IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 13, Issue 6 Ver. II (Nov. - Dec. 2017), PP 01-05
www.iosrjournals.org

Steiner Triple Systems Of Order 25

Awatef Ahmed Omar Aughnai(PH.Dscolar,ID.No:13PHMATH202)[1],
Dr. Mohit James (Assistant Professor)[2]
[1,2](Department Of Mathematics & Statistics, School Of Basic Sciences Sam Higginbottom University Of Agriculture, Technology & Sciences, Allahabad, Faculty Of Sciences. Allahabad -211007, U.P., India)

Abstract: *The higher order Steiner system is very complex in nature. Various methodologies can be used to construct and enumerate the STS for order of 25. The various combinatory theory and design theory can be used for the enumeration of the desired STS. Here, in this paper, we have presented a detailed method of the enumeration, methodology and construction of the STS systems. Detailed numerical analysis has been done for the enumeration of the STS of order 25. The algorithm discussed will here generate the total possible STS combinations. Finally, construction methodology has also been presented. Graphical construction has been discussed with the reference to the kirkman systems. The properties of the STS system equations have been discussed. The various theorems have been represented using the order 9 subsystems. Results have shown the steps of the enumeration and combinations of various pairs showing the properties of STS of the order 25.*

Date of Submission: 07-10-2017 Date of acceptance: 21-11-2017

I. Introduction

Steiner defined two kinds of structure: S_n ,what we have called a Steiner triple system of order n ;and S_n^0 whose exact details don't concerns here. It shows:

(a) S_1 exists;

(b)if S_n exists, then S_{2n+1} exists;

(c)if S_n exists, n exists for $n > 29$ ndnS^0

II. Enumeration Of Problems

Problem-1: To construct the disjoint points of the STS using generating points [15]and generating function (GF) [11][12] of order 25using polynomial formulation.

To show that $Z^2 - Z + 1$ is irreducible over GF(5), it is enough to show that none of the five element so fGF(5) is a root:

$$0^2 - 0 + 1 = 1 = 0,$$

$$1^2 - 1 + 1 = 1 = 0,$$

$$2^2 - 2 + 1 = 3 = 0,$$

$$3^2 - 3 + 1 = 2 = 0,$$

$$4^2 - 4 + 1 = 3 = 0,$$

A root of the quadratic a primitive 6th root of unity (since $Z^2 - Z + 1$ divides $Z^6 - 1$ but doesn't divide $Z^3 - 1$ or $Z^2 - 1$); but the multiplicative group of GF(5)hasorder4andhasnoprimitive6throotofunity. So GF(25)[15] can be represented as claimed.

The subgroup eneratedbyzis $\{1, Z, Z-1, -1, -Z, -Z+1\}$. Now choose an element,say2,andformthecloset $\{2, 2Z, 2Z-2, -2, -2Z, -2Z+2\}$; another, say $Z+1$,and form $\{Z+1, 2Z-1, Z-2, -Z-1, -2Z+1, -Z+2\}$; and finally, say $Z+2$, which gives there mainingsixnon-zero elements.

So the four base triples are $\{0,1,Z\}, \{0,2,2Z\}, \{0, Z+1, 2Z-1\}$, and $\{0, z+2, -2z-1\}$. Thus, other triples are obtained by adding the elements of GF (25) to each of these.

More generally, the number of triples of a $STS(n)$ disjoint from a given triple is $(n-3)(n-7)/6$: for there are $n(n-1)/6$ triples altogether, of which each point of a triple T lies in $(n-1)/2$ triples, $n(n-3)/2$ of which meet T_i none point[15]

For $n = 7$, let1, 2, 4 be any three points not for mingatriple. There are unique triples through any two of them, which we may label123, 145, 246. Then the triples through 16,25 and 43 must each contain the one remaining point, say 7. Finally, the three pairs 36, 35, 56 must each lieonatriple , and since only one triple remains to be found, Itmustbe356[11]

For $n = 9$, the two triples disjoint from a given on eared is joint from one another. For if T_i T is a triple and $x \in T$, then x lies on four triples, three of which meet T, so just one is disjoint from T. Hence the twelve triples fall in to four sets of three mutually disjoint triples, so that triples from different sets intersect. Then two of these sets form as quaregrid, and we can number the points so that these triples are [11]

III. Deatiled Enumeration

Starting with the analysis used above for $n = 9$: there are ten triples disjoint from a given one, each of the ten points lying on three triples. Classify the configurations formed by the set en triples [13]

An *automorphism* of a Steiner triple system [13] is a nisomorphism from the system to itself.
For $n = 7$, any three points x, y, z not forming a triple can be mapped to any other three such points by a unique isomorphism. If we take the two systems to be the same, then isomorphisms are auto-morphisms. So the number of Auto-isomorphism [13] is equal to the number of choices of x, y, z, which is $7 \cdot 6 \cdot 4 = 168$.For $n = 9$, the argument is similar, and yields $9 \cdot 8 \cdot 6 = 432$ as the number of auto-isomorphisms.

Theorom (Kirkman): A triple system, each triangle lies in a subsystem of order9 [15]

(a) Prove that an affine triple system is a Kirkman system [15]
(b) Given a triangle x, y, z, consider the set
$\{x + \mu y + z : + \mu + = 1\}$.

This set contains nine points. (There are nine choices of x, μ, y, z with sum zero in the integer's mod3.If two choices lead to the same vector, we have $x + y + z = 0$. Up to permutation, there are two cases:

Moreover, these nine points form a subsystem. For suppose that
$a_i = x_i + \mu_i y + z_i, \quad x_i + \mu_i + z_i = 0$
For $i = 1,2,$ and let $a_3 = -(a_1 + a_2)$ be the third point of the triple through $a_1 a_2$
The entry agrees with the bound given by last equation .So we are required just to exhib it packing and coverings attaining the bounds. This is clear for the values $n = 3,7,9$ for which STS exist, and also for $p(6)$ and $p(8)$, by[2].For the other values we have:

$p(4): 123$
$c(4): 123,124,134$
$c(5): 123,145,234,235$
$c(6): 123,124,345,346,5651,562$
$c(8): STS(7)$ *to gether with* $128,348,568,678.$

We see that $p(5) \le 2$ as follows: two3-sets can have at most one point in common, so are 123, 145 without loss of generality. A further 3-set could contain at most oneof1, 2, 3 and at most one of 4,5, so cannot exist. Apackingofsize2is

If a SQS of order n exists, with $n \ge 2$, then $n \equiv 2 \, or \, 4 (\bmod 6)$.

If (X, B) is a SQS and $x \in X$, let $Y = X \setminus \{x\}$ and let C be the set of triples T such that $T \cup \{x\} \in B$. Then (Y, C) is a STS. Hence if a SQS of order n exists, then $n-1 \equiv 1 or 3 \bmod 6$, when $cen \equiv 2 or 4 \bmod 6$.

If (X, B) is a SQS of order n, then $|B| = n(n-1)(n-2)/24$.

Count pairs (x, U), where x is a point and U a quadruple containing it. We see that $4|B| = n.(n-1)(n-2)/6$. (By the remarks in the preceding solution, the number of quadruples containing x is equal to the number of triples in a STS of order $n-1$.)

Let X be a vectors pace over $Z(2)$, and let B bethesetof4-subsets $\{x, y, z, w\}$
Of X for which $x + y + z + w = 0$. Show that (X, B) is a SQS.

Given three distinct vectors x, y, z, let $w = x + y + z$. Since−1=1,we have $x + y + z + w = 0$. Could w be equal to one of (x, y, z) If $w = x$, then $y + z = 0$, when $cey = z$, contrary to assumption. The other cases are similar. So $\{w, x, y, z\}$ is a quadruple, clearly the unique a quadruple containing x, y, z.
Now let $Y = X \cup X^0$, and $C = B \cup B^0 \cup R$, where R isthesetof4-sets $\{x, y, z^0, w^0\}$ such that $x, y \in X$, $z^0, w^0 \in X^0$;
for some $i (1 \le i \le n-1)$, $\{x, y\} \in R_i$ and $\{z^0, w^0\} \in R^0$. i
Show that (Y, C) is a SQS of order $2n$.

Clearly all elements of C are 4-sets. We have to show that any three points x, y, z
Lie in exactly one quadruple. There are several cases:

(a) $x, y, z \in X$. Then a unique quadruple in B contains them.

(b) $x, y \in X, z \in X^0$. There is a unique i such that $\{x, y\} \in R_i$, and then a unique w
Such that $\{z, w\} \in R^0$. then $\{x, y, z, w\}$ is the unique quadruple containing the three points.

Problem-2: To validate and find the resolution for STS 25 systems:

Theorem [11]: To validate and find the resolution of the STS 25 system of higher order n

Let (X, B) be a STS of order n, and Y a subsystem of order m, where $m<n$. Prove that $n \ge 2m+1$. Show further that $n = 2m+1$ if and only if every triple in B contains either 1 or 3points of Y

Let $x \in X \setminus Y$. Then x lies in $(n-1)/2$ triples. But the points of Y lie on distinct triples through x, since a triple containing two points of Y is contained within y. As there are m such triples, we have $m \le (n-1)/2$, or $n \ge 2m+1$.

Equality holds if and only if every triple containing x meets Y. Since this is true for each point outside Y, equality holds if and only if no triple is disjoint from Y. By the definition of a subsystem, no triple can meet Y in two points. So, finally, equality holds if and only if every triple meets Y in one or three points.

Let (X,B) be a STS of order $n = 2m+1$, and Y a subsystem of order m; say $Y = \{y_1,\ldots,y_m\}$. For $i = 1,\ldots,m$, let R_i be the set of all pairs $\{z, z^0\} \subseteq X \setminus Y$ for which $\{y_i, z, z^0\} \in B$. Show that $\{R_1,\ldots,R_m\}$ is a tournament schedule on $X \setminus Y$. Show further that this construction can be reversed: a $STS(m)$ and a tournament schedule of order $m+1$ can be used to build a $STS(2m+1)$.

Given an $y_i, z, z^0 \in X \setminus Y$, he triple containing z and z^0 meets Y in a point y_i (see Question11);so $\{z, z^0\} \in R_i$. On the other hand, y_i lies on m triples, of which $(m-1)/2$ are in the subsystem Y, so $(m+1)/2$ have the form $\{y_i, z, z^0\}$ for some $z, z^0 \in X \setminus Y$. Since the pairs $\{z, z^0\}$ are disjoint, they cover the $m+1$ points of $x \setminus Y$. So we do have a tournament schedule.

Theorem [11]-[13] :Pairing properties of the STS systems for finding the unique triples
And let D consist of all triples of the following types:

• $\{(x,y_1),(x,y_2),(x,y_3)\}$ for $x \in X, \{y_1,y_2,y_3\} \in C$;
$\{(x_1,y),(x_2,y),(x_3,y)\}$ for $\{x_1,x_2,x_3\} \in B, y \in Y$;
$\{(x_1,y_1),(x_2,y_2),(x_3,y_3)\}$ for $\{x_1,x_2,x_3\} \in B$, $\{y_1.y_2,y_3\} \in C$
(Note that a triple in Band one in C give rise to six triples of the third type, corresponding to the six possible bijtions from one to the other.) Show that (Z,D) is a STS of order mn. Show further that, if $m > 1$ and $n > 1$, then (Z,D) contains subsystemoforder9

(a)If $x_1 = x_2$ and $y_1 = y_2$, there is a unique triple $\{y_1,y_2,y_3\} \in C$: then (x_1, y_3)
Is the third point of the triple through the two given points.

(b) $x_1 = x_2$, $y_1 = y_2$: dualto(a).

(c) $x_1 = x_2$ and $y_1 = y_2$. There are unique triples $\{x_1,x_2,x_3\} \in B$ and $\{y_1,y_2,y_3\} \in C$; then (x_3,y_3) is the third point of the triple.

If $\{x_1,x_2,x_3\} \in B$ and $\{y_1,y_2,y_3\} \in C$, then $\{(x_i,y_j): i, j = 1,2,3\}$ is a subsystem of order9.

IV. Results

Here, the grouping of the pairs for the STS has been selected. The non-cyclic STS has been evaluated. Thus, the total number of STS pairs presented for STS of order 25 for the non-cyclic is found to be 75[18].
Thus, the total isomeric generations can be obtained as:
Non-cyclic Generation: 75[18]
Non-isomorphic generation seeds: 784
Isomorphic generation: 576

CPU Time – 2 Hrs

Triple systems in general. Although direct constructions have been known previously for many cases, notably those of Bose [2] for Steiner triple systems and the elegant construction for (v, 2) triple systems we have succeeded in pushing through the direct construction for all cases. For many values of (0, A) whose designs have already been constructed directly by others, our constructions give non isomorphic designs in general, and we present them anew here both for completeness and for use in the construction of triple systems with other parameters, as in the case for the direct construction of designs for h = 3 and h = 6.

References

[1]. R. C. Bose, Strongly regular graphs, partial geometries and partially balanced designs, Paci_c J. Math. 13 (1963), 389{419. MR 28:1137

[2]. G. Butler, Fundamental Algorithms for Permutation Groups, Lecture Notes in ComputerScience, Vol. 559, Springer-Verlag, Berlin, 1991. MR 94d:68049

[3]. C. J. Colbourn, S. S. Magliveras, and D. R. Stinson, Steiner triple systems of order 19 with nontrivial automorphism group, Math. Comp. 59 (1992), 283{295. MR 92k:05022

[4]. C. J. Colbourn and A. Rosa, Triple Systems, Clarendon Press, Oxford, 1999. MR

[5]. F. N. Cole, L. D. Cummings, and H. S.White, The complete enumeration of triad systems in 15 elements, Proc. Nat. Acad. Sci. U.S.A. 3 (1917)

[6]. G. P. Egorychev, The solution of van der Waerden's problem for permanents, Adv. in Math.42 (1981), 299{305. MR 83b:15002b

[7]. D. I. Falikman, Proof of the van der Waerden conjecture on the permanent of a doubly stochastic matrix (in Russian), Mat. Zametki29 (1981), 931{938, 957. MR 82k:15007

[8]. M. J. Grannell, T. S. Griggs, and E. Mendelsohn, A small basis for four-line con_gurations in Steiner triple systems, J. Combin. Des.3 (1995), 51{59. MR 95j:05041

[9]. B. D. Gray and C. Ramsay, On the number of Pasch configurations in a Steiner triple system, Bull. Inst. Combin.Appl. 24 (1998), 105{112. MR 99c:05030

[10]. M. Hall, Jr. and J. D. Swift, Determination of Steiner triple systems of order 15, Math. Tables Aids Comput.9 (1955), 146{152. MR 18:192d

[11]. Xiao-Yi Li, Zhao-Di Xu, "Construction and Enumeration of the Steiner Triple systems with order sxt", 2011 IEEE

[12]. Xiaoyi li, Wanxi Chou, "A method of constructing Kirkman Triple system of higher order",2010 IEEE.

[13]. David Guichard, "A Introduction of the combinatorics and Graph theory".

[14]. A.D. Forbes, M.J. Garnell, T.S. Griggs, "Steiner Triple systems and existentially closed graphs", The electric journal of combinatorics, 2005

[15]. KeijoRuohonen "Graph Theory", 2013

[16]. MariuzMeszka, Alexander Rosa " Embedding Steiner Triple systems into Steiner systems S(2,4,u)

[17]. Milazzo ,Tuza, Voloshin "Strict Colorings of the Steiner triple system and Quadruple system",Hungarian Scientific Reaseach fund

[18]. Tonchev ,"Transitive order of the Steiner systems",Discrete Mathematics,1990

Biography

 **Dr. Mohit James** **Assistant professor** jamesmohit@yahoo.co.in	**Ph.D** SHIATS-DU, Allahabad(2011) M.Sc MJP Rohit Khand, Chandauli(2006) B. Ed Lucknow University, Lucknow (2005) B.Sc UP, Badaun (2003) **Expertise:** Graph Theory, Operations Reserach **Conferences:** National 13, International 1
Awatef Ahmed Omar Aughnai **Ph.D. scolar (13 phmath 202)** **gamal.kalefa@yahoo.com** abobkrklefa@yahoo.ccom	**P.hd. Research Student,** School Of Basic Sciences Sam Higginbottom University Of Agriculture, Technology & Sciences, Allahabad, Faculty Of Sciences. Allahabad -211007, U.P., India **M.Sc. Mathematics** 2013, School Of Basic Sciences Sam Higginbottom University Of Agriculture, Technology & Sciences, Allahabad, Faculty Of Sciences. Allahabad -211007, U.P., India **B.Sc**. (2005-2006 Libya)

Awatef Ahmed Omar Aughnai Steiner Triple Systems Of Order 25. IOSR Journal of Mathematics (IOSR-JM) , vol. 13, no. 6, 2017, pp. 01-05.