

$\hat{\beta}g$ - Soft semi closed sets in Soft Topologic spaces

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Abstract: In 1999 Molodstov introduced the main concept of Soft set theory for Uncertainties Problems. They shows that Soft sets are a class of extraordinary information systems. So it's important to study the structure of soft sets for information systems. In this piece we carry on to investigate the Properties of Soft semi closed sets in Soft topological spaces. The objective of this work is to describe the basic concept of $\hat{\beta}g$ - Soft semi closed sets in Soft topological spaces.

Key Words: Soft semi open sets – Soft semi closed sets - $\hat{\beta}g$ - Soft semi closed sets.

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I. Introduction

Soft set theory concept has been introduced by Molodstov [9] this Soft set Content has been functional to many branches of applied mathematics such as Riemann Integration, Operations Research etc. The notion of a topological spaces for Soft sets was formed by Shabir and Naz which is clear over an initial Universe with a fixed set of Parameters. Levine [8] introduced generalized closed sets in general topology . Kannan [7] introduced Soft set generalized closed and Open sets in Soft topological spaces. Kohila.S and M.Kamaraj[5] introduced the Characterization of $\hat{\beta}g$ - Soft closed sets in Soft topological spaces. In this paper we established a new category of sets namely $\hat{\beta}g$ - Soft semi closed sets in Soft topological spaces. Further more ,these research not only can form the speculative concept for further applications of topology on Soft set but also lead to the enlargement of informations.

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II. Preliminaries

2.1 Definition:[1]

A pair off (L,E) is called a Soft set over X if and only if F is a mapping of E into the set of all subsets of the set X_E . In other words, the Soft set is a Parameterized family of Soft subsets of the set X_E . Every set $F(e)$, $e \in E$.

From this family may be considered as the set of e - approximate elements of the Soft set.

Example:

A Soft set (L,E) describes the attractiveness of the houses which Mr. Y is going to buy

- X – is the set of houses under consideration .
- E – is the set of parameters ,Each Parameter is a word or a sentence is given below
- {expensive, wooden, beautiful, cheap, in the green surrounding, Modern, in good repair, in bad repair }

To define a Soft set means to point out a collection of houses as in E.

In this case the sets F (may be arbitrary, Some of them may be empty, Some of them may be non-empty.

2.2 Definition:[6]

Any two Soft sets (F,H) and (G,J) over a common Universe X_E , (F,H) is a Soft set subset of (G,J) over a common Universe X_E , If $H \subseteq J$ and $\forall a \in H$, $F(a) \subseteq G(a)$

It is denoted by $(F,H) \subseteq (G,J)$.

2.3 Definition: [6]

Two Soft sets (F,H) and (G,J) over a common Universe X_E is said to be Soft equal if $(F,H) \subseteq (G,J)$ and $(G,J) \subseteq (F,H)$.

II. Result

If $(F,H) = (G,J)$ then $H = B$

2.4 Definition:[6]

The complement of a Soft set (F,H) denoted by $(F,H)^c$, It is defined by $F^c : H \rightarrow P (X)$ is a mapping given by $F^c (a) = X_E - F (a)$, $\forall a \in H$.

F^c is called the Soft complement function of F .clearly $(F^c)^c$ is the same as F and $((F,H)^c)^c = (F, H)$.

2.5 Definition:[6]

A Soft set (F, E) over X_E is said to be a Null Soft set , It is defined by,

if $\forall e \in E, F(e) = \emptyset$. It is denoted by ϕ_E

2.6 Definition:[6]

A Soft set (L,E) over X_E is said to be Absolute Soft set denoted by X_E . If $\forall e \in E, L(e) = X$. Clearly $X_E^c = \phi_E$ and $\phi_E^c = X_A$

2.7 Definition:[12]

Let Y be a non-empty subset of X , then Y denotes the Soft set (Y,E)

Over X_E for which $Y(e) = Y_E$, for all $e \in E$. Especially , (X,E) will denoted by X_E .

2.8 Definition :[12]

Let (L,E) be a Soft set over X and $x \in X$. $x \in (F,E)$ whenever

$x \in F(e)$ for all $e \in E$. For any $x \in X$, $x \notin (F, E)$, if $x \notin F(e)$ for some $e \in E$.

2.9 Definition:[7]

The Union of two (F,H) and (G,J) over a common Universe X_E is the Soft set

(I,C) where $C = H \cup J$ and for all $e \in C$, $I(e) = H(e)$ if $e \in H \setminus J$,

$I(e) = J(e)$ if $e \in B \setminus A$ and $I(e) = F(e) \cup G(e)$ if $e \in H \cap J$.

It is denoted by $(F, H) \cup (G, J) = (I,C)$.

2.10 Definition:[7]

The Intersection of two Soft sets (F,H) and (G B) over a common Universe X_E is

the Soft set (H ,C) where $C = A \cap B$ and for all $e \in C, H(e) = F(e) \cap G(e)$.

This relationship is written as $(F,A) \cap (G,J) = (H,C)$.

We denote the family of these Soft sets denoted by $SSE (X)_E$

2.11 Definition: [12]

Let τ be the collection of a Soft set over a Universe X_E with a fixed set of Parameter E , then

$\tau \subseteq SS (X)_E$ is called a Soft set topology on X if

- $X, \phi \in \tau$.
- The union of any number of Soft sets in τ belongs to τ
- The Intersection of any two Soft set s in τ belongs to τ

(X , τ , E) is called a Soft set topological space.

2.12 Definition:[7]

Let (X, τ, E) be a Soft set topological space over X_E and (F, E) be a Soft set over X . then the closure of (F, E) denoted by $\text{cl}(F, E)$. It is the intersection of all Soft set closed super set of (F, E) .

2.13 Definition:[12]

Let (X, τ, E) be a Soft set topological space and $(F, E) \in \text{Soft set}(X)_E$, (F, E) is said to be,

- Soft set pre open set if $(F, E) \subseteq \text{int}(\text{cl}(F, E))$.
- Soft set semi open set if $(F, E) \subseteq \text{cl}(\text{int}(F, E))$.
- Soft set α - open set if $(F, E) \subseteq \text{int}(\text{cl}(\text{int}(F, E)))$.

2.14 Definition:[12]

A Soft set (F, E) in a Soft set topological space (X, τ, E) is said to be Soft set pre generalized closed (in short Soft set pg closed) set, if Soft set $\text{cl}(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and (G, E) is a Soft open set in X .

2.15 Definition:[12]

A Soft set (F, E) in a Soft set topological space (X, τ, E) is said to be Soft set generalized pre closed set (in short Soft set gp closed) sets, if Soft set $\text{cl}(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and (G, E) is Soft preopen set in X .

2.16 Definition:[12]

A Soft set (F, E) in a Soft set topological space (X, τ, E) is said to be Soft set generalized -closed set (in short Soft set g closed) set, if Soft set $\text{cl}(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and (G, E) is Soft open set.

2.17 Definition:[12]

A Soft set (F, E) in a Soft set topological space (X, τ, E) is said to be Soft set sg- closed sets, if Soft set $\text{cl}(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and (G, E) is Soft semi-open set.

2.18 Definition:[12]

A Soft set (A, E) in a Soft topological space (X, τ, E) is called

1. A Soft generalized closed set (Soft g- closed) in a Soft topological space (X, τ, E) if $\text{cl}(A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is Soft open in (X, τ, E)
1. A Soft sg closed set if $\text{S.scl}(A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is Soft semi-open.&
A Soft gs closed set if $\text{S.scl}(A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is Soft open.
2. A Soft rwg closed set if $\text{cl}(\text{int}(A, E)) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is Soft regular open.
3. A Soft wg closed set if $\text{cl}(\text{int}(A, E)) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is Soft open.

2.19 Definition:[12]

A Soft set (F, E) in a Soft topological space (X, τ, E) is called Soft generalized preregular closed (in short Soft gpr closed) set, if Soft $\text{pcl}(F, E) \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and (G, E) is Soft regular open set.

2.20 Definition:[12]

A Soft set (F, E) in a Soft topological space (X, τ , E) is called a Soft regular generalized closed (in short Soft rg closed) set, if $\text{Soft cl}(F, E) \subseteq (G, E)$ whenever (F, E) \subseteq (G, E) and (G, E) is Soft regular open set.

2.21 Definition:[12]

A Soft set (F, E) in a Soft topological space (X, τ , E) is called Soft Weakly closed (in short Soft SW closed) set, if $\text{Soft cl}(F, E) \subseteq (G, E)$ whenever (F, E) \subseteq (G, E) and (G, E) is Soft semi open

III. $\hat{\beta}G$ - Soft Semi Closed Set

3.1 Definition:

A Soft subset (L, E) of a Soft topological Space (X, τ , E) is called $\hat{\beta}G$ - Soft semi closed set, if $\text{cl int cl}(L, E) \subseteq (U, E)$ whenever (F, E) \subseteq (U, E) and (U, E) is Soft semi Open in X_E .

3.2 Example:

The Following examples shows that the few $\hat{\beta}G$ - Soft semi closed sets

Let $X = \{h_1, h_2, h_3\}$; $E = \{e_1, e_2\}$ and

$\tau = \{\Phi, X, (F_1, E), (F_2, E)\}$ where $(F_1, E) = \{\{h_1\}, \{h_1\}\}$, $(F_2, E) = \{\{h_1, h_3\}, \{h_1, h_3\}\}$.

Closed sets are $\{\{h_2, h_3\}, \{h_2, h_3\}\}$, $\{\{h_2\}, \{h_2\}\}$,

Soft Set	Interior of the Soft Set
$\{h_1\}, \{h_1\}$, $\{h_1, h_3\}, \{h_1, h_3\}$. $\{h_1, h_2\}, \{h_1, h_2\}$ $\{h_1\}, \{h_1, h_2\}$ $\{h_1, h_3\}, \{h_1\}$	(F ₁ , E)
$\{h_3\}, \{h_1, h_3\}$ $\{h_1, h_3\}, \{h_1, h_3\}$. $\{h_2, h_3\}, \{h_2, h_3\}$ $\{h_2\}, \{h_2\}$ $\{h_3\}, \{h_3\}$	$\{\{\Phi\}, \{\Phi\}\}$

Soft Set	Closure of a Soft Set
$\{h_1\}, \{h_1\}$, $\{h_1, h_3\}, \{h_1, h_3\}$. $\{h_1, h_2\}, \{h_1, h_2\}$ $\{h_1\}, \{h_1, h_2\}$ $\{h_1, h_3\}, \{h_1\}$ $\{h_3\}, \{h_1, h_3\}$ $\{h_1, h_3\}, \{h_1, h_3\}$.	$\{X_E, X_E\}$
$\{h_2\}, \{h_2\}$	$\{h_2\}, \{h_2\}$
$\{h_3\}, \{h_3\}$	$\{h_2, h_3\}, \{h_2, h_3\}$

Soft semi Open in X	$\hat{\beta}_G$ - Soft semi closed set
$\{\{h_1\}, \{h_1\}\}$	$\{\{h_2\}, \{h_2\}\}$
$\{\{h_1, h_3\}, \{h_1, h_3\}\}$	$\{\{h_3\}, \{\Phi\}\}$
$\{\{h_1, h_2\}, \{h_1, h_2\}\}$	$\{\Phi, \{h_3\}\}$
$\{\{h_1\}, \{h_1, h_2\}\}$	$\{\{h_3\}, \{h_3\}\}$
$\{\{h_2\}, \{h_1, h_2\}\}$	$\{\{\Phi\}, \{\Phi\}\}$
	$\{\{h_2\}, \{\Phi\}\}$
	$\{\{X\}, \{X\}\}$

3.3 Example :

$X = \{h_1, h_2, h_3\}$ $E = \{e_1, e_2\}$ $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$ Where $(F_1, E) = \{\{h_1\}, \{h_1\}\}$; $(F_2, E) = \{\{h_2\}, \{h_2\}\}$; $(F_3, E) = \{\{h_1, h_2\}, \{h_1, h_2\}\}$ $(F_4, E) = \{\{h_2, h_3\}, \{h_2, h_3\}\}$; $(F_5, E) = \{\{h_1, h_3\}, \{h_1, h_3\}\}$; $(F_6, E) = \{\{h_3\}, \{h_3\}\}$

Solution :

Soft Set	Interior of the Soft Set
$\{\{h_1\}, \{h_1\}\}$	(F_1, E)
$\{\{h_2\}, \{h_2\}\}$	(F_2, E)
$\{\{h_1\}, \{h_1, h_2\}\}$ $\{\{h_1, h_2\}, \{h_1\}\}$ $\{\{h_1\}, \{h_1, h_2\}\}$ $\{\{h_1\}, \{h_1\}\}$	(F_1, E)
$\{\{h_2\}, \{h_2, h_3\}\}$ $\{\{h_2, h_3\}, \{h_2\}\}$ $\{\{h_2, h_3\}, \{h_2, h_3\}\}$ $\{\{h_2\}, \{h_1, h_2\}\}$ $\{\{h_2\}, \{h_2\}\}$	(F_2, E)
$\{\{h_1\}, \{h_1, h_3\}\}$ $\{\{h_3\}, \{h_1, h_3\}\}$ $\{\{h_1, h_3\}, \{h_3\}\}$ $\{\{h_1, h_3\}, \{h_1\}\}$	(F_1, E)

Soft Set	Closure of the Soft Set
$\{\{h_1\}, \{h_1\}\}$	(F_5, E)
$\{\{h_2\}, \{h_2\}\}$	(F_4, E)
$\{\{h_1\}, \{h_1, h_2\}\}$ $\{\{h_1, h_2\}, \{h_1\}\}$ $\{\{h_1\}, \{h_1, h_2\}\}$	$\{\{X\}, \{X\}\}$
$\{\{h_2\}, \{h_2, h_3\}\}$ $\{\{h_2, h_3\}, \{h_2\}\}$ $\{\{h_2, h_3\}, \{h_2, h_3\}\}$ $\{\{h_2\}, \{h_1, h_2\}\}$ $\{\{h_2\}, \{h_2\}\}$	(F_4, E)
$\{\{h_1\}, \{h_1, h_3\}\}$ $\{\{h_3\}, \{h_1, h_3\}\}$ $\{\{h_1, h_3\}, \{h_3\}\}$ $\{\{h_1, h_3\}, \{h_1\}\}$	(F_5, E)

SOFT SEMI OPEN SET	$\hat{\beta}_g$ - Soft semi closed set
$\{\{h_1, h_1\}\}$ $\{\{h_2, \{h_2\}\}\}$ $\{\{h_1, h_2\},$ $\{h_1, h_2\}\}$ $\{\{h_2, h_3\}, \{h_2, h_3\}\}$ $\{\{h_1, h_3\}, \{h_1, h_3\}\}$ $\{\{h_2\}, \{\Phi\}\},$ $\{\{\Phi\}, \{h_2\}\}$ $\{\{h_1\}, \{\Phi\}\},$ $\{\{\Phi\}, \{h_1\}\}$ $\{\{h_1, h_2\}, \{h_1\}\}$ $\{h_1, h_2\} \{h_2\}\}$ $\{\{h_1\}, \{h_1, h_2\}\}$ $\{\{h_2\}, \{h_1, h_2\}\}$ $\{\{h_1\}, \{h_1\}\}$ $\{\{h_2\}, \{h_2\}\}$ $\{\{h_2\}, \{h_2, h_3\}\}$ $\{\{h_2, h_3\}, \{h_2, h_3\}\}$ $\{\{h_2, h_3\}, \{h_2\}\}$ $\{\{h_1\}, \{h_1, h_3\}\}$ $\{\{h_3\}, \{h_1, h_3\}\}$ $\{h_1, h_3\}, \{h_3\}\} \{\{h_1, h_3\}, \{h_1\}\}$	$\diamond \{\{h_1, h_2\}, \{\Phi\}\}$ $\diamond \{\{\Phi\}, \{h_1, h_2\}\}$ $\diamond \{\{h_2, h_3\}, \{\Phi\}\}$ $\diamond \{\{\Phi\}, \{h_2, h_3\}\}$ $\diamond \{\{h_2\}, \{h_2, h_3\}\}; \{\{h_3\}, \{h_2, h_3\}\}$ $\diamond \{\{h_3\}, \{h_3\}\}$ $\diamond \{\{h_2, h_3\}, \{h_3\}\}$ $\diamond \{\{h_2, h_3\}, \{h_2\}\}$ $\diamond \{\{\Phi\}, \{\{\Phi\}\}\}$ $\diamond \{\{h_2, h_3\}, \{h_2, h_3\}\}$ $\diamond \{\{h_1, h_3\}, \{\Phi\}\}$ $\diamond \{\{\Phi\}, \{h_1, h_3\}\}$ $\diamond \{\{h_1\}, \{h_1, h_3\}\}; \{\{h_3\}, \{h_1, h_3\}\}$ $\diamond \{\{h_3\}, \{h_3\}\}; \{\{h_1, h_3\}, \{h_3\}\}$ $\diamond \{\{h_1, h_3\}, \{h_1\}\}$ $\diamond \{\{\Phi\}, \{\{\Phi\}\}\}$ $\diamond \{\{h_1, h_3\}, \{h_1, h_3\}\}$ $\diamond \{\{h_3\}, \{\Phi\}\};$ $\diamond \{\{\Phi\}, \{\{h_3\}\}\}$ $\diamond \{\{\Phi\}, \{\{h_1\}\}\}$ $\diamond \{\{h_1\}, \{\{h_1\}, \{h_1\}\}\{h_1\}\}$ $\diamond \{\{h_3\}, \{h_3\}\};$

Remark 3.4

➤ If $(F, E) \subseteq (U, E)$ is proper Soft semi open set in X_E , and $cl(F, E) = X_E$.

Then (F, E) is not $\hat{\beta}_g$ - Soft semi closed set in X .

➤ If (F, E) is Soft semi open set in X and $cl(F, E) = \emptyset$. Then (F, E) is $\hat{\beta}_g$ - Soft semi closed set in X .

3.5 Theorem:

The Union of two $\hat{\beta}_g$ - Soft semi closed Subsets of X_E is also $\hat{\beta}_g$ - Soft semi closed Subsets of X .

Proof :

Suppose that (A, E) and (B, E) are two $\hat{\beta}_g$ - Soft semi closed Subsets of X_E .

Let $(A, E) \cup (B, E) \subseteq (U, E)$ and (U, E) is Soft semi open in X .

Since $(A, E) \cup (B, E) \subseteq (U, E)$.

we have $(A, E) \subseteq (U, E)$ and $(B, E) \subseteq (U, E)$. Since (U, E) is Soft semi open in X_E and (A, E) and (B, E) are two $\hat{\beta}_g$ - Soft semi closed Subsets of X .

We have $cl(A, E) \subseteq (U, E)$ and $cl(B, E) \subseteq (U, E)$.

Therefore $cl(A, E) \cup cl(B, E) = cl((A, E) \cup (B, E))$.

This implies that $int cl(A, E) \cup int cl(B, E) = int cl((A, E) \cup (B, E))$.

Hence $cl int cl(A, E) \cup cl int cl(B, E) = cl int cl((A, E) \cup (B, E))$.

Therefore $(A, E) \cup (B, E)$ are also $\hat{\beta}_g$ - Soft Semi closed set in X_E .

3.6 Remark :

The following example shows that the intersection of two $\hat{\beta}_g$ - Soft semi closed Subsets of X_E is also $\hat{\beta}_g$ - Soft semi closed Subsets of X .

Example :

The above Example 3.3 says that the intersection of two $\hat{\beta}_g$ - Soft semi closed Subsets of X_E is also $\hat{\beta}_g$ - Soft semi closed Subsets of X .

Solution :

Let $X = \{h_1, h_2, h_3\}$ $E = \{e_1, e_2\}$ $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$
 Where $(F_1, E) = \{\{h_1\}, \{h_1\}\}$; $(F_2, E) = \{\{h_2\}, \{h_2\}\}$; $(F_3, E) = \{\{h_1, h_2\}, \{h_1, h_2\}\}$

$(F_4, E) = \{\{h_2, h_3\}, \{h_2, h_3\}\}$; $(F_5, E) = \{\{h_1, h_3\}, \{h_1, h_3\}\}$; $(F_6, E) = \{\{h_3\}, \{h_3\}\}$

Soft subsets of $(F_4, E) = \{\{h_2\}, \{h_2, h_3\}\}$; $\{\{h_3\}, \{h_2, h_3\}\}$ is $\hat{\beta}_G$ - Soft semi closed sets.

the intersection of $\{\{h_2\}, \{h_2, h_3\}\}$; $\{\{h_3\}, \{h_2, h_3\}\}$ is $\{\{\Phi\}, \{h_2, h_3\}\}$ is also $\hat{\beta}_G$ - Soft semi closed sets.

3.7 Theorem:

If a Soft subset (F, E) of X is $\hat{\beta}_G$ - Soft semi closed Subsets of X_E , Then $\text{clintcl}(F, E) - (F, E)$ does not contain any non-empty Soft semi open set in X .

Proof :

Suppose that (F, E) is $\hat{\beta}_G$ - Soft semi closed Subsets of X . We Prove this result by contradiction. Let (U, E) be Soft semi open set Such that $\text{clintcl}(F, E) - (F, E) \supseteq (U, E)$ and $(U, E) \neq \varphi_E$

Now $(U, E) \subseteq \text{clintcl}(F, E) - (F, E)$.

Therefore $(U, E) \subseteq X - (U, E)$. Since $(X - (U, E))$ is also Soft open set in X .

Since (F, E) is $\hat{\beta}_G$ - Soft semi closed Subsets of X . By defn of $\hat{\beta}_G$ - Soft semi closed Subsets of X_E
 $\text{clintcl}(F, E) \subseteq X - (U, E)$, So $(U, E) \subseteq X - \text{clintcl}(F, E)$.

Also $(U, E) \subseteq \text{clintcl}(F, E)$.

Therefore $(U, E) \subseteq \text{clintcl}(F, E) \cap (X - \text{clintcl}(F, E))$. (ie) $(U, E) = \varphi$.

This is contradicts to $(U, E) \neq \varphi$.

Hence $\text{clintcl}(F, E) - (F, E)$ does not contain any non-empty Soft semi open set in X .

It does not contain any non-empty Soft Semi open set in But (F_1, E) is not $\hat{\beta}_G$ - Soft semi closed of X .

3.8 Remark :

The Converse of the above theorem need not be true as seen from the following Example .

3.9 Example :

Let $X = \{h_1, h_2, h_3\}$ $E = \{e_1, e_2\}$ $\tau = \{\Phi, X, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$

Where $(F_1, E) = \{\{h_1\}, \{h_1\}\}$; $(F_2, E) = \{\{h_2\}, \{h_2\}\}$; $(F_3, E) = \{\{h_1, h_2\}, \{h_1, h_2\}\}$
 $(F_4, E) = \{\{h_2, h_3\}, \{h_2, h_3\}\}$; $(F_5, E) = \{\{h_1, h_3\}, \{h_1, h_3\}\}$; $(F_6, E) = \{\{h_3\}, \{h_3\}\}$

Solution :

We take the Soft set $(F_1, E) = \{\{h_1\}, \{h_1\}\}$; $\text{clintcl}(F_1, E) = (F_5, E)$

Therefore $\text{clintcl}(F_1, E) - (F_1, E) = \{\{h_3\}, \{h_3\}\}$;

Here $\{\{h_3\}, \{h_3\}\}$ is not Soft Semiopen. Therefore

$\text{clintcl}(F_1, E) - (F_1, E)$ does not contain any nonempty Soft semi open set in X . But it is not $\hat{\beta}_G$ - Soft semi closed set in X .

3.10 Definition:

Let (X, τ, E) be a Soft topological space and define a Soft set

$$(F, E) = (X, E) - \{x, e_0\} \text{ is defined by } F(e) = \begin{cases} X - \{x\} & \text{if } e = e_0 \\ X & \text{otherwise} \end{cases}$$

3.11 Example :

Let $X = \{h_1, h_2, h_3\}$ $E = \{e_1, e_2\}$ and define a Soft set

$$(F, E) \in \text{SS}(X)_E$$

Therefore

$$\checkmark \quad (X, E) - \{\{h_1\}, e_1\} = (\{h_2, h_3\} \cdot X)$$

✓ $(X,E) - \{ \{ h_1 \}, e_2 \} = (X, \{ h_2, h_3 \})$

✓ $(X,E) - \{ h_2 \}, e_1 \} = (\{ h_1, h_3 \} . X)$

3.12 Theorem :

For an element $\{x, e_0\} \in (X,E)$, the set $(X,E) - \{x, e_0\}$ is $\hat{\beta}g$ - Soft semi closed Subsets of X or Soft open .

Proof :

Suppose $(X,E) - \{x, e_0\}$ is not Soft open . Then X is the only Soft open set containing $X - \{x, e_0\}$. This implies $\text{cl}(\text{int}(\{X_E - \{x, e_0\}\}) \subset X$. Hence $(X,E) - \{x, e_0\}$ is $\hat{\beta}g$ -Soft semi closed Subsets of X .

3.13 Theorem:

If (F,E) is Soft regular open and $\hat{\beta}g$ - Soft semi closed Subsets of X_E then (F,E) is Soft regular closed and hence clopen.

Proof :

Suppose (F,E) is Soft regular open and $\hat{\beta}g$ - Soft semi closed Subsets of X.

As every Soft regular open set is Open and $(F,E) \subset (F,E)$.

We have $\leftrightarrow \text{cl}(\text{int}(F,E)) \subset (F,E)$. Since $\text{cl}(F,E) \subset \text{cl}(\text{int}(F,E))$.

We have $\text{cl}(F,E) \subseteq (F,E)$. Also $(F,E) \subseteq \text{cl}(F,E)$. Therefore $\text{cl}(F,E) = (F,E)$

which means (F,E) is closed . Since (F,E) is Soft regular open, (F,E) is open .

Now $\text{cl}(\text{int}(F,E)) = \text{cl}(F,E) = (F,E)$. Therefore (F,E) is Soft regular closed and clopen.

3.14 Theorem:

If (F,E) is Soft regular open and Soft rg- closed, then (F,E) is $\hat{\beta}g$ - Soft semi closed Subsets of X

Proof :

Let (F,E) is Soft regular open and Soft rg- closed in X . We prove that (F,E) is an $\hat{\beta}g$ - Soft semi closed Subsets of X. Let (U,E) be any Soft semi open set in X such that $(F,E) \subset (U,E)$. Since (F,E) is Soft regular open and Soft rg- closed, we have $\text{cl}(F,E) \subset (F,E)$.

Then $\text{cl}(F,E) \subset (F,E) \subset (U,E)$. (ie) $\text{cl}(\text{int}(F,E)) = (F,E)$ Hence (F,E) is $\hat{\beta}g$ - Soft semi closed Subsets of X .

3.15 Theorem:

If (F,E) is an $\hat{\beta}g$ - Soft semi closed Subsets of X such that $(F,E) \subset (G,E) \subset \text{cl}(F,E)$. then (G,E) is an $\hat{\beta}g$ - Soft semi closed Subsets of X .

Proof :

Let (F,E) be an $\hat{\beta}g$ - Soft semi closed Subsets of X such that $(F,E) \subset (G,E) \subset \text{cl}(F,E)$.

Let (U,E) be an Soft semi open set of X such that $(G,E) \subset (U,E)$. then $(F,E) \subset (U,E)$.

Since (F,E) be an $\hat{\beta}g$ - Soft semi closed. We have $\text{cl}(F,E) \subset (U,E)$.

Now $\text{cl}(G,E) \subset \text{cl}(\text{cl}(F,E)) = \text{cl}(F,E) \subset (U,E)$. Therefore (G,E) is $\hat{\beta}g$ - Soft semi closed Subsets of X

3.16 Theorem:

Let (F,E) be $\hat{\beta}g$ - Soft semi closed Subsets of X. Then (F,E) is Soft semi closed if and only if $\text{cl}(F,E) - (F,E)$ is Soft semi open.

Proof :

Suppose (F,E) is Soft semi closed Subsets of X. then $\text{cl}(F,E) - (F,E) = \emptyset_E$, which is Soft semi open set in X.

Conversly ,

Suppose $cl(F,E) - (F,E)$ is Soft semi open in X. Since (F,E) $\hat{\beta}_g$ - Soft semi closed .

By Theorem , $cl(F,E) - (F,E)$ does not contain any non-empty Soft semi open set in X.

Then $cl(F,E) - (F,E) = \emptyset$. Hence (F,E) is Soft semi closed.

3.17 Theorem:

If (F,E) is both Soft semi open and Soft generalized closed then it is $\hat{\beta}_g$ - Soft semi closed Subsets set in X.

Proof :

Let (F,E) is Soft semi open and Soft generalized closed in X . Let $(F,E) \subset (U,E)$ where (U,E) Soft semi open in X. Now $(F,E) \subset (F,E)$, By hypothesis $cl(F,E) \subseteq (F,E)$.

(ie) $cl(F,E) \subseteq (U,E)$.

Hence (F,E) is $\hat{\beta}_g$ - Soft semi closed set in X_E .

IV. Conclusion

In the present work , a new class of sets called $\hat{\beta}_g$ - Soft semi closed in Subsets set in X Soft topological spaces which are defined over an initial universe with a fixed set of parameters and some of their properties are studied. We have presented it 's basic properties with the help of some counter examples. This new class of sets widen the scope to do further research in the areas like $g \alpha$ semi- closed sets in Soft topological spaces.

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