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Indrajeet's Law

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Abstract: *Indrajeet's Law is a general formula for n^{th} degree polynomial equation. We can find all real roots of n^{th} degree polynomial equation by Indrajeet's Law . In this law we used divisors and coefficients which make it easy to find the roots of n^{th} degree polynomial equation.*

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I. Introduction

The general form of the n^{th} degree polynomial equation is given by $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots+a_n = 0, a_0\neq0$ This equation has n roots, some of which may be real numbers, may be conjugate numbers or may be complex numbers (or imaginary numbers). As we know that , yet no any formula is given for find the roots of n^{th} degree polynomial equation (or fifth degree polynomial equation or grater than fifth degree polynomial equation), but now Indrajeet's Law is available for find the roots of n^{th} degree polynomial equation. As we know that a linear equation $ax + b = 0$ can be solve direct as $x = -\frac{b}{a}$ and a quadratic equation $ax^2 + bx + c = 0, a \neq 0$ also can be solve by many methods that are $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ we can find the roots of quadratic equation by using this formula and also we can find the roots of quadratic equatin by factor method and complete the square method. One most important method is 'Indrajeet's Rule for Quadratic Equation' this is developed by me which published in IOSR Journal of Mthematics in march 2016. Indrajeet's Law came from 'Indrajeet's Rule for Quadratic Equation'. Indrajeet's Law is generalization form of 'Indrajeet's Rule for Quadratic Equation' which works for n^{th} degree polynomial equation. We can find roots of n^{th} degree polynomial equation by Indrajeet's Law using coefficients and divisors of $(a_0{\cdot}a_n)$. We can find the roots of cubic equation $ax^3 + bx^2 + cx + d = 0, a \neq 0$ by factor theorem, if $(x - a)$ is a factor of above cubic equation, then we say that 'a' is a root of above cubic equation and then we using factor theorem we can find roots of cubic equation. One impotant method is cardano's method we can find the roots of cubic equation by using cardano's method. We can also find the roots of quartic equation $ax^4 + bx^3 + cx^2 + dx + e = 0, a \neq 0$ by Ferrari's method.

Finally, we know that any equation of which degree is 5 (or grater than 5) has (have) no any formula to solve that equation, but now Indrajeet's Law is available which works for all polynomial equations. This Indrajeet's Law does not work only for a linear equation.

II. Indrajeet's Law

Rule:

Step1: Find $(a_0{\cdot}a_n)$ and $(a_0)^{n-1}{\cdot}a_n$

Step2: Write all divisors of $(a_0{\cdot}a_n)$, where not required all divisors of $(a_0{\cdot}a_n)$, if you feel write all, then you can write.

Step3: Choose n numbers from the divisors of which multiplication is $(a_0)^{n-1}{\cdot}a_n$ and sum or difference is a_1

Step4: Change the sign of all chosen numbers.

Step5: Divide all the numbers by a_0

Keywords: Divisors and coefficients

Remarks:

1: This law will not work for linear equation.

2: If at least one root is conjugate or complex, then this law may be difficult.

3: If polynomial equation is in standard form in descending order and no any term is absent or no any term is equal to zero, then this law will work. (The last term a_n can be zero).

III. Factorization of n^{th} Degree Polynomial Equation

If we have $\alpha_1, \alpha_2, \alpha_3,\cdots,\alpha_n$ are roots of n^{th} degree polynomial equation $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n = 0$, $a_0\neq0$, then we can write n^{th} degree polynomial equation in factor form according to factor theorem, so n^{th} degree polynomial equation can be written in the factor form as $(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)\cdots(x-\alpha_n) = 0$.

IV. Examples

Example1: Solve the equation $4x^9 - 8x^8 - 93x^7 + 132x^6 + 714x^5 - 672x^4 - 1957x^3 + 1268x^2 + 1332x - 720 = 0$ and also factorise it.

Solution: Given equation is of the form $ax^9 + bx^8 + cx^7 + dx^6 + ex^5 + fx^4 + gx^3 + hx^2 + ix + j = 0$ this equation is colled 9th degree polynomial equation which is in standard form and no any term is equal to zero, now we will compare given question with this equation

By comparing we can write

$a = 4, \quad b = -8, \quad j = -720$

Now we use Indrajeet's Law,

Step1: $a \cdot j = 4 \times (-720) = -2880$

And $a^8 \cdot j = 4^8 \times (-720) = -47185920$

Step2: $\pm 1, \pm 2, \pm 3, \pm 4, \cdots$

Step3: $-2, -4, -8, -12, -16, 4, 8, 10, 12$

Here, sum of these numbers $= b = -8$

And multiplication of these numbers $= a^8 \cdot j = -47185920$

Step4: $2, 4, 8, 12, 16, -4, -8, -10, -12$

Step5: $\frac{2}{4}, \frac{4}{4}, \frac{8}{4}, \frac{12}{4}, \frac{16}{4}, \frac{-4}{4}, \frac{-8}{4}, \frac{-10}{4}, \frac{-12}{4}$

Hence, roots of given nth degree polynomial equation are given below

$$x = \left\{\frac{1}{2}, 1, 2, 3, 4, -1, -2, \frac{-5}{2}, -3\right\}$$

We can check the roots of the given example are correct or wrong, so we let $P(x) = 0$ is given example, then we checked that

$P\left(\frac{1}{2}\right) = 0,\ P(1) = 0,\ P(2) = 0,\ P(3) = 0,\ P(4) = 0,\ P(-1) = 0,\ P(-2) = 0,\ P\left(\frac{-5}{2}\right) = 0,\ P(-3) = 0$

So above roots are correct.

2nd part of question

Since, when $x = a$ is a root of any equation, then $(x - a)$ will be a factor of that equation according to factor theorem

Similarly, $x = \left\{\frac{1}{2}, 1, 2, 3, 4, -1, -2, \frac{-5}{2}, -3\right\}$ are the roots of the given equation, so factorization form of given equation be

$$\left(x - \frac{1}{2}\right)(x-1)(x-2)(x-3)(x-4)(x+1)(x+2)\left(x + \frac{5}{2}\right)(x+3) = 0$$

Example2: Solve the equation $2x^5 - 41x^4 + 306x^3 - 1015x^2 + 1396x - 480 = 0$

Solution: Given equation is of the form $ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$

Comparing the given question with above quintic equation,

Here, $a = 2, \quad b = -41, \quad f = -480$

Now, we use Indrajeet's Law

Step1: $a \times f = 2 \times (-480) = -960$

And $a^4 \times f = 2^4 \times (-480) = -7680$

Step2: $\pm 1, \pm 2, \pm 3, \cdots$

Step3: $-16, -8, -6, -10, -1$

Here, sum of these numbers$= b = -41$

And multiplication of these numbers $= a^4 \times f = -7680$

Step4: $16, 8, 6, 10, 1$

Step5: $\frac{16}{2}, \frac{8}{2}, \frac{6}{2}, \frac{10}{2}, \frac{1}{2}$

Hence, $x = \left\{8, 4, 3, 5, \frac{1}{2}\right\}$ are the roots of required equation.

Example3: Find the roots of the polynomial equation equation $4x^7 - 4x^6 - 49x^5 + 35x^4 + 161x^3 - 91x^2 - 116x + 60 = 0$

Solution: Given equation is of the form $ax^7 + bx^6 + cx^5 + dx^4 + ex^3 + fx^2 + gx + h = 0$

Comparing the given question with above 7th degree polynomial equation,

Here, $a = 4, \ b = -4, \ h = 60$

Now, we use Indrajeet's Law

Step1: $a \times h = 4 \times 60 = 240$

And $a^6 \times h = 4^6 \times 60 = 245760$

Step2: $\pm 1, \ \pm 2, \ \pm 3, \ \pm 4, \cdots$

Step3: $4, -12, -2, -4, -8, 8, 10$

Here, sum of these numbers $= b = -4$

and multiplication of these numbers $a^6 \times h = 245760$

Step4: $-4, 12, 2, 4, 8, -8, -10$

Step5: $-\frac{4}{4}, \ \frac{12}{4}, \ \frac{2}{4}, \ \frac{4}{4}, \ \frac{8}{4}, \ \frac{-8}{4}, \ -\frac{10}{4}$

Hence, $x = \left\{-1, \ 3, \ \frac{1}{2}, \ 1, \ 2, \ -2, \ -\frac{5}{2}\right\}$ are roots of the required equation.

Example4: Factorise the polynomial equation $8x^8 - 4x^7 - 102x^6 + 21x^5 + 357x^4 - 21x^3 - 323x^2 + 4x + 60 = 0$.

Solution: Given equation is of the form $ax^8 + bx^7 + cx^6 + dx^5 + ex^4 + fx^3 + gx^2 + hx + i = 0$

For factorization, first of all we use Indrajeet's Law and then we will use factor theorem

Comparing the given question with above 8th degree polynomial equation,

Here, $a = 8, \ b = -4$ and $i = 60$

Step1: $a \times i = 8 \times 60 = 480$

And $a^7 \times i = 8^7 \times 60 = 125829120$

Step2: $\pm 1, \ \pm 2, \ \pm 3, \cdots$

Step3: $8, -24, -4, -8, -16, 16, 20, 4$

Here, sum of these numbers $= b = -4$

and multiplication of these numbers $= a^7 \times i = 125829120$

Step4: $-8, 24, 4, 8, 16, -16, -20, -4$

Step5: $-\frac{8}{8}, \ \frac{24}{8}, \ \frac{4}{8}, \ \frac{8}{8}, \ \frac{16}{8}, \ -\frac{16}{8}, \ -\frac{20}{8}, \ -\frac{4}{8}$

Hence, $x = \left\{-1, \ 3, \ \frac{1}{2}, \ 1, \ 2, \ -2, \ -\frac{5}{2}, \ -\frac{1}{2}\right\}$ are roots of the given equation.

Now, according to factor theorem, if $x = a$ is a root of any equation, then $(x - a)$ will be a factor of that equation.

Similarly, we can write required equation into factor form which is given below

or $(x+1)(x-3)\left(x-\frac{1}{2}\right)(x-1)(x-2)(x+2)\left(x+\frac{5}{2}\right)\left(x+\frac{1}{2}\right) = 0$

or $(x+1)(x-1)(x+2)(x-2)\left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right)(x-3)\left(x+\frac{5}{2}\right) = 0$ this is required

V. Conclusions

Indrajeet's Law is a unique rule for nth degree polynomial equation, because five degree equation (or more than five degree equations) has (have) no general formula yet, but Indrajeet's Law is a method which works for quadratic equation, cubic equation, quartic equation , $\cdots$ to nth degree polynomial equation. This law does not work only for linear equation. Indrajeet's Law is a good method to fond the roots of nth degree polynomial equation, because in this law we have not done calculation and simplification. In this law I used coefficients and divisors only. Indrajeet's Law came from 'Indrajeet's Rule for Quadratic Equation' which is also developed by me and published in IOSR Journal of Mathematics in March 2016. If once we find roots of nth degree polynomial equation by Indrajeet's Law, then we can write nth degree polynomial equation into factor form, so we can say that nth degree polynomial equation can be factorised by using Indrajeet's Law. Hence we can say that Indrajeet's Law is a best method for nth degree polynomial equation.

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