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# Comparison of Euclidean and Non-Euclidean Geometry

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***Abstract :*** *This paper described the comparison of Euclidean and non- Euclidean geometry. Geometry was extreme important to ancient societies and was used for surveying, astronomy, navigation, and building. Geometry, mainly divided in two parts:*
*1. Euclidean geometry*
*2. Non- Euclidean geometry*
*Also non –Euclidean geometry is divided into two sub parts.*

- *Hyperbolic geometry*
- *Spherical geometry*

*The intention of this article is to compare Euclidean and non –Euclidean geometry.*
***Keywords:*** *Euclidean geometry, hyperbolic geometry, non –Euclidean geometry, spherical geometry,*

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## I. Introduction

For this purpose I furnish two geometries and then compare them. For that first of all I furnish Euclidean geometry and his book elements and then I illustrate Euclid failure and discovery of non –Euclidean geometry and then furnish non –Euclidean geometry after that I discussed about some similarities and differences between Euclidean and non Euclidean geometry. *Geometry* is a branch of mathematics that is concerned with the properties of configurations of geometric objects -points, (straight) lines, and circles, being the most basic of these. Although the word *geometry* derives from the Greek *geo* (earth) and *metron* (measure) it concerned with the properties of space and figures.

## II. Euclidean geometry

Euclidean geometry is a mathematical system attributed to the Alexandrian Greek mathematician Euclid, which he described in his textbook on geometry: the *Elements*. Euclid's method consists in assuming a small set of intuitively appealing axioms, and deducing many other propositions (theorems) from these. Although many of Euclid's results had been stated by earlier mathematicians, Euclid was the first to show how these propositions could fit into a comprehensive deductive and logical system. The *Elements* begins with plane geometry, still taught in secondary school as the first axiomatic system and the first examples of formal proof. It goes on to the solid geometry of three dimensions. Much of the *Elements* states results of what are now called algebra and number theory, explained in geometrical language. Euclid The elements are mainly a systematization of earlier knowledge of geometry. Its superiority over earlier treatments was rapidly recognized, with the result that ratios between the volume of a cone and a cylinder with the same height and base.

## III. The parallel postulate

If two lines intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.

## IV. Euclide Failure

Since the dawn of time, Man has always been concerned to understand the world he lived in. He observed carefully and accurately the shapes of nature and felt the need to theorize, and later, to find mathematical proofs for various empirical elements. If we examine with some attention, in fact, we have a daily need to use Geometry. When we say something is far away, we are referring to a length and this is Geometry. When we discuss about the area of a football field, we use Geometry. If we say that a dress is wide, we are thinking of volumes, and therefore thinking in Geometry. Geometry accompanies us all the time. One of the most important books ever written is probably Euclid's *Elements*. Its volumes have provided a model for the rigorous development of mathematical ideas, which is still used today. The Euclidean Geometry defines the situations of the plan. However, when we are dealing with different surfaces, we are faced with the impossibility of solving problems through the same geometry. Unlike what happens with the initial four postulates of Euclid, the Fifth Postulate, the famous Parallel Postulate, revealed a lack intuitive appeal, and several were the mathematicians who, throughout history, tried to show it. Many retreate before the findings that this would be untrue; some had the courage and determination to make such a falsehood, thus opening new doors to Geometry. One puts up, then, two questions. Where can be found the clear concepts of such Geometries? And how important is the knowledge and study of Geometries, beyond the Euclidean, to a better understanding of the world around us? The study, now developed, seeks to answer these questions.

Since the primary objective is a response to these earlier questions, this study is divided into three phases. The first phase focuses on the historical evolution of Geometry, from its beginnings to the work of the Greek Euclid. In a second phase, the main precursors of Geometry are presented and, subsequently, the discoverers of Non-Euclidean Geometries, the Elliptic and Hyperbolic Geometries themselves, being the most outstanding among all the Non-Euclidean, and even some models of its representations. The third and final phase is related to the analysis of the presence of Non-Euclidean Geometries in Art and in the Real, the study of Geometry in Secondary Education and Non-Euclidean Geometries in Higher Education, ending up with some philosophical implications that one understands be relevant, given all the controversy generated around these Non-Euclidean theories of Geometry.

## V. The Forerunners of Non-Euclidean Geometries

**(The fifth postulate of Euclidean geometry)**

Several mathematicians tried to prove the correctness of Euclid's $5^{th}$ *P*ostulate for a long time. Although they could get close to real conclusions, they failed, as its primary objective was to prove the Postulate, and not conclude that this could be false (Saccheri, Legendre, Farkas Bolyai, Gauss). Moreover, even with assurances regarding their results, the fear of facing the mathematical community, and the shame of being marginalized by their act of courage, always stopped them from publishing such findings. As Greenberg said, it is delightfully instructive to observe the mistakes made by capable people as they struggled with the strange possibility that they or their culture might not accept their conclusions. , but it was finally shown to be impossible .Postulate 5, the Parallel Postulate If a straight line meets two straight lines, so as to make the two interior angles on the same side of it taken together less than two right angles, these straight lines, being continuously produced, shall at length meet on the Alternative, but equivalent, version of the Parallel Postulate Given a line l and a point P not on l, there is only one line m containing P such that l||m. These alternative versions the most commonly used version, but there are several others. It is interesting that one of the other equivalent versions is the statement that "the sum of the angles in a triangle is 180°".

We prove this as a theorem, and the Alternative version of the Parallel Postulate will be a very important piece of the proof. They are equivalent because if you started with the "the sum of angles in triangle is 180°" you could prove the parallel postulate. All theorems whose proofs relyon the Parallel Postulate, plus the Absolute Geometry theorems, are what is known as Euclidean Geometry or Flat Geometry.

Euclid's famous treatise, the Elements, was most probably a summary of side on which are the angles that are less than two right angle what was known about geometry in his time, rather than being his original work. In it, he sets out five geometric "postulates", the fifth of which is this:

If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

## VI. Discovery of non-Euclidean geometry

The beginning of the 19th century would finally witness decisive steps in the creation of non-Euclidean geometry. Circa 1813, Carl Friedrich Gauss and independently around 1818, the German professor of law Ferdinand Karl Schweikart had the germinal ideas of non-Euclidean geometry worked out, but neither published any results. Then, around 1830, the Hungarian mathematician János Bolyai and the Russian mathematician Nikolai Ivanovich Lobachevsky separately published treatises on hyperbolic geometry. Consequently, hyperbolic geometry is called Bolyai-Lobachevskian geometry, as both mathematicians, independent of each other, are the basic authors of non-Euclidean geometry. Gauss mentioned to Bolyai's father, when shown the younger Bolyai's work, that he had developed such a geometry several years before,[10] though he did not publish. While Lobachevsky created a non-Euclidean geometry by negating the parallel postulate, Bolyai worked out a geometry where both the Euclidean and the hyperbolic geometry are possible depending on a parameter *k*. Bolyai ends his work by mentioning that it is not possible to decide through mathematical reasoning alone if the geometry of the physical universe is Euclidean or non-Euclidean; this is a task for the physical sciences.

Bernhard Riemann, in a famous lecture in 1854, founded the field of Riemannian geometry, discussing in particular the ideas now called manifolds, Riemannian metric, and curvature. He constructed an infinite family of geometries which are not Euclidean by giving a formula for a family of Riemannian metrics on the unit ball in Euclidean space. The simplest of these is called elliptic geometry and it is considered to be a non-Euclidean geometry due to its lack of parallel lines. By formulating the geometry in terms of a curvature tensor, Riemann allowed non-Euclidean geometry to be applied to higher dimensions

## VII. Axiomatic basis of non-Euclidean geometry

Euclidean geometry can be axiomatically described in several ways.
Unfortunately, Euclid's original system of five postulates (axioms) is not one of these as his proofs relied on several unstated assumptions which should also have been taken as axioms. Hilbert's system consisting of 20 axioms most closely follows the approach of Euclid and provides the justification for all of Euclid's proofs. Other systems, using different sets of undefined terms obtain the same geometry by different paths. In all approaches, however, there is an axiom which is logically equivalent to
Euclid's fifth postulate, the parallel postulate. Hilbert uses the Playfair axiom form, while Birkhoff, for instance, uses the axiom which says that
"there exists a pair of similar but not congruent triangles."
In any of these systems, removal of the one axiom which is equivalent to the parallel postulate, in whatever form it takes, and leaving all the other axioms intact, produces absolute geometry. As the first 28 propositions of Euclid (in *The Elements*) do not require the use of the parallel postulate or anything equivalent to it, they are all true statements in absolute geometry. axiom form, since it is a compound statement (... there exists one and only one ...), can be done in two ways. Either there will exist more than one line through the point parallel to the given line or there will exist no lines through the point parallel to the given line. In the first case, replacing the parallel postulate (or its equivalent) with the statement "In a plane, given a point P and a line $\ell$ not passing through P, there exist two lines through P which do not meet $\ell$" and keeping all the other axioms, yields hyperbolic geometry. The second case is not dealt with as easily. Simply replacing the parallel postulate with the statement, "In a plane, given a point P and a line $\ell$ not passing through P, all the lines through P meet $\ell$", does not give a consistent set of axioms. This follows since parallel lines exist in absolute geometry, but this statement says that there are no parallel lines. This problem was known (in a different guise) to Khayyam, Saccheri and Lambert and was the basis for their rejecting what was known as the "obtuse angle case". In order to obtain a
consistent set of axioms which includes this axiom about having no parallel lines, some of the other axioms must be tweaked. The adjustments to be made depend upon the axiom system being used. Among others these tweaks will have the effect of modifying Euclid's second postulate from the statement that line segments can be extended indefinitely to the statement that lines are unbounded. Riemann's elliptic geometry emerges as the most natural geometry satisfying this axiom.

To obtain a non-Euclidean geometry, the parallel postulate (or its equivalent) *must* be replaced by its negation. Negating the Playfair's axiom form, since it is a compound statement (..there exists one and only one ...), can be done in two ways. Either there will exist more than one line through the point parallel to the given line or there will exist no lines through the point parallel to the given line. In the first case, replacing the parallel postulate (or its equivalent) with the statement "In a plane, given a point P and a line $\ell$ not passing through P, there exist two lines through P which do not meet $\ell$" and keeping all the other axioms, yields hyperbolic geometry. The second case is not dealt with as easily. Simply replacing the parallel postulate with the statement, "In a plane, given a point P and a line $\ell$ not passing through P, all the lines through P meet $\ell$", does not give a consistent set of axioms. This follows since parallel lines exist in absolute geometry, but this statement says that there are no parallel lines. This problem was known (in a different guise) to Khayyam, Saccheri and Lambert and was the basis for their rejecting what was known as the "obtuse angle case". In order to obtain a
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**1 Comparison of Euclidean geometry and non-Euclidean geometry**

A. Euclidean geometry
B. Parabolic geometry
C. Spherical geometry

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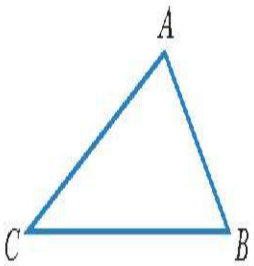
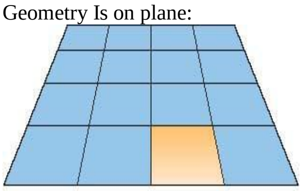
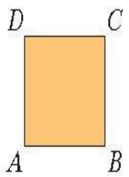
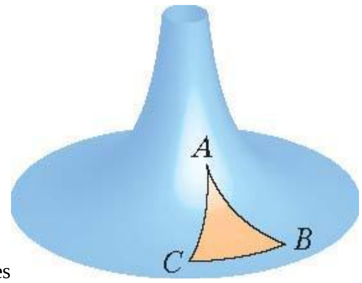
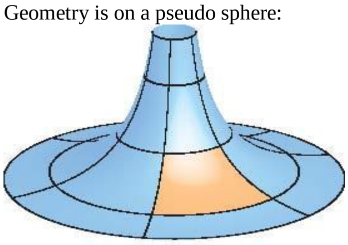
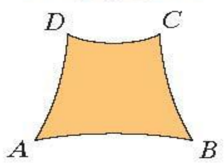
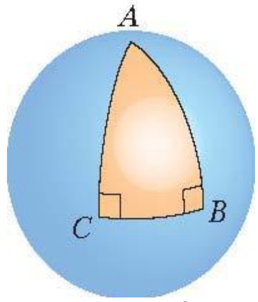
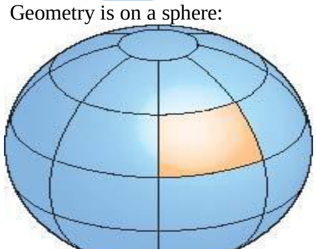
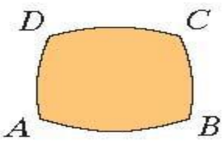
|---|---|---|
| **Euclidean** | **Non- Euclidean** | |
| **Parabolic geometry** | **Hyperbolic geometry** | **Spherical geometry** |
| **Euclid (300 B.C)** | **Lobatchevski , Bloyai (1830)** | **G .F .B .Riemann (1850)** |
| Euclidean geometry in this classification is parabolic geometry, through the name is less often used. | The negatively curved non-Euclidean geometry is called hyperbolic geometry | Spherical geometry is called elliptical geometry, but the space of elliptic geometry is really has points = antipodal pairs on the sphere. |
| Euclidean Geometry is what we're familiar with on a day to day basis and follows Euclid's Parallel postulate; given a Straight Line and a Point not on that Line, there is only one Line you can draw that passes through that Point AND is parallel to the first Line. | Hyperbolic Geometry can be derived from the answer to Saccheri's Quadrilateral where the two remaining angles are smaller than 90 degrees. It's typically only used to very high level math and physics and some models of the universe revolve around the use of Hyperbolic Geometry. This leads to a variation of Euclid's Parallel Postulate, and in Hyperbolic Geometry this new version states; given a Straight Line and a Point not on that Line, there are at least two lines parallel to the initial Line. | Elliptic Geometry is derived from the last answer to Saccheri's Quadrilateral where the two remaining angles are larger than 90 degrees. Unlike Hyperbolic Geometry, Elliptic Geometry is widely used by pilots and ship captains because it describes the Geometry on the surfaces of Spheres. Similar to Euclidean and Hyperbolic Geometries, it too has it's own variation on the original Parallel Postulate which states; given a Straight Line and a Point not on that Line, there are no lines parallel to the initial Line. |
| Euclidean geometry is flat so its curvature is zero. | Negatively curved | Positively curved |
| 5th axiom/parallel axiom: given a straight line and a point not on the line, there exists one and only one straight line through the pioint which is parallel to the original line. | Given a straight line and a point not on the line .there exists an infinite number of straight lines through the point parellal to the original line. | Given a straight line and the point not on the line, there are no straight lines through the point parallel to the original line. |
| The sum of the angles of a triangle is 180 degrees.   Geometry Is on plane:    | The sum of the angles of a triangle is less than 180   degrees <br> Geometry is on a pseudo sphere:    | The sum of the angles of a triangle is always greater than 180 degrees.   Geometry is on a sphere:    |
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| In Euclidean geometry, given a point and a line, there is exactly one line through the point that is in the same plane as the given line | In hyperbolic geometry there are at least two distinct lines that passes through the point and are parallel to (in the same plane as and do not intersect) the given line. | In spherical geometry there are no such lines. |

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| and never intersects it. | | |
| Definition of a line is "breadth less length" and a straight line being a line "which lies evenly with the points on itself". | | Lines are defined such that the shortest distance between two points lies along them. Lines in spherical geometry are *great circles*. A great circle is the largest circle that can be drawn on a sphere. Great circles are lines that divide a sphere into two equal. |
| If two lines are parallel to a third line, then the two lines are parallel to each other. | This is false in hyperbolic geometry. | |
| If two lines are parallel then, two lines are equidistant. | This is false in hyperbolic geometry. | |
| Lines that do not have an end (infinite lines),also do not have a boundary (a point that they are headed toward, yet never reach) | This is false in hyperbolic geometry. | |
| Euclidean geometry is geometry on a plane(like the surface of a piece of paper), and deals with points and lines | Non Euclidean geometry are on a sphere. Hyperbolic geometry is associated with 'curved' space | Spherical geometry is plane geometry on the surface of a sphere. So a straight line on the surface of a sphere would become an arc, and the longest line would equate to the diameter of the sphere. |
| In Euclidean geometry<br>.<br><br>. .<br>is a triangle | In non-Euclidean geometry<br>. . .<br>is a triangle. | |
| euclidean geometry deals with more structured mathematics, such as triangles. | while non euclidean geometry has more abstract shapes, ones with no describable, or permanent name | |
| Euclidean geometry has 1 parallel line through any given point not on a line | Hyperbolic geometry has at least 2 parallel lines | Spherical geometry has no parallel lines |
| Euclidean geometry is hold in two dimensions. | Hold in two and three dimension. | Hold in two and three dimension |
| Euclidean geometry is used in surveying and building | A modern use of Hyperbolic geometry is in the theory of special Relativity ,particularly minskowski space time ,and gyro vector space | Concepts of spherical geometry may also be applied to the oblong sphere, though minor modifications must be implemented on certain formulas.<br>Two practical application of the principal of spherical geometry are to navigation and astronomy |
| It has simple trigonometry | | It has spherical geometry |

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