IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 13, Issue 6 Ver. II (Nov. - Dec. 2017), PP 52-59
www.iosrjournals.org

Applying Laminar and Turbulent Flow and measuring Velocity Profile Using MATLAB

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Abstract: Review works Laminar flow and Turbulent flow in pipes. Some discuses and measuring velocity profile using mat lab.

Keywords: *Laminar flow, Turbulent flow, velocity profile.*

I. Introduction

Fluid flow is as defined external and internal, Internal and external flows exhibit very different characteristics. We consider internal flow where the conduit is completely filled with the fluid, and flow is driven primarily by a pressure difference. This should not be confused with open-channel flow where the conduit is partially filled by the fluid and thus the flow is partially bounded by solid surfaces, as in an irrigation ditch, and flow is driven by gravity alone. a general physical description of laminar flow and turbulent flow and the velocity boundary layer. We continue with a discussion of the pressure drop correlations associated with it for both laminar and turbulent flows. Then we present the minor losses and determine the pressure drop and pumping power requirements for real-world piping systems.

Finally, we present an MATLAB code to show the relative position of velocity profile of laminar and turbulent flow.

II. Laminar Flow in Pipes

Flow in pipes is laminar for $R_e \leq 2300$ and that the flow is fully developed if the pipe is sufficiently long (relative to the entry length). we consider the steady laminar flow of an incompressible fluid with constant properties in the full developed region of a straight circular pipe. Fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile u(r) remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to flow is everywhere zero. There is no acceleration since the flow is steady and fully developed. Now consider a ring-shaped differential volume element of radius r, thickness *dr*, and length dx oriented coaxially with the pipe, as shown in Fig.1. The volume element involves only pressure and viscous effects and thus the pressure and shear forces must balance each other.

A force balance on the volume element in the flow direction gives

$$(2\pi\, rdrP)_x - (2\pi\, rdrP)_{x+dx} + (2\pi\, rdr\,\tau)_r - (2\pi\, rdr\,\tau)_{r+dr} = 0 \qquad \text{................(1)}$$

Which indicates that in fully developed flow in a horizontal pipe.

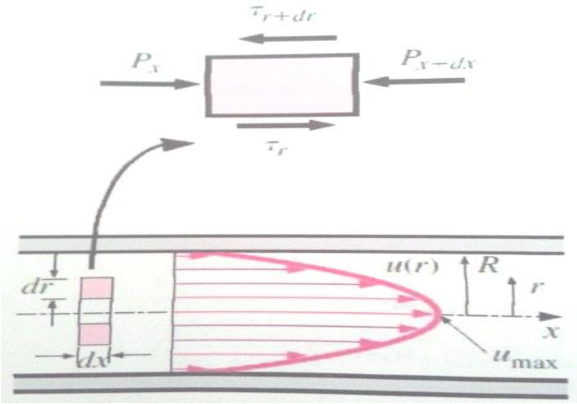

Figure 1: Free-body diagram of a ring-shaped .

Dividing by $2\pi\ drdx$ and rearranging,

$$r\frac{P_{x+dx}-P_x}{dx}+\frac{(r\tau)_{r+dr}-(r\tau)_r}{dr}=0 \qquad \text{............(2)}$$

Taking the limit as $dr, dx \to 0$ gives

$$r\frac{dP}{dx}+\frac{d(r\tau)}{dr}=0 \qquad \text{..................(3)}$$

Substituting $\tau=-\mu\left(du/dr\right)$ and taking μ = constant gives the desired equation,

$$\frac{u}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)=\frac{dP}{dx} \qquad \text{...............(4)}$$

The quantity $\frac{du}{dr}$ is negative in pipe flows, and the negative sign is included to obtain positive values for τ (or, $du/dr=-du/dy$). The left side of Equation (4) is a function of *r*, and the right side is a function of *x*. The equality must hold for any value of *r* and *x*, and an equality of the form $f(r)=g(x)$ can be satisfied only if both $f(r)$ and $g(x)$ are equal to the same constant. Thus we conclude that $\frac{dP}{dx}$ = constant. This can be verified by writing a force balance on a volume element of radius *R* and thickness *dx* (a slice of the pipe), which gives (Fig.2)

$$\frac{dP}{dx}=-\frac{2\tau_w}{R} \qquad \text{.................(5)}$$

Here τ_w is constant since the viscosity and the velocity profile are constants. Therefore, $\frac{dP}{dx}$ =constant.

Equation (4) can be solved

$$u(r)=\frac{1}{4\mu}\left(\frac{dP}{dx}\right)+C_1\ln r+C_2 \qquad \text{.........................(6)}$$

Applying the boundary conditions $\frac{\partial u}{\partial r}=0$ at $r=0$ and $u=0$ at $r=R$. We get,

$$u(r)=\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right) \qquad \text{............................(7)}$$

Therefore, the velocity profile in fully developed laminar flow in a pipe is parabolic with a maximum at the centerline and minimum (zero) at the pipe wall. Also, the axial velocity *u* is positive for any *r*, and thus the axial pressure gradient $\frac{dP}{dx}$ must be negative.

III. Force balance

$$\pi R^2 P-\pi R^2(P+dP)-2\pi Rdx\ \tau_w=0$$

Simplifying: $\frac{dP}{dx} = -\frac{2\tau_w}{R}$

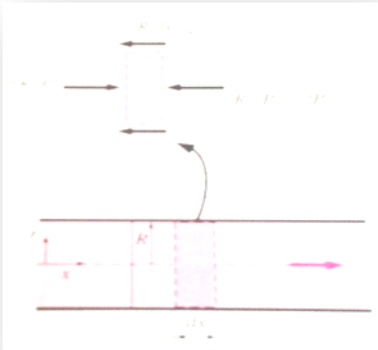

Figure 2: force balance on a volume element of radius *R* and thickness *dx*

The average velocity is determined from its definition by substituting Equation (7) and equation

$V_{avg} = \frac{2}{R^2}\int_0^R u(r)r\,dr$ performing the integration . It gives

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r)r\,dr \Rightarrow V_{avg} = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1-\frac{r^2}{R^2}\right)r\,dr \qquad \text{................(8)}$$

Combining the last two equations,

$$u(r) = 2V_{avg}\left(1-\frac{r^2}{R^2}\right) \qquad \text{..................(9)}$$

Since V_{avg} can be determined easily from the flow rate information. The maximum velocity occurs at the centerline and is determined from Equation (9) by substituting $r = 0$,

$$u_{max} = 2V_{avg} \qquad \text{..............(10)}$$

Therefore, the average velocity in fully developed laminar pipe flow is one half of the maximum velocity.

IV. Turbulent Flow in Pipes

Turbulent flow is characterized by random and rapid fluctuations of swirling regions of fluid, called eddies, throughout the flow. These fluctuations provide an additional mechanism for momentum and energy transfer. In laminar flow, fluid particles flow in an orderly manner along path lines, and momentum and energy are transferred across streamlines by molecular diffusion. In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer. As a result, turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients (Fig. 3).

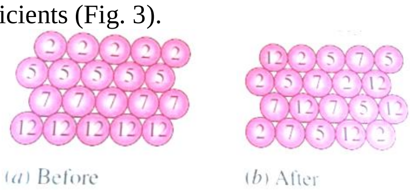

Figure 3: Turbulent Flow in Pipes

Even when the average flow is steady, the eddy motion in turbulent flow causes significant fluctuations in the values of velocity, temperature, pressure, and even density (in compressible flow). Following figure shows the variation of the instantaneous velocity component u with time at a specified location, as can be measured with a hot-wire anemometer probe or other sensitive device.

Figure 4**:** Fluctuations of the velocity component u with time at a specified location in turbulent flow.

We observe that the instantaneous values of the velocity fluctuate about an average value, which suggests that the velocity can be expressed as the sum of an average value $\bar{u}$ and a fluctuating component u',

$$u = \bar{u} + u' \qquad \text{.....................................(11)}$$

This is also the case for other properties such as the velocity component v in the y-direction, and thus, $v = \bar{v} + v'$, $P = \bar{P} + P'$, and $T = \bar{T} + T'$. The average value of a property at some location is determined by averaging it over a time interval that is sufficiently large so that the time average levels off to a constant. Therefore, the time average of fluctuating components is zero, e.g., $\bar{u} = 0$. The magnitude of u' is usually just a few percent of $\bar{u}$, but the high frequencies of eddies makes them very effective for the transport of momentum, thermal energy, and mass. In time-averaged stationary turbulent flow, the average values of properties are independent of time.

Perhaps the first thought that comes to mind is to determine the shear stress in an analogous manner to laminar flow from $-\mu \frac{d\bar{u}}{dr}$, where $\bar{u}(r)$ is the average velocity profile for turbulent flow. But the experimental studies show that this is not the case, and the shear stress is much larger due to the turbulent fluctuations. Therefore, it is convenient to think of the turbulent shear stress as consisting of two parts: the laminar component, which accounts for the friction between layers in the flow direction (expressed as $\tau_{lam} = -\mu \frac{d\bar{u}}{dr}$), and the turbulent component, which accounts for the friction between the fluctuating fluid particles and the fluid body (denoted as τ_{turb} and is related to the fluctuation components of velocity). Then the total shear stress in turbulent flow can be expressed as

$$\tau_{total} = \tau_{lam} + \tau_{turb} \qquad \text{..................(12)}$$

The typical average velocity profile and relative magnitudes of laminar and turbulent components of shear stress for turbulent flow in a pipe are given in Fig.5. Note that although the velocity profile is approximately parabolic in laminar flow, it becomes flatter or "fuller" in turbulent flow with a sharp drop near the pipe wall.

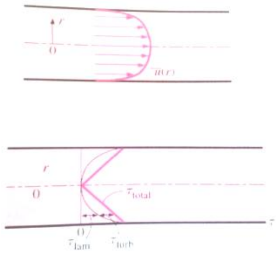

Figure 5: The velocity profile and the variation of shear stress with radial distance for turbulent flow in a pipe.

V. Turbulent Flow in a Pipe

Many fluid flow applications involve the transport of a gas or liquid through circular pipes. Reynolds number, R_e, where

$$R_e = \frac{\rho v D}{\mu} \qquad \text{.......................... (13)}$$

With ρ = fluid density, v = average velocity, D = pipe diameter, and μ = fluid viscosity. For $R_e \le 2300$, the flow is said to be laminar and it is usually associated with the smooth deterministic flow of a highly viscous or low density fluid. For turbulent flow, where $R_e > 4000$, one observe irregular random fluctuations in the fluid velocity caused by local mixing (or turbulence) within the fluid ,which usually has a low viscosity and or a high velocity.

For flow in a pipe ,the " no-slip" boundary condition at the pipe walls forces the fluid velocity to be zero at $r = R$,where *R* is the pipe radius .Thus, the velocity will increase from zero at the wall to some maximum value at the center of the pipe (at $r = 0, u(r) = u_{\max}$).

Now, for laminar flow, the actual velocity profile can be developed analytically and it is given by

$$u_L(r) = u_{L_{\max}}\left[1-\left(r/R\right)^2\right] \quad \text{(Laminar flow)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

And a formal averaging process gives

$$u_{ave} = \frac{1}{A}\int_0^R u_L(r)\,2\pi\, r dr = \frac{u_{L_{\max}}}{2} \qquad \ldots\ldots\ldots\ldots\ldots (15)$$

Where *A* is the flow area ($A = \pi R^2$ for flow in a circular pipe).

For turbulent flow, the exact velocity profile cannot be determined analytically.

$$u_T(r) = u_{T_{\max}}\left[1-\left(\frac{r}{R}\right)\right]^m \qquad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

And again. Taking a formal average gives,

$$u_{ave} = \frac{1}{A}\int_0^R u_T(r)\,2\pi\, r dr = \frac{2u_{T_{\max}}}{(m+1)(m+2)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Where *m* is an exponent that is only weakly dependent on the degree of turbulence. For fully developed turbulent flow a value of $m = 1/7$ is commonly used to represent the turbulent flow profile in equation (17). In the above equations, the average velocity, u_{ave} , is usually available from the volumetric flow rate r (which can easily measured in most cases). Since $Q = vA$, we have

$$v = u_{ave} = \frac{Q}{A} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

VI. Solving with MATLAB

Now, our goal in this example is simply to evaluate, plot and compare the laminar and turbulent velocity profiles associated with fluid flow in a pipe for a typical situation. An algorithm to do this is given below:

1. Define the volumetric flow rate, Q and the pipe diameter, *D*.
2. Compute the pipe area and the average velocity via Eq. (8).
3. Knowing u_{ave} compute u_{max} from Eq. (17) and Eq. (10), for laminar and turbulent flow.
4. Define a discrete spatial grid, *r*, and evaluate the laminar and turbulent flow profiles via Eq. (14) and Eq. (16), respectively (use element-by-element vector arithmetic).
5. Plot both curves on a single axis for ease of comparison.
6. Print some summary data (*Q*, u_{ave} , u_{max} for both cases etc.) and interpret the overall results.

This algorithm assumes that the average velocity is the same for both the laminar and turbulent profiles (same *Q* and *D*). This was done so that we could focus on the shape of the profiles (not the actual magnitude of the velocity). In many cases, however, turbulent flow is associated with a higher average velocity (and higher *Q*) – although it doesn' t have to be, since we could have the same *Q* and *D* but a different viscosity.

In any case, the above algorithm was implemented in pipe_flow_1.m, as shown in Table 1. The program is quite straightforward and it produces a single plot that is given in Fig.6,and a brief summary table of results that is reproduced in Table 2. Since the velocity profile is symmetric, the visualization focuses on only half of the pipe over the range $0 \le r \le R$.

As apparent, the laminar flow case has a parabolic profile with a peak velocity that is twice the average (note that the flow rate was chosen to give $u_{ave} = 2m/s$).The turbulent flow profile is much flatter over most of the pipe , with a larger gradient near the pipe wall. This is characteristic of the local mixing that causes a more uniform profile in the center of the pipe. Since the turbulent profile is more uniform, the maximum to average value is much smaller than for the laminar flow case (about 1.22 for turbulent flow compared to 2.0 for laminar flow).

TABLE 1:
Program listing for pipe_flow_1.m

```
%PIPE_FLOW_1.M Plot Laminar and Turbulent Velocity Profiles in a pipe
%This illustrates function evaluation and plotting in Mat lab.
%The goal here is simply to evaluation and plot the velocity profiles for
% laminar and turbulent flow in a circular pipe geometry.
%For laminar flow, the velocity profile is parabolic, where u(r) is given
%by  u(r) = u max * [1 - (r / R)^2]
%For turbulent flow the velocity profile is given by
% u(r) = u max * [1 - (r / R)]^ m
%Where  m = 1/7 for fully developed turbulent flow.
%In evaluating these functions, we will again illustrate Mat lab' s vector processing capabilities %and some simple 2-D plotting function available in MATLAB.
%getting started
clc
 clear all, close all, nfig=0;
%
%define problem parameters
Q1 = 64.25  %volumetric flow rate
Q2 = Q1/15850  ;                    %Convert Q to
D = 2 * 0.0254                      %Pipe diameter (inches converted to meter)
R = D / 2;                          %Pipe radius (m)
m = 1/7 ;                           %exponent in turbulent flow equations
%
%Compute some derived quantities
A= pi*R^2;                          %pipe flow area
uave= Q2/A;                         %ave velocity
uLmax= 2*uave;                       %max velocity for turbulent flow
uTmax= (m+1)*(m+2)*uave/2;                 %max velocity for turbulent flow
%
%Evaluate the laminar and turbulent velocity profiles
Nr=101;         r=linspace (0, R, Nr);   %discrete r vector
uL=uLmax*(1-(r/R) ^2);   %laminar velocity profile
uT=uTmax*(1-(r/R)^m;     %turbulent velocity profile
%
%Now plot the results(both profiles on the same plot)
nfig=nfig+1; figure (nfig)
Plot (r/R,uL,'  r-' , r/R, uT, '  g- -' , '  Line Width' , 2), grid
title ('  Pipe\_Flow\_1: Laminar vs. Turbulent Velocity Profiles' );
xlabel('  Normalized Radial Position- (r/R)' ), ylabel ('  Fluid Velocity (m/s)' )
legend("  Laminar Flow' ,'  Turbulent Flow' )
%
%Finally, let' s print out a few summary results
fprintf(1,'  \n   Summary Results for the Pipe Flow Problem \n\n' )
fprintf(1,'   Volumetric flow rate (gal/min):          %10.3f \n' , Q1)
fprintf(1,'  Volumetric flow rate (m^3/s):             %10.3e \n' , Q2)

fprintf(1,'  Pipediameter (inches):          %10.3f \n'  ,D/0.0254)
fprintf(1,'  Average velocity (m/s):                          %10.3f \n' , uave)
fprintf(1,'  LAMINAR flow peak velocity (m/s):  %10.3f   \n' , uLmax)
fprintf(1,'  TURBULENT flow peak velocity  (m/s):  %10.3f \n' , uTmax)
%
%end program
```

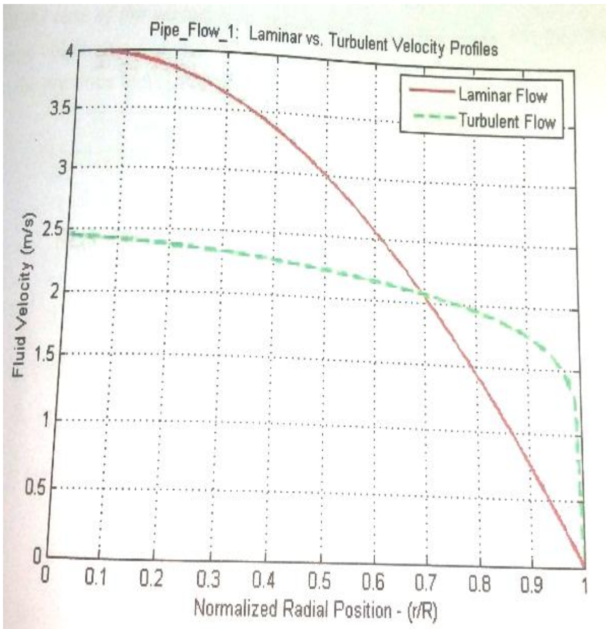

Figure 6: Laminar and turbulent velocity profiles in a circular pipe.

TABLE 2:

Summary results from the pipe_flow_1.m program.

Summary Results for the Pipe flow Problem
Volumetric flow rate (gal/min): 64.250
Volumetric flow rate (m^3/s): 4.054e-003
Pipe diameter (inches): 2.000
Average velocity (m/s): 2.000
LAMINAR flow peak velocity(m/s): 4.000
TURBULENT flow peak velocity(m/s): 2.449

In the case of the variation in volumetric flow rate: Using comparatively large volumetric flow rate such as 70 gal/min at the place of 64.25 gal/min in the previous **MATLAB** code we get,

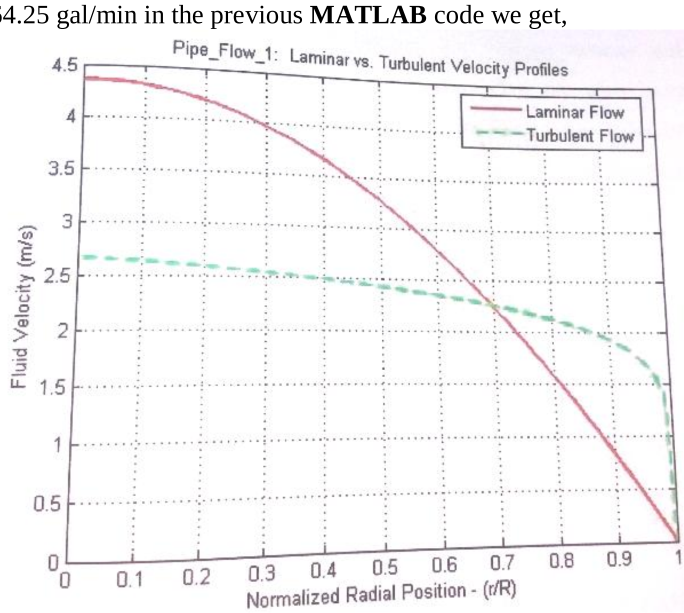

Figure 7: Laminar and Turbulent velocity profiles in a circular pipe when Q_1 =70 gal/min.

TABLE 3:

Summary results from the pipe_flow _1.m program when Q_1 =70 gal/min.

Summary Results for the Pipe flow Problem

Volumetric flow rate (gal/min): 70.000
Volumetric flow rate (m^3/s): 4.416e-003
Pipe diameter (inches) : 2.000
Average velocity (m/s) : 2.179
LAMINAR flow peak velocity(m/s): 4.358
LAMINAR flow peak velocity(m/s): 2.668

From the Fig. 6, Fig. 7 and Table 2, Table 3. We observed that, if the volumetric flow rate is increase , the average velocity and the profile pick velocity of laminar and turbulent flow are also increase. Similarly, if the volumetric flow rate is decrease, the average velocity and the profile pick velocity of laminar and turbulent flow are also decrease.

In the case of the variation in volumetric flow rate: Using comparatively large pipe diameter such as 3 inches at the place of 2 inches in the MATLAB code we get,

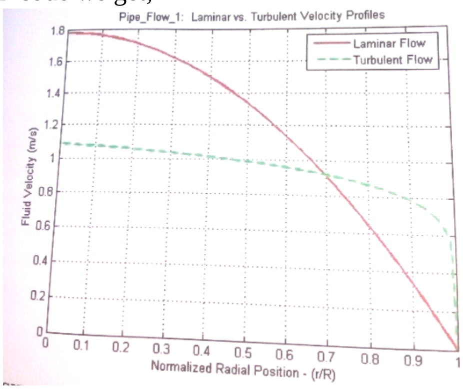

Figure 8: Laminar and Turbulent velocity profiles in a circular pipe when *D* =3 inches.

TABLE 4:

Summary results from the pipe_flow _1.m program when D=3 inches.
Summary Results for the Pipe flow Problem
Volumetric flow rate (gal/min): 64.250
Volumetric flow rate (m^3/s): 4.054e-003
Pipe diameter (inches) : 3.000
Average velocity (m/s) : 0.889
LAMINAR flow peak velocity(m/s): 1.778
LAMINAR flow peak velocity(m/s): 1.088

From the Fig. 6, Fig. 8 and Table 2,Table 4, we observed that, if the volumetric flow rate is increase , the average velocity and the profile pick velocity of laminar and turbulent flow are also decrease. Similarly, if the volumetric flow rate is decrease, the average velocity and the profile pick velocity of laminar and turbulent flow are also increase.

VII. Conclusion

We have calculated the velocity profiles that occur during laminar and turbulent flow in a circular pipe.

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Md. Alamin Applying Laminar and Turbulent Flow and measuring Velocity Profile Using MATLAB. IOSR Journal of Mathematics (IOSR-JM) , vol. 13, no. 6, 2017, pp. 52-59.