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Manufacturer's Profit Maximization Policy in a Supply Chain Management System with Price Dependent Demand under Fuzzy Environment

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Abstract: *In this paper we have developed a pricing model under fuzzy environment where in-house manufacturing and outsourcing occur simultaneously so as to meet the customers' demand and maximize the profit of the manufacturer. Demand is assumed to be an exponential function of price and cost involved - holding cost, shortage cost and set-up cost are considered as trapezoidal fuzzy numbers. Under these circumstances, a profit function of the model has been formulated. Finally the proposed model has been demonstrated taking a numerical example and the sensitivity analysis of the optimal solution is provided with respect to key parameters of the system.*

Keywords: *Supply chain management, outsourcing, manufacturing, shortages, price dependent demand, trapezoidal fuzzy numbers, signed distance method.*

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I. Introduction

In any supply chain system it is very important to manage the flow of materials or products from manufacturer to supplier and then to retailer. Hence the key concern of any business organization is to coordinate between these different stages for smooth and efficient functioning of the system by providing the right quantity of product at the right time. For this purpose proper managing of inventory can significantly boost up a company's total profit. Lee and Wu (2006) [1]developed a study on inventory replenishment policies in a two-echelon supply chain system. Chen and Kang (2007) [2] studied integrated vendor-buyer cooperative inventory models with variant permissible delay payments. Tripathi and Misra (2012) [3]have developed an optimal inventory policy for items having constant demand and constant deterioration rate. Kim and Park (2008) [4] have assumed development of a three-echelon SC model to optimize coordination costs. Chung and Wee (2007) [5] developed economic lot size model of a three-stage supply chain with backordering derived without derivatives. Rau and Ouyang (2008) [6] have introduced an optimal batch size for integrated production-inventory policy in a supply chain. Goyal, S.K. and Nebebe, F. (2000) [7] determined economic production-shipment policy for a single-vendor-single-buyer system. Huang, Y.F. (2004) [8] presents optimal retailer's replenishment policy for the EPQ model under supplier's trade credit policy. Li et al [9] studied the impact of supply chain management practices on competitive advantage and organizational performance. Meixell and Gargeya [10] developed global supply chain design.

In any production system we find uncertainty associated with demand, various relevant cost, cycle time of the system etc., which has adverse effect on the outcome of the system. We solve the problems of this type using fuzzy set theory. A. Gupta, C.D. Maranas [11] studied demand uncertainty in supply chain planning. L.F. Escudero, E. Galindo, G. Garcia, E. Gomez, V. Sabau, Schumann [12] presented a modeling framework for supply chain management under uncertainty.Kao, C.K., Hsu, W.K. [13] studied a single-period inventory model with fuzzy demand.Türkşen and Fazel Zarandi [14] developed production planning and scheduling both crisp and fuzzy approaches. Saha (2017) [15] studied fuzzy inventory model for deteriorating Items in a supply chain system with price dependent demand and without backorder. Xu and Wang [16] presented an economic ordering policy model for deteriorating items with time proportional demand. Kaur et al. [17] developed an optimal ordering policy for inventory model with non-instantaneous deteriorating items and stock dependent demand. Srivastava and Gupta [18] presented an EPQ model for deteriorating items with time and price dependent demand under markdown policy. Chung and Ting [19] studied economic ordering policy for deteriorating items with a linearly increasing demand.In recent time, some researchers considered outsourcing decision in their studies. They took this decision based on capacity allocation or manufacturing capacity of the manufacturer. Kok et al [20] presents capacity allocation and outsourcing in a process industry. Lee et al [21] presents advanced planning and scheduling with outsourcing in a manufacturing supply chain.

In this paper we have developed a simple supply chain pricing model considering holding cost, shortage cost and setup cost as trapezoidal fuzzy numbers. The manufacturer meets the demand of the product by in-house manufacturing and by outsourcing from the supplier. Demand is assumed to be an exponential function of price. For defuzzification we have used the signed distance method.

This paper is organized as follows. In section 2, the assumption and notations are given. In section 3, we develop the mathematical models and analysis. In section 4, we provide numerical examples to illustrate the results. In addition, the sensitivity analysis of the optimal solution with respect to parameters of the system is carried out in section 5. Finally, we draw the conclusions in section 6 and acknowledgement in section 7.

II. Notations And Assumptions

The mathematical model in this paper is based on the following notations and assumptions.

2.1 Notations**:**

i) D = demand rate
ii) C_0 = set-up cost per order
iii) C_1 = the inventory holding cost per unit item per unit time.
iv) C_2 = shortage cost per unit item per unit time.
v) c = manufacturing cost per unit item.
vi) m = procurement cost per unit item.
vii) r = manufacturer's selling price to the retailer.
viii) TP = manufacturer's total profit
ix) $\widetilde{C_0}$ = fuzzy set-up cost per order.
x) $\widetilde{C_1}$ = fuzzy holding cost per unit item per unit time
xi) $\widetilde{C_2}$= fuzzy shortage cost per unit item per unit time
xii) $\widetilde{\mathrm{TP}}$ = fuzzy total profit of the manufacturer.

2.2 Assumptions**:**

(i) The inventory system involves production of single item.
(ii) Demand is assumed to be an exponential function of price and is of the form $D = ae^{-br}$, where a is the scaling factor and b is price elasticity.
(iii) Demand is met by both in-house manufacturing and by outsourcing from the supplier. Let f fraction of the total demand D is met by in-house manufacturing and remaining (1 – f) fraction is fulfilled by outsourcing.
(iv) Shortages are allowed and are backlogging.
(v) The manufacturer produces the product at a unit price c and supply the product to the retailer at a unit price r. If the product is outsourced, he procure the product at a unit price m (m > c). Also the manufacturer would have to incur a cost of capacity enhancement for the increasing value of f, which can be taken as zf^2, where z is the cost incurred by the manufacturer for producing increasing number of units in-house.
(vi) In the reality it is seen that the various relevant costs involved in the inventory may defer variously during the business period which occur naturally without any prior intimation. Therefore in this paper it is assumed that the holding cost, shortage costs and setup cost are fuzzy.
(vii) The cycle time t is divided into two parts t_1 and t_2 such that $t = t_1 + t_2$. During t_1 the product is supplied to the retailer and during t_2 order for the product is accumulated but not filled. When the amount Q of the produced or procured it is divided into two parts Q_1 and Q_2, such that $Q = Q_1 + Q_2$. The quantity Q_1 goes in the inventory and the quantity Q_2 is immediately taken to satisfy the unfulfilled demand.

III. The Model Description And Analysis

3.1 Crisp model:

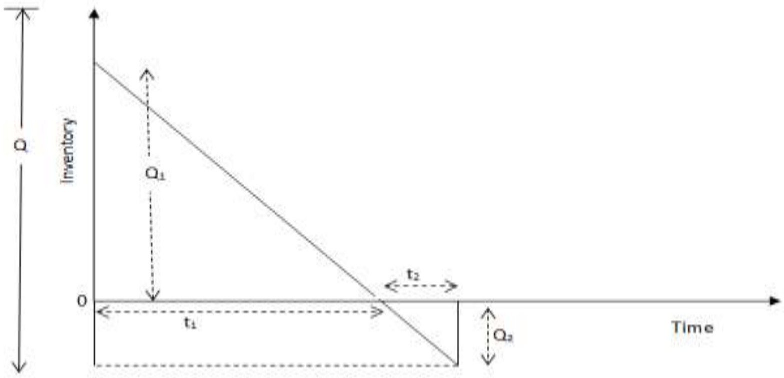

Figure – 1: Time – Inventory graph

Annual set up cost = $C_0\frac{D}{Q}$

Total inventory holding cost = $C_1 (\frac{1}{2} Q_1t_1)/t$

Annual shortage cost = $C_2 (\frac{1}{2} Q_2t_2)/t$

Now from Fig-1, using the relationship for similar triangles we have-

$\frac{t_1}{t} = \frac{Q_1}{Q}$ and $\frac{t_2}{t} = \frac{Q_2}{Q}$

$\Rightarrow$ $t_1 = \frac{Q_1}{Q} t$ and $t_2 = \frac{Q_2}{Q} t$

Using the above relationship we have-

Total inventory holding cost = $\frac{1}{2}C_1\frac{Q_1^2}{Q}$and total shortage cost = $\frac{1}{2}C_2\frac{Q_2^2}{Q} = \frac{1}{2}C_2\frac{(Q-Q_1)^2}{Q}$

Manufacturing cost of the items = cfD

Procurement cost of the items = $m(1- f)D$

Therefore, total cost of the manufacturer-

$= cfD + m(1- f)D + C_0\frac{D}{Q} + \frac{1}{2}C_1\frac{Q_1^2}{Q} + \frac{1}{2}C_2\frac{(Q-Q_1)^2}{Q} + zf^2$

Again, the total revenue obtained by selling all the items to the retailer –

$= r(1- f)D + rfD$

Therefore, manufacturer's profit function can be written as-

$TP = (r-m)(1-f)D + (r-c)fD - C_0\frac{D}{Q} - \frac{1}{2}C_1\frac{Q_1^2}{Q} - \frac{1}{2}C_2\frac{(Q-Q_1)^2}{Q} - zf^2$

$= (r-m)(1-f)ae^{-br} + (r-c)fae^{-br} - C_0\frac{ae^{-br}}{Q} - \frac{1}{2}C_1\frac{Q_1^2}{Q} - \frac{1}{2}C_2\frac{(Q-Q_1)^2}{Q} - zf^2$ …………….. (1)

3.2 Fuzzy model:

In reality it is not always possible to define certain parameters with certainty for which we fuzzify some parameters. Here we fuzzify the parameters C_0, C_1, and C_2.

Then the total profit can be written as-

$\widetilde{TP} = (r-m)(1-f)ae^{-br} + (r-c)fae^{-br} - \widetilde{C_0}\frac{ae^{-br}}{Q} - \frac{1}{2}\widetilde{C_1}\frac{Q_1^2}{Q} - \frac{1}{2}\widetilde{C_2}\frac{(Q-Q_1)^2}{Q} - zf^2$

Now wedefuzzify the total profit we use sign distance method.

Let $\widetilde{C_0} = (a_0, b_0, c_0, d_0)$, $\widetilde{C_1} = (a_1, b_1, c_1, d_1)$ and $\widetilde{C_2} = (a_2, b_2, c_2, d_2)$, where $a_0, b_0, c_0, d_0, a_1, b_1, c_1, d_1, a_2, b_2, c_2, d_2$ are known positive numbers.

Then, set-up cost + holding cost + shortage cost

$= \frac{D}{Q} \times \widetilde{C_0} + \frac{Q_1^2}{2Q} \times \widetilde{C_1} + \frac{(Q-Q_1)^2}{2Q} \times \widetilde{C_2}$

$=[a_0\frac{D}{Q} + a_1\frac{Q_1^2}{2Q} + a_2\frac{(Q-Q_1)^2}{2Q},\ b_0\frac{D}{Q} + b_1\frac{Q_1^2}{2Q} + b_2\frac{(Q-Q_1)^2}{2Q},\ c_0\frac{D}{Q} + c_1\frac{Q_1^2}{2Q} + c_2\frac{(Q-Q_1)^2}{2Q},\ d_0\frac{D}{Q} + d_1\frac{Q_1^2}{2Q} + d_2\frac{(Q-Q_1)^2}{2Q}]$

= (a, b, c, d) (say)

Now, $A_L(\alpha) = a + (b-a)\alpha$

$= a_0\frac{D}{Q} + a_1\frac{Q_1^2}{2Q} + a_2\frac{(Q-Q_1)^2}{2Q} + [(b_0-a_0)\frac{D}{Q} + (b_1-a_1)\frac{Q_1^2}{2Q} + (b_2-a_2)\frac{(Q-Q_1)^2}{2Q}]\alpha$

and $A_R(\alpha) = d - (d-c)$

$= d_0\frac{D}{Q} + d_1\frac{Q_1^2}{2Q} + d_2\frac{(Q-Q_1)^2}{2Q} - [(d_0-c_0)\frac{D}{Q} + (d_1-c_1)\frac{Q_1^2}{2Q} + (d_2-c_2)\frac{(Q-Q_1)^2}{2Q}]\alpha$

Then the defuzzified value of (set-up cost + holding cost + shortage cost)

$= \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)]d\alpha$

$= \frac{1}{2}[(a_0+d_0)\frac{D}{Q} + (a_1+d_1)\frac{Q_1^2}{2Q} + (a_2+d_2)\frac{(Q-Q_1)^2}{2Q}] + \frac{1}{4}[(b_0+c_0-a_0-d_0)\frac{D}{Q} + (b_1+c_1-a_1-d_1)\frac{Q_1^2}{2Q} +$
$(b_2+c_2-a_2-d_2)\frac{(Q-Q_1)^2}{2Q}]$

$= \frac{1}{4}(a_0+b_0+c_0+d_0)\frac{D}{Q} + \frac{1}{8}(a_1+b_1+c_1+d_1)\frac{Q_1^2}{Q} + \frac{1}{8}(a_2+b_2+c_2+d_2)\frac{(Q-Q_1)^2}{Q}$

Therefore, fuzzy total profit is given by-

$\widetilde{TP} = (r-m)(1-f)ae^{-br} + (r-c)fae^{-br} - \frac{1}{4}(a_0+b_0+c_0+d_0)\frac{D}{Q} - \frac{1}{8}(a_1+b_1+c_1+d_1)\frac{Q_1^2}{Q} - \frac{1}{8}(a_2+b_2+c_2+$
$d_2)\frac{(Q-Q_1)^2}{Q} - zf^2$

For maximizing the total profit we partially differentiate the equation (1) with respect to f, Q, Q_1 and r to obtain the respective optimal values as-

$f^* = \frac{(m-c)}{2z}ae^{-br}$

$$Q^*= \sqrt{\frac{2\{(a_1+b_1+c_1+d_1)+(a_2+b_2+c_2+d_2)\}(a_0+b_0+c_0+d_0)ae^{-br}}{(a_1+b_1+c_1+d_1)(a_2+b_2+c_2+d_2)}}$$

$$Q_1^*= \sqrt{\frac{2(a_0+b_0+c_0+d_0)(a_2+b_2+c_2+d_2)ae^{-br}}{(a_1+b_1+c_1+d_1)\{(a_1+b_1+c_1+d_1)+(a_2+b_2+c_2+d_2)\}}}$$

$$r^*= \mathrm{m} + \frac{1}{b} - \frac{(m-c)^2}{2z} ae^{-br} + \frac{1}{4}\sqrt{\frac{(a_0+b_0+c_0+d_0)(a_1+b_1+c_1+d_1)(a_2+b_2+c_2+d_2)}{2\{(a_1+b_1+c_1+d_1)+(a_2+b_2+c_2+d_2)\}ae^{-br}}}$$

IV. Numerical Analysis

To illustrate the following model we consider the following numerical values of the parameters.
A = 100, b = 0.01, z = 250, m = 6, c = 2, C_0 = (9, 10, 11, 12), C_1 = (1, 1.5, 2, 2.5), C_2 = (4, 4.5, 5, 5.5).
We obtain, $\widetilde{TP}$ = 3453.339, r = 105.3225, f = 0.279, Q = 23.9329, Q_1 = 17.4894.

V. Sensitivity Analysis

Table-1: sensitivity on m

m	r	f	D	fD	(1-f)D	$\widetilde{TP}$
4	104.1538	0.1412	35.29	4.98	30.31	3508.739
6	105.3225	0.2790	34.88	9.73	25.15	3453.339
8	105.9442	0.4160	34.66	14.42	20.24	3407.976
10	106.0059	0.5543	34.64	19.20	15.44	3372.293

Table-2: Sensitivity on b

b	r	f	$\widetilde{TP}$	Q	Q_1
0.01	105.3225	0.2790	3453.339	23.9329	17.4894
0.02	55.394	0.2642	1618.948	23.2877	17.018
0.03	38.7978	0.2498	1010.771	22.6441	16.5476
0.04	30.5337	0.2359	709.102	22.003	16.079

Table-3: Sensitivity on c

c	r	f	TP	Q	Q_1
2	105.3225	0.2790	3453.339	23.9329	17.4894
3	105.815	0.2082	3444.864	23.874	17.446
4	106.163	0.1384	3438.861	23.8324	17.416
5	106.372	0.069	3435.279	23.8077	17.3979

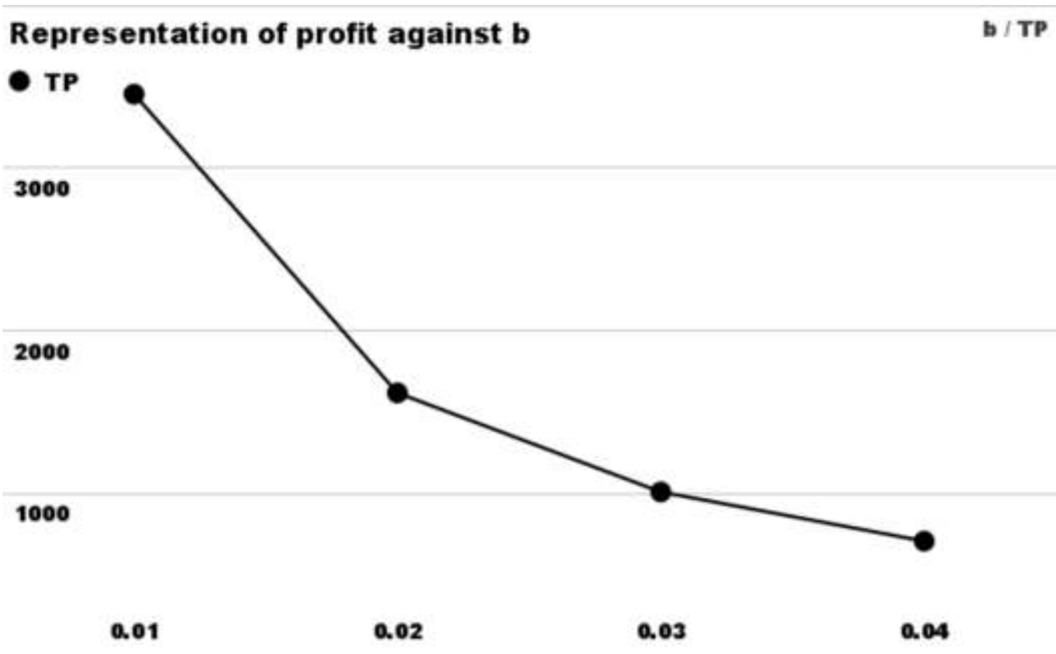

Figure – 2

Figure – 3

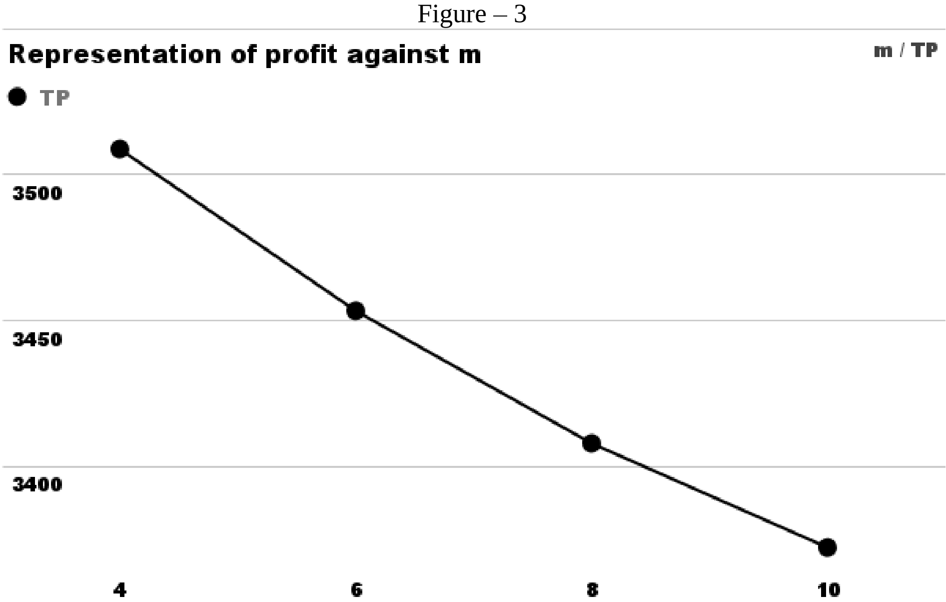

Figure – 4

3.2 Observations**:**

i) From table-1 it is seen that, with the increasing values of m (outsourcing cost) the amount of in-house manufactured product (fD) increases and the amount of product outsourced from the supplier decreases, which indicates that the manufacturer should meet the demand more by in-house manufacturing than outsourcing. Besides,it is also clear from this table that, the increasing cost of outsourced product has an adverse effect on the profit.

ii) Table-2 depicts the effect of price elasticity on different parameters, such as- r, f, TP, Qand Q_1. It is observed that, as the value of b increases, the selling price and the total profit of the manufacturer decreases dramatically.

iii) Table-3 shows that, as the manufacturing cost increases, the rate at which the retailer gets the product increases. Moreover, the total profit of the manufacturer and the values of the decision variables f,Q and Q_1 also decrease with the increasing values of c.

VI. Conclusions

In this paper we have considered a supply chain model under fuzzy environment, in which we considered both of in-house manufacturing and outsourcing so as to meet the customers demand and to maximize manufacturer's total profit. Realistically it is observed that some parameters cannot be defined with certainty, so we have described holding cost, shortage cost and set-up cost as trapezoidal fuzzy numbers.This model can help the manufacturer to take the decision on the amount of product to be outsourced and manufactured so as to gain maximum profit and also the manufacturer will be able to set up the selling price of the product.

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