection of all distribution functions and the value of F at $(u,v) \in X \times X$ is represented by $F_{u,v}$. The function $F_{u,v}$ is assumed to satisfy the following conditions:

(PM- 1) $F_{u,v}(x) = 1$,for all $x > 0$ if and only if $u = v$

(PM- 2) $F_{u,v}(0) = 0$;

(PM- 3) $F_{u,v} = F_{v,u}$;

(PM- 4) $F_{u,v}(x) = 1$and $F_{v,w}(y) = 1$ then $F_{u,w}(x+y) = 1$for all u,v,w in X and $x, y \geq 0$.

Definition 2.2. [23] A mapping $*$:[0, 1] x [0, 1] →[0, 1] is called a t- norm if

(a) $*(a,1) = a$ for all $a \in [0,1]$

(b) $*(a,b) = *(b,a)$ (symmetric property)

(c) $*(c,d) \geq *(a,b)$ for $c \geq a$, $d \geq b$

(d) $*(*(a,b),c) = *(a,*(b,c))$

Definition 2.3.[23] A Menger space is a triplet (X, F, $*$) where (X, F) is a PM- space and $*$ is a T-norm such that the inequality

$F_{u,w}(x+y) \geq * \{ F_{u,v}(x) , F_{v,w}(y) \}$ for all u, v, w in X and $x, y > 0$

Definition 2.4. [18] The self maps A and B of a Menger space $(X, F, *)$ are said to be compatible if

$F_{ABx_n,BAx_n}(t) \rightarrow 1$ for all $t > 0$ whenever $\{x_n\}$ is a sequence in X such that Ax_n , $Bx_n \rightarrow z$ for some $z \in X$ as $n \rightarrow \infty$.

Definition 2.5.[28] Self –maps A and S of a Menger space (X, F, t) are said to be weak compatible (or coincidentally commuting) if they commute at their coincidence points i.e. if $Ap = Sp$ for some $p \epsilon$ N then $ASp = SAp$

Remark: Two compatible self mappings are weakly compatible, however the converse is not true in general.

Definition 2.6.[4]: A pair of self mappings (A, S) defined on a Menger space $(X, F, *)$ is said to be sub compatible if there exists a sequence $\{x_n\}$ such that

$$\mathrm{Lim}_{n\rightarrow\infty} A\,x_n = \lim_{n\rightarrow\infty} S\,x_n = z,$$

For some $z \in X$ and $\lim_{n\rightarrow\infty} F_{ASx_n,SAx_n}(t) = 1$, for all $t > 0$.

Definition 2.7.[4]: A pair of self mappings (A, S) defined on a Menger space $(X, F, *)$ is called subsequentially continuous if there exists a sequence $\{x_n\}$ such that

$$\mathrm{Lim}_{n\rightarrow\infty} A\,x_n = \lim_{n\rightarrow\infty} S\,x_n = z$$

For some $z \in X$ and $\mathrm{Lim}_{n\rightarrow\infty} AS\,x_n = Az$ and $\mathrm{Lim}_{n\rightarrow\infty} SA\,x_n = Sz$,

Implicit Relations [2.8] : In [17], Mihet established a fixed point theorem concerning probabilistic contractions satisfying an implicit relation. This implicit relation is similar to that in [22], In [22] Popa used the family F_4 of implicit real functions to find the fixed points of two pairs of semi compatible mapping in a d- compatible topological space . Here F_4denote the family of all real continuous functions $F : (R^+)^4 \rightarrow R$ satisfying the following properties:

(F_h) There exists $h \geq 1$ such that every $u \geq 0, v \geq 0$ with $F(u, v, u, v) \geq 0$ or $F(u, v, v, u) \geq 0$, we have $u \geq hv$.

$(F_u) F(u, u, 0, 0,) < 0$for all $u > 0$.

In our result , we deal with the class $\boldsymbol{\Phi}$ of all real continuous functions $\varphi : (R^+)^4 \rightarrow R$, non-decreasing in the first argument and satisfying the following conditions :

I For $u, v \geq 0, \varphi(u, v, u, v) \geq 0$ or $\varphi(u, v, v, u) \geq 0$ implies that $u \geq v$.

II $\varphi(u, u, 1, 1) \geq 0$ for all $u \geq 1$.

Example : Define $\varphi(t_1, t_2, t_3, t_4) = at_1 + bt_2 + ct_3 + dt_4, a, b, c, d \in R$ with $a + b + c + d = 0, a > 0, a + c > 0, a + b > 0$ and $a + d > 0$. Then $\varphi \in \Phi$.

Example : Define $\varphi(t_1, t_2, t_3, t_4) = 20t_1 - 18t_2 + 6t_3 - 8t_4$. Then $\varphi \in \Phi$.

The following theorem proved by Chauhan Sunny and Pant B.D.[9]

Theorem : Let A, B, S and T be self maps of a Menger space (X, F, Δ), where Δ is a continuous t-norm. If the pairs (A, S) and (B, T) are compatible and sub sequentially continuous, then

1. The pair (A, S) has a coincident point,
2. The pair (A, S) has a coincident point,
3. There exists a constant $k \in (0, 1)$such that

$$F_{Ax,By}(kt) \geq min\{F_{Sx,Ty}(t), F_{Ax,Sx}(t), F_{By,Ty}(t), F_{Ax,Ty}(t), F_{By,Sx}(t)\}$$

For all $x, y \in X$ and $t > 0$, then A, B, S and T have a unique common fixed point in X.

In this section we extend the theorem for six mapping by using implicit relation.

III. Main Result

Theorem (3.1): Let A, B, P, Q, S and T be self maps of a Menger space $(X, F, *)$, where $*$ is a continuous t-norm defined by $t * t \geq t$ for all $t \in [0, 1]$. If the pairs (P, AB) and (Q, ST) are sub compatible and sub sequentially continuous and satisfying

3.1.1 The pairs $(P,T),(AB,T),(Q,B),(ST,B)$ are commutes,

3.1.2 For some $\varphi \in \Phi$, there exists $k \in (0,1)$ such that for all $x,y \in X$ and $t > 0$

$$\varphi\left(F_{Px,Qy}(kt), F_{ABx,STy}(t), F_{Px,ABx}(t), F_{Qy,STy}(kt)\right) \geq 0$$

Then the pairs (P,AB) and (Q,ST) have a coincident point each. Moreover A,B,P,Q,S and T have a unique common fixed point in X.

Proof : Since the pair (P,AB) is sub compatible and sub sequentially continuous then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} ABx_n = z, z \in X \text{ and satisfy}$$

$$\lim_{n\to\infty} F_{P(AB)x_n,(AB)Px_n}(t) = \lim_{n\to\infty} F_{Pz,ABz}(t) = 1$$

For all $t > 0$ then $Pz = ABz$, whereas in respect of the pair (Q,ST) is sub compatible and sub sequentially continuous then there exists a sequence $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Qy_n = \lim_{n\to\infty} STy_n = w, w \in X \text{ and satisfy}$$

$$\lim_{n\to\infty} F_{Q(ST)y_n,(ST)Qy_n}(t) = \lim_{n\to\infty} F_{Qw,STw}(t) = 1$$

For all $t > 0$ then $Qw = STw$. Hence z is a coincident point of the pair (P,AB) and w is a coincident point of the pair (Q,ST).

Now we prove that $z = w$, By putting$x = x_n$ and $y = y_n$ in 3.1.2 we get

$$\varphi\left(F_{Px_n,Qy_n}(kt), F_{ABx_n,STy_n}(t), F_{Px_n,ABx_n}(t), F_{Qy_n,STy_n}(kt)\right) \geq 0$$

Taking the limit as $n \to \infty$, we get

$$\varphi\left(F_{z,w}(kt), F_{z,w}(t), F_{z,z}(t), F_{w,w}(kt)\right) \geq 0$$

$$\varphi\left(F_{z,w}(kt), F_{z,w}(t), 1, 1\right) \geq 0$$

As φ is non-decreasing in the first argument, we have

$$\varphi\left(F_{z,w}(t), F_{z,w}(t), 1, 1\right) \geq 0$$

In view of implicit relation 2.8 we get

$F_{z,w}(t) \geq 1$ for all $t > 0$. This gives $F_{z,w}(t) = 1$

we get $z = w$

Now we prove that $Pz = z$ then we putting $x = z$ and $y = y_n$ in 3.1.2 we get

$$\varphi\left(F_{Pz,Qy_n}(kt), F_{ABz,STy_n}(t), F_{Pz,ABz}(t), F_{Qy_n,STy_n}(kt)\right) \geq 0$$

Taking the limit as $n \to \infty$, and using $Pz = ABz$ we get

$$\varphi\left(F_{Pz,w}(kt), F_{Pz,w}(t), F_{Pz,Pz}(t), F_{w,w}(kt)\right) \geq 0$$

$$\varphi\left(F_{Pz,w}(kt), F_{Pz,w}(t), 1, 1\right) \geq 0$$

As φ is non-decreasing in the first argument, we have

$$\varphi\left(F_{Pz,w}(t), F_{Pz,w}(t), 1, 1\right) \geq 0$$

In view of implicit relation 2.8 we get

$F_{Pz,w}(t) \geq 1$ for all $t > 0$. This gives $F_{Pz,w}(t) = 1$

we get $Pz = w = z$ Therefore $Pz = ABz = z$, Now we assert that $Qz = z$, then by putting $x = x_n$ and $y = z$ in 3.1.2 we get

$$\varphi\left(F_{Px_n,Qz}(kt), F_{ABx_n,STz}(t), F_{Px_n,ABx_n}(t), F_{Qz,STz}(kt)\right) \geq 0$$

Taking the limit as $n \to \infty$, and using $Qz = STz$ we get

$$\varphi\left(F_{z,Qz}(kt), F_{z,Qz}(t), F_{z,z}(t), F_{Qz,Qz}(kt)\right) \geq 0$$

$$\varphi\left(F_{z,Qz}(kt), F_{z,Qz}(t), 1, 1\right) \geq 0$$

As φ is non-decreasing in the first argument, we have

$$\varphi\left(F_{z,Qz}(t), F_{z,Qz}(t), 1, 1\right) \geq 0$$

In view of implicit relation 2.8 we get

$F_{z,Qz}(t) \geq 1$ for all $t > 0$. This gives $F_{z,Qz}(t) = 1$ thus $z = Qz$

we get, $Qz = STz = z$In all $Pz = ABz = Qz = STz = z$

Now we claim that $Tz = z$, by putting $x = Tz$ and $y = z$ in 3.1.4 we get

$$\varphi\left(F_{PTz,Qz}(kt), F_{ABTz,STz}(t), F_{PTz,ABTz}(t), F_{Qz,STz}(kt)\right) \geq 0$$

Using 3.1.1 the pairs (P, T) and (AB, T) are commutes, i.e. $PT = TP$ and $(AB)T = T(AB)$ we have

$$\varphi\left(F_{TPz,Qz}(kt), F_{TABz,STz}(t), F_{TPz,TABz}(t), F_{Qz,STz}(kt)\right) \geq 0$$

$$\varphi\left(F_{Tz,z}(kt), F_{Tz,z}(t), F_{Tz,Tz}(t), F_{z,z}(kt)\right) \geq 0$$

$$\varphi\left(F_{Tz,z}(kt), F_{Tz,z}(t), 1, 1\right) \geq 0$$

As φ is non-decreasing in the first argument, we have

$$\varphi\left(F_{Tz,z}(t), F_{Tz,z}(t), 1, 1\right) \geq 0$$

In view of implicit relation 2.8 we get

$F_{Tz,z}(t) \geq 1$ for all $t > 0$. This gives $F_{Tz,z}(t) = 1$ thus $Tz = z$ and $STz = z$ implies $Sz = z$

Now we claim that $Bz = z$, by putting $x = z$ and $y = Bz$ in 3.1.2 we get

$$\varphi\left(F_{Pz,QBz}(kt), F_{ABz,STBz}(t), F_{Pz,ABz}(t), F_{QBz,STBz}(kt)\right) \geq 0$$

Using 3.1.1 the pairs (Q, B) and (ST, B) are commutes, i.e. $QB = BQ$ and $(ST)B = B(ST)$ we have

$$\varphi\left(F_{Pz,BQz}(kt), F_{ABz,BSTz}(t), F_{Pz,ABz}(t), F_{BQz,BSTz}(kt)\right) \geq 0$$

$$\varphi\left(F_{z,Bz}(kt), F_{z,Bz}(t), F_{z,z}(t), F_{Bz,Bz}(kt)\right) \geq 0$$

$$\varphi\left(F_{z,Bz}(kt), F_{z,Bz}(t), 1, 1\right) \geq 0$$

As φ is non-decreasing in the first argument, we have

$$\varphi\left(F_{z,Bz}(t), F_{z,Bz}(t), 1, 1\right) \geq 0$$

In view of implicit relation 2.8 we get

$F_{z,Bz}(t) \geq 1$ for all $t > 0$. This gives $F_{z,Bz}(t) = 1$ *Thus* $Bz = z$ and $ABz = z$ implies $Az = z$

Thus in all $z = Pz = Qz = Az = Bz = Sz = Tz$, i.e. z is a common fixed point of P, Q, A, B, S and T.

Uniqueness of z: Let $z^{'}(z \neq z^{'})$ be another common fixed point of P, Q, A, B, S and T ; then $z^{'} = Pz^{'} = Qz^{'} = Az^{'} = Bz^{'} = Sz^{'} = Tz^{'}$

Putting $x = z$ and $y = z^{'}$ in 3.1.2 we get

$$\varphi\big(F_{Pz,Qz^{'}}(kt), F_{ABz,STz^{'}}(t), F_{Pz,ABz}(t), F_{Qz^{'},STz^{'}}(kt)\big) \geq 0$$

$$\varphi\big(F_{z,z^{'}}(kt), F_{z,z^{'}}(t), F_{z,z}(t), F_{z^{'},z^{'}}(kt)\big) \geq 0$$

$$\varphi\big(F_{z,z^{'}}(kt), F_{z,z^{'}}(t), 1, 1\big) \geq 0$$

As φ is non-decreasing in the first argument, we have

$$\varphi\big(F_{z,z^{'}}(t), F_{z,z^{'}}(t), 1, 1\big) \geq 0$$

In view of implicit relation 2.8 we get

$F_{z,z^{'}}(t) \geq 1$ for all $t > 0$. This gives $F_{z,z^{'}}(t) = 1$ we get $z = z^{'}$

Thus z is a unique common fixed point of P, Q, A, B, S and T. This completes the proof of the theorem.

Corollary (3.2): Let A, P, Q and S be self maps of a Menger space $(X, F, *)$, where $*$ is a continuous t-norm defined by $t * t \geq t$ for all $t \in [0,1]$. If the pairs (P, A) and (Q, S) are sub compatible and sub sequentially continuous then

1. P and A have a coincidence point,
2. Q and S have a coincidence point,
3. For some $\varphi \in \Phi$, there exists $k \in (0,1)$ such that for all $x, y \in X$ and $t > 0$

$$\varphi\big(F_{Px,Qy}(kt), F_{Ax,Sy}(t), F_{Px,Ax}(t), F_{Qy,Sy}(kt)\big) \geq 0$$

Then A, P, Q and S have a unique common fixed point in X.

Corollary (3.3): Let A, P and S be self maps of a Menger space $(X, F, *)$, where $*$ is a continuous t-norm defined by $t * t \geq t$ for all $t \in [0,1]$. If the pairs (P, A) and (P, S) are sub compatible and sub sequentially continuous then

1. P and A have a coincidence point,
2. P and S have a coincidence point,
3. For some $\varphi \in \Phi$, there exists $k \in (0,1)$ such that for all $x, y \in X$ and $t > 0$

$$\varphi\big(F_{Px,Py}(kt), F_{Ax,Sy}(t), F_{Px,Ax}(t), F_{Py,Sy}(kt)\big) \geq 0$$

Then A, P and S have a unique common fixed point in X.

Corollary (3.4): Let P and S be self maps of a Menger space $(X, F, *)$, where $*$ is a continuous t-norm defined by $t * t \geq t$ for all $t \in [0,1]$. If the pairs (P, S) is sub compatible and sub sequentially continuous then

1. P and S have a coincidence point,
2. For some $\varphi \in \Phi$, there exists $k \in (0,1)$ such that for all $x, y \in X$ and $t > 0$

$$\varphi\big(F_{Px,Py}(kt), F_{Sx,Sy}(t), F_{Px,Px}(t), F_{Py,Sy}(kt)\big) \geq 0$$

Then P and S have a unique common fixed point in X. This completes the proof of the theorem.

Example 3.5. Let $X = [0, \infty)$ and d be the usual matric on X and for each $t \in [0,1]$ define

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|} & if\ t > 0 \\ 0, & if\ \ t = 0 \end{cases}$$

For all $x, y \in X$. Clearly $(X, F, *)$ is a Menger space. Now we define a self maps P and S on X by

$$P(X) = \begin{cases} \frac{x}{5} & if\ \ 0 \leq x \leq 3 \\ 4x - 9 & if\ \ 3 < x < \infty \end{cases} \quad \text{and} \quad S(X) = \begin{cases} \frac{x}{6} & if\ \ 0 \leq x \leq 3 \\ 3x - 6 & if\ \ 3 < x < \infty \end{cases}$$

Consider a sequence $x_n = \frac{1}{n}$ in X.

We have $\lim\limits_{n\to\infty} P(x_n) = \lim\limits_{n\to\infty} \frac{1}{5n} = 0 = \lim\limits_{n\to\infty} \frac{1}{6n} = \lim\limits_{n\to\infty} S(x_n)$

Next $\lim\limits_{n\to\infty} PS(x_n) = \lim\limits_{n\to\infty} P\left(\frac{1}{6n}\right) = \lim\limits_{n\to\infty} \left(\frac{1}{30n}\right) = 0 = P(0)$

$$\lim_{n\to\infty} SP(x_n) = \lim_{n\to\infty} S\left(\frac{1}{5n}\right) = \lim_{n\to\infty} \left(\frac{1}{30n}\right) = 0 = S(0)$$

And $\lim\limits_{n\to\infty} F_{PSx_n,SPx_n}(t) = 1$, for all $t > 0$.

Consider another sequence $x_n = 3 + \frac{1}{n}$. Then

$$\lim_{n\to\infty} P(x_n) = \lim_{n\to\infty} P\left(3 + \frac{1}{n}\right) = \lim_{n\to\infty} \left\{4\left(3 + \frac{1}{n}\right) - 9\right\} = 3$$

$$\lim_{n\to\infty} S(x_n) = \lim_{n\to\infty} S\left(2 + \frac{1}{n}\right) = \lim_{n\to\infty} \left\{3\left(3 + \frac{1}{n}\right) - 6\right\} = 3$$

Also

$$\lim_{n\to\infty} PS(x_n) = \lim_{n\to\infty} P\left(3 + \frac{1}{n}\right) = \lim_{n\to\infty} \left\{4\left(3 + \frac{3}{n}\right) - 9\right\} = 3 \neq P(3)$$

$$\lim_{n\to\infty} SP(x_n) = \lim_{n\to\infty} S\left(2 + \frac{1}{n}\right) = \lim_{n\to\infty} \left\{3\left(3 + \frac{4}{n}\right) - 6\right\} = 3 \neq S(3)$$

But $\lim\limits_{n\to\infty} F_{PSx_n,SPx_n}(t) = 1$. Thus the pair (P, S) is compatible as well as subsequential continuous, but not reciprocally continuous. Therefore all the conditions of Corollary 3.4 are satisfied for some $k \in (0,1)$. Here , 0 is a coincidence as well as unique common fixed point of the pair (P, S).

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References

[1] Ali J., Imdad M. and Bahaguna D., Common fixed point theorems in Menger spaces with common property (E.A.) Comput. Math. Appl. 60(12) (2010), 3152-3159.

[2] Ali J., Imdad M., Mihet D. and Tanveer M. Common fixed points of strict contractions in Menger spaces, Acta Math. Hungar. 132(4) (2011), 367-386.

[3] BouhadjeraH.,andDjoudi A., Common fixed point theorems for subcompatible D-maps of integral type. General Math. 18(4) (2010), 163-174.

[4] BouhadjeraH.,andGodet-Thobie C., Common fixed theorems for pairs of subcompatible maps, arXiv:0906.3159v 1[math. FA] 17 June (2009) [Old version].

[5] BouhadjeraH.,andGodet-Thobie C., Common fixed theorems for pairs of subcompatible maps, arXiv:0906.3159v 2[math. FA] 23 May (2011) [New version].

[6] Chang S. S. ,Cho.Y. J., and Kang. S. M. Nonlinear Operator Theory in Probabilistic Metric spaces. NovaScience Publishers, Huntington, USA, (2001).

[7] Chauhan S., and Kim J.K. Common fixed point theorems for compatible and sub sequentially continuous mappings in Menger spaces, Nonlinear Functional Anal. Appl. 18(2) (2013), 177-192.

[8] Chauhan S., and Kumar S. Common fixed point theorems for compatible and sub sequentially continuous mappings in fuzzy metric spaces, Kragujevac J. Math. 36(2) (2012), 225-235.

[9] Chauhan Sunny, Pant B.D., Fixed point theorems for compatible and subsequentially continuous mappings in Menger space J. Nonlinear Sci. Appl. 7(2014), 78-89.

[10] Fang J.-X. andGao Y., Common fixed point theorems under strict contractive conditions in Menger spaces, Nonlinear Anal. 70(1) (2009), 184-193.

[11] Gopal D. and Imdad M. Some new common fixed point theorems in fuzzy metric spaces, Ann. Univ. Ferrara Sez VII Sci. Mat. 57(2) (2011), 303-316.

[12] Imdad M., Ali J., and Tanveer M., Remarks on some recent material fixed point theorems, Appl. Math. Lett. 24(7) (2011), 1165-1169.

[13] Jungck G. Compatible mappings and common fixed points.Int. J. math.Math. Sci., 9:771:773, (1986)

[14] Jungck G. and Rhoades B.E. Fixed points for set valued functions without continuity. Indian J. Pure Appl. Math., 29:227:238,(1998)

[15] Khan M.A. and Sumitrasubcompatible and subsequentially continuous maps in fuzzy metric spaces, Appl. Math. Sci. 5(29) (2011), 1421-1430.

[16] Menger K. Statistical metric. Proc. Nat. Acad. (USA) 28:535:537, (1942).

[17] Mihet D.A generalization of a contraction principle in Probabilistic metric spaces, Part II, Int. J. Math.Math.Sci 2005(2005), 729-736.

[18] Mishra S.N., Common fixed points of compatible mappings in PM-spaces, Math. Japon. 36(1991), 283-289.

[19] Nashine H.K. and Imdad M., Common fixed point and invariant approximations for subcompatible mappings in convex metric spaces, Math.Commun. 16(1) (2011), 1-12.

[20] Pant B.D. and Kumar S., A common fixed point theorem in probabilistic metric using implicit relation, Filomat 22(2) (2008), 43-52.

[21] Pant R.P., Common fixed points of four mappings, Bull. Cal. Math. Soc. 90 (1998), 281-286

[22] Popa, V., 'A general coincidence theorem for compatible multi valued mappings satisfying an implicit relation', Demonsratio Math., No.32 pp159-164, (2000).

[23] Schweizer B. and Sklar A. Statistical metric spaces. Pacific J. Math., 10:313:334, (1960).

[24] Schweizer B. and Sklar A. Probabilistic metric spaces. Elsevier, North-Holland, New York, (1983).

[25] Sehgal V. M.andBharucha A. T. -Reid. Fixed point of contraction mappings on probabilistic metric spaces.Math. Syst. Theory, 6:97:102, (1972).

[26] Sessa S., On a weak commutativity condition of mappings in fixed point considerations, publ. Inst. Math. 32(1982), 149-153.

[27] Singh B., Jain A. and Wani A.A., Sub-compatibility and fixed point theorem in fuzzy metric space, Int. J. Math. Anal. Appl. (Ruse) 5 (27) (2011), 1301-1308.

[28] Singh B. and Jain S. A fixed point theorem in Menger space through weak compatibility. J. Math. Anal. Appl., 301 (2005), 439-448.

Dilip Kumar Gupta. "Common Fixed Point Theorem under Sub Compatibility and Sub Sequentially Continuous Mappings in Menger Spaces by Using Implicit Relation." IOSR Journal of Mathematics (IOSR-JM) 13.4 (2017): 92-99.