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Nano Regular Generalized Star Star B-Normal Space In Nano Topological Spaces

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Abstract: *In this paper we introduce the concept of nano $rg^{**}b$-regular and nano $rg^{**}b$-normal spaces in nano topological spaces. Also some characterizations and several properties concerning of these spaces.*
Key Words: *nano $rg^{**}b$-regular, nano $rg^{**}b$-normal.*

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I. Introduction

The concept of nano topology was introduced by Lellis Thivagar [5] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established the week forms of nano open sets namely nano α-open sets, nano-semi open sets and nano pre-open sets in a nano topological space.

Maheshwari and Prasad [2,3] introduced and studied s-regular and s-normal spaces in topological spaces. Munshi [4] introduced g-regular and g-normal spaces using g-closed sets in topological spaces. Dhanis Arul Mary and Arockiarani [1] discussed Nano b normal space and Nano gb normal spaces in Nano topological spaces.

This chapter contains Nano regular generalized star star b- regular spaces and Nano regular generalized star star b-normal space are introduced, and characterization of these spaces are studied.

II. Preliminaries

Definition 2.1 Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation, elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \bigcup_{x\in U}\{R(x): R(x) \subseteq X$, where Rx denotes the equivalence class determined by x.

ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{x\in U}\{R(x): R(x) \cap X \neq \emptyset$ where Rx denotes the equivalence class determined by x.

iii) The boundary region of X with respect to R is set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.That is $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\emptyset \in \tau_R(X)$.
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on U called as the nano topology on U with respect to X We call $(U, \tau_R(X))$ as the Nano topological space.

Definition 2.3 A map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is said to be nano $rg^{**}b$-continuous if $f^{-1}(V)$ is nano $rg^{**}b$- closed in $(U, \tau_R(X))$ for each nano closed set Vin $(V, \tau_R(Y))$.

Definition 2.4 A map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is said to be nano $rg^{**}b$-closed if the image of every nano closed set in $(U, \tau_R(X))$ is nano $rg^{**}b$-closed in $(V, \tau_R(Y))$.

Definition 2.5 A map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is said to be nano $rg^{**}b$-open if $f(A)$ is nano $rg^{**}b$-open for each nano open set A in $(U, \tau_R(X))$.

Definition 2.6 A map $f:(U,\tau_R(X)) \longrightarrow (V,\tau_R(Y))$ is said to be strongly nano $rg^{**}b$-closed if the image $f(A)$ is nano $rg^{**}b$-closed in $(V,\tau_R(Y))$ for each nano $rg^{**}b$-closed in $(U,\tau_R(X))$.
Definition 2.7 A map $f:(U,\tau_R(X)) \longrightarrow (V,\tau_R(Y))$ is said to be strongly nano $rg^{**}b$-open if the image $f(A)$ is nano $rg^{**}b$-open in $(V,\tau_R(Y))$ for each nano $rg^{**}b$-open in $(U,\tau_R(X))$.
Definition 2.8 A map $f:(U,\tau_R(X)) \longrightarrow (V,\tau_R(Y))$ is said to be Contra nano $rg^{**}b$-closed map (resp. Contra nano $rg^{**}b$-open) if the image of every nano closed (resp. nano open) set in $(U,\tau_R(X))$ is nano $rg^{**}b$-open (resp. nano $rg^{**}b$-closed) set in $(V,\tau_R(Y))$.
Definition 2.9 A map $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is said to be $Nrg^{**}b$-Irresolute if the inverse image of every $Nrg^{**}b$-closed set in $(V,\tau_{R'}(Y))$ is $Nrg^{**}b$-closed set in $(U,\tau_R(X))$.

III. Nano Regular Generalized Star Star b-Regular Space

Definition 3.1 A nano topological space $(U,\tau_R(X))$ is said to be nano regular generalized star star b-regular space(briefly $Nrg^{**}b$-regular), if for each nano-closed subset ρ of U and each point $x \in U-\rho$, there exist two disjoint $Nrg^{**}b$-open sets one containing ρ and other containing x.
Example 3.2 Let $U=V=\{a,b,c\}$ with $U/R=\{\{a\},\{b,c\}\}$ and $X=\{a,b\} \subseteq U$. Then the nano topology is defined as $\tau_R(X)=\{U,\emptyset,\{a\},U,\{b,c\}\}$.
i) Let $\rho=\{b,c\} \subseteq U$ and the point $a \in U-\rho$ there exist $Nrg^{**}b$-open sets are $\{b,c\} \subseteq \{b,c\}$, $a \in \{a\}$ and $\{b,c\} \cap \{a\}=\emptyset$.
ii) Let $\rho=\{a\} \subseteq U$ and the point $b \in U-\rho$ there exist $Nrg^{**}b$-open sets are $\{a\} \subseteq \{a\}, b \in \{b,c\}$ and $\{b,c\} \cap \{a\}=\emptyset$ and the point $c \in U-\rho$ there exist $Nrg^{**}b$-open sets are $\{a\} \subseteq \{a\}, c \in \{b,c\}$ and $\{b,c\} \cap \{a\}=\emptyset$. Therefore $(U,\tau_R(X))$ $Nrg^{**}b$-regular.

Theorem 3.3 Every nano regular space is $Nrg^{**}b$-regular.
The proof is obvious.
Example 3.4 Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{b\},\{c,d\}\}$ and $X=\{a,b\} \subseteq U$. Then the nano topology is defined as $\tau_R(X)=\{U,\emptyset,\{a,b\},\emptyset\}$. here only one nano-closed proper subsets of U consists of $\rho=\{c,d\}$ and corresponding to $\{c,d\} \subseteq \{b,c,d\}$ and the point $a \in \{a\}$ such that $\{b,c,d\} \cap \{a\}=\emptyset$. Again $\{c,d\} \subseteq \{a,c,d\}$ and the point $b \in \{b\}$ such that $\{a,c,d\} \cap \{b\}=\emptyset$. Therefore $(U,\tau_R(X))$ $Nrg^{**}b$-regular but not nano-regular.
Theorem 3.5 If $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is $Nrg^{**}b$- irresolute, nano-closed injection and V is $Nrg^{**}b$-regular space then U is $Nrg^{**}b$-regular space.
Proof: Let ρ be nano-closed set in U and $u \notin \rho$. since f is nano-closed injection, $f(\rho)$ is nano-closed set in V such that $f(u) \notin f(\rho)$. Now V is $Nrg^{**}b$-regular, there exist disjoint $Nrg^{**}b$-open sets E and F such that $f(u) \in G$ and $f(\rho) \subset H$. which implies $u \in f^{-1}(G)$ and $\rho \subset f^{-1}(H)$. Since f is $Nrg^{**}b$- irresolute, where $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $Nrg^{**}b$-open sets in U. Therefore U is $Nrg^{**}b$-regular space.

Theorem 3.6 If $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is nano-continuous surjective, strongly $Nrg^{**}b$-open function and U is $Nrg^{**}b$-regular space then V is $Nrg^{**}b$-regular space.
Proof: Let ρ be nano-closed set in V and $v \notin \rho$. Take $v=f(u)$ for some $u \in U$. since f is nano-continuous surjective, $f^{-1}(\rho)$ is nano-closed set in U such that $u \notin f^{-1}(\rho)$. Now U is $Nrg^{**}b$-regular, there exist disjoint $Nrg^{**}b$-open sets G and H such that $u \in G$ and $f^{-1}(\rho) \subset H$. That is $v=f(u) \in f(G)$ and $\rho \subset f(H)$. Since f is strongly $Nrg^{**}b$-open function, $f(G)$ and $f(H)$ are disjoint $Nrg^{**}b$-open sets in V. Therefore V is $Nrg^{**}b$-regular space.
Theorem 3.7 If $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is $Nrg^{**}b$- continuous, nano-closed injection and V is ultra regular space then U is $Nrg^{**}b$-regular space.
Proof: Let ρ be nano-closed set in U and $u \notin \rho$. since f is nano-closed injection, $f(\rho)$ is nano-closed set in V such that $f(u) \notin f(\rho)$. Now V is ultra regular, there exist disjoint nano clopen-open sets E and F such that $f(u) \in G$ and $f(\rho) \subset H$. which implies $u \in f^{-1}(G)$ and $\rho \subset f^{-1}(H)$. Since f is $Nrg^{**}b$- continuous, where $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $Nrg^{**}b$-open sets in U. Therefore U is $Nrg^{**}b$-regular space.
Theorem 3.8 If $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ is nano-continuous bijective, $Nrg^{**}b$-open function and U is nano regular space then V is $Nrg^{**}b$-regular space.

Proof: Let ρ be nano-closed set in V and $v \notin \rho$. Take $v = f(u)$ for some $u \in U$. since f is nano-continuous bijective, $f^{-1}(\rho)$ is nano-closed set in U such that $u \notin f^{-1}(\rho)$. Now U is nano regular space, there exist disjoint open sets G and H such that $u \in G$ and $f^{-1}(\rho) \subset H$. That is
$v = f(u) \in f(G)$ and $\rho \subset f(H)$. Since f is $Nrg^{**}b$-open function , $f(G)$ and $f(H)$ are
disjoint $Nrg^{**}b$-open sets in V. Therefore V is $Nrg^{**}b$-regular space.

Theorem 3.9 If $f: (U, \tau_R(X)) \to \left(V, \tau_{R'}(Y)\right)$ is nano-continuous bijective, contra $Nrg^{**}b$-closed function and U is ultra regular space then V is $Nrg^{**}b$-regular space.

Proof: Let ρ be nano-closed set in V and $v \notin \rho$. Take $v = f(u)$ for some $u \in U$. since f is nano-continuous bijective, $f^{-1}(\rho)$ is nano-closed set in U such that $u \notin f^{-1}(\rho)$. Now U is ultra regular, there exist disjoint clopen sets G and H such that $u \in G$ and $f^{-1}(\rho) \subset H$. That is
$v = f(u) \in f(G)$ and $\rho \subset f(H)$. Since f is contra $Nrg^{**}b$-closed function, $f(G)$ and $f(H)$ are
disjoint $Nrg^{**}b$-open sets in V. Therefore V is $Nrg^{**}b$-regular space.

Theorem 3.10 If $f: (U, \tau_R(X)) \to \left(V, \tau_{R'}(Y)\right)$ is $Nrg^{**}b$- continuous, nano-closed injection and V is nano regular space then U is $Nrg^{**}b$-regular space.

Proof: Let ρ be nano-closed set in U and $u \notin \rho$. since f is nano-closed injection, $f(\rho)$ is nano-closed set in V such that $f(u) \notin f(\rho)$. Now V is nano regular, there exist disjoint nano open-open sets E and F such that $f(u) \in G$ and $f(\rho) \subset H$. which implies $u \in f^{-1}(G)$ and
$\rho \subset f^{-1}(H)$. Since f is $Nrg^{**}b$- continuous, where $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $Nrg^{**}b$-open sets in U. Therefore U is $Nrg^{**}b$-regular space.

Theorem 3.11 If $f: (U, \tau_R(X)) \to \left(V, \tau_{R'}(Y)\right)$ is nano-continuous bijective, $Nrg^{**}b$-open function and U is ultra regular space then V is $Nrg^{**}b$-regular space.

Proof: Let ρ be nano-closed set in V and $v \notin \rho$. Take $v = f(u)$ for some $u \in U$. since f is nano-continuous bijective, $f^{-1}(\rho)$ is nano-closed set in U such that $u \notin f^{-1}(\rho)$. Now U is ultra regular, there exist disjoint clopen sets G and H such that $u \in G$ and $f^{-1}(\rho) \subset H$. That is
$v = f(u) \in f(G)$ and $\rho \subset f(H)$. Since f is $Nrg^{**}b$-open function, $f(G)$ and $f(H)$ are
disjoint $Nrg^{**}b$-open sets in V. Therefore V is $Nrg^{**}b$-regular space.

Theorem 3.12 If $f: (U, \tau_R(X)) \to \left(V, \tau_{R'}(Y)\right)$ is nano-continuous surjective, strongly $Nrg^{**}b$-open function and U is ultra regular space then V is $Nrg^{**}b$-regular space.

Proof: Let ρ be nano-closed set in V and $v \notin \rho$. Take $v = f(u)$ for some $u \in U$. since f is nano-continuous surjective, $f^{-1}(\rho)$ is nano-closed set in U such that $u \notin f^{-1}(\rho)$. Now U is $Nrg^{**}b$-regular, there exist disjoint $Nrg^{**}b$-open sets G and H such that $u \in G$ and $f^{-1}(\rho) \subset H$. We know that every nano-open set is $Nrg^{**}b$-open. That is $v = f(u) \in f(G)$ and $\rho \subset f(H)$. Since f is strongly $Nrg^{**}b$-open function, $f(G)$ and $f(H)$ are disjoint $Nrg^{**}b$-open sets in V. Therefore V is $Nrg^{**}b$-regular space.

IV. Nano Regular Generalized Star Star b-Normal Space

Definition 4.1 A nano topological space $(U, \tau_R(X))$ is said to be nano regular generalized star star b-normal space(briefly $Nrg^{**}b$-normal), if for every pair of disjoint nano-closed subsets ρ_1
and ρ_2 of U there exist $Nrg^{**}b$-open sets G and H such that $\rho_1 \subseteq G$, and $\rho_2 \subseteq H$ and
$G \cap H = \emptyset$.

Example 4.2 Let $U = V = \{a, b, c\}$ with $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a, b\} \subseteq U$. Then the nano topology is defined as $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, c\}\}$. Here the only one pair of non-empty disjoint nano-closed proper subset of U consists of $\rho_1 = \{b, c\}$ and $\rho_2 = \{a\}$ and corresponding to $Nrg^{**}b$-open sets $\{b, c\}$ and $\{a\}$ such that $\{b, c\} \subseteq \{b, c\}, \{a\} \subseteq \{a\}$ and $\{b, c\} \cap \{a\} = \emptyset$.

Theorem 4.3 If $f: (U, \tau_R(X)) \to \left(V, \tau_{R'}(Y)\right)$ is $Nrg^{**}b$- irresolute, nano-closed injection and V is $Nrg^{**}b$-normal space then U is $Nrg^{**}b$-normal space.

Proof: Let ρ_1 and ρ_2 be disjoint nano-closed sets in U then $f(\rho_1)$ and $f(\rho_2)$ are disjoint nano-closed sets in V. Since f is nano-closed injection. Now V is $Nrg^{**}b$-normal, there exist disjoint $Nrg^{**}b$-open sets G and H such that $f(\rho_1) \subset G$ and $f(\rho_2) \subset H$. which implies $\rho_1 \subset f^{-1}(G)$ and $\rho_2 \subset f^{-1}(H)$. Since f is $Nrg^{**}b$- irresolute, where $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $Nrg^{**}b$-open sets in U. Therefore U is $Nrg^{**}b$-normal space.

Theorem 4.4 If $f: (U, \tau_R(X)) \to \left(V, \tau_{R'}(Y)\right)$ is nano-continuous surjective, strongly $Nrg^{**}b$-open function and U is $Nrg^{**}b$-normal space then V is $Nrg^{**}b$-normal space.

Proof: Let ρ_1 and ρ_2 be disjoint nano-closed sets in V then $f^{-1}(\rho_1)$ and $f^{-1}(\rho_2)$ are disjoint nano-closed set in U. Since f is nano-continuous. Now U is $Nrg^{**}b$-normal, there exist disjoint $Nrg^{**}b$-open sets G and H such that $f^{-1}(\rho_1) \subseteq G$ and $f^{-1}(\rho_2) \subset H$. That is $\rho_1 \subset f(G)$ and $\rho_2 \subset f(H)$, $f(G)$ and $f(H)$ are disjoint $Nrg^{**}b$-open sets in V. Since f is strongly $Nrg^{**}b$-open function. Therefore V is $Nrg^{**}b$-regular space.

Theorem 4.5 If $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is $Nrg^{**}b$- continuous, nano-closed injection and V is ultra normal space then U is $Nrg^{**}b$-normal space.

Proof: Let ρ_1 and ρ_2 be disjoint nano-closed sets in U then $f(\rho_1)$ and $f(\rho_2)$ are disjoint nano-closed sets in V. Since f is nano-closed injection. Now V is ultra normal, there exist disjoint nano clopen sets G and H such that $f(\rho_1) \subset G$ and $f(\rho_2) \subset H$. which implies $\rho_1 \subset f^{-1}(G)$ and $\rho_2 \subset f^{-1}(H)$. Since f is $Nrg^{**}b$- continuous, where $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $Nrg^{**}b$-open sets in U. Therefore U is $Nrg^{**}b$-normal space.

Theorem 4.6 If $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano-continuous bijective, $Nrg^{**}b$-open function and U is nano normal space then V is $Nrg^{**}b$-normal space.

Proof: Let ρ_1 and ρ_2 be disjoint nano-closed sets in V then $f^{-1}(\rho_1)$ and $f^{-1}(\rho_2)$ are disjoint nano-closed set in U. Since f is nano-continuous bijective. Now U is ultra normal, there exist disjoint nano clopen sets G and H such that $f^{-1}(\rho_1) \subseteq G$ and $f^{-1}(\rho_2) \subset H$. That is $\rho_1 \subset f(G)$ and $\rho_2 \subset f(H)$, $f(G)$ and $f(H)$ are disjoint $Nrg^{**}b$-open sets in V. Since f is $Nrg^{**}b$-open function. Therefore V is $Nrg^{**}b$-normal space.

Theorem 4.7 If $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano-continuous bijective, contra $Nrg^{**}b$-closed function and U is ultra normal space then V is $Nrg^{**}b$-normal space.

Proof: Let ρ_1 and ρ_2 be disjoint nano-closed sets in V then $f^{-1}(\rho_1)$ and $f^{-1}(\rho_2)$ are disjoint nano-closed set in U. Since f is nano-continuous bijective. Now U is ultra normal, there exist disjoint nano clopen sets G and H such that $f^{-1}(\rho_1) \subseteq G$ and $f^{-1}(\rho_2) \subset H$. That is $\rho_1 \subset f(G)$ and $\rho_2 \subset f(H)$, $f(G)$ and $f(H)$ are disjoint $Nrg^{**}b$-open sets in V. Since f is contra $Nrg^{**}b$-closed function. Therefore V is $Nrg^{**}b$-normal space.

Theorem 4.8 If $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is $Nrg^{**}b$- continuous, nano-closed injection and V is nano normal space then U is $Nrg^{**}b$-normal space.

Proof: Let ρ_1 and ρ_2 be disjoint nano-closed sets in U then $f(\rho_1)$ and $f(\rho_2)$ are disjoint nano-closed sets in V. Since f is nano-closed injection. Now V is normal, there exist disjoint nano open sets G and H such that $f(\rho_1) \subset G$ and $f(\rho_2) \subset H$. which implies $\rho_1 \subset f^{-1}(G)$ and $\rho_2 \subset f^{-1}(H)$. Since f is $Nrg^{**}b$- continuous, where $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint $Nrg^{**}b$-open sets in U. Therefore U is $Nrg^{**}b$-normal space.

Theorem 4.9 If $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano-continuous bijective, $Nrg^{**}b$-open function and U is normal space then V is $Nrg^{**}b$-normal space.

Proof: Let ρ_1 and ρ_2 be disjoint nano-closed sets in V then $f^{-1}(\rho_1)$ and $f^{-1}(\rho_2)$ are disjoint nano-closed set in U. Since f is nano-continuous bijective. Now U is normal, there exist disjoint nano-open sets G and H such that $f^{-1}(\rho_1) \subseteq G$ and $f^{-1}(\rho_2) \subset H$. That is $\rho_1 \subset f(G)$ and $\rho_2 \subset f(H)$, $f(G)$ and $f(H)$ are disjoint $Nrg^{**}b$-open sets in V. Since f is $Nrg^{**}b$-open function. Therefore V is $Nrg^{**}b$-normal space.

Theorem 4.10 If $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano-continuous surjective, strongly $Nrg^{**}b$-open function and U is ultra regular space then V is $Nrg^{**}b$-regular space.

Proof: Let ρ_1 and ρ_2 be disjoint nano-closed sets in V then $f^{-1}(\rho_1)$ and $f^{-1}(\rho_2)$ are disjoint nano-closed set in U. Since f is nano-continuous bijective. Now U is normal, there exist disjoint nano-clopen sets G and H such that $f^{-1}(\rho_1) \subseteq G$ and $f^{-1}(\rho_2) \subset H$. We know that every nano-open set is $Nrg^{**}b$-open. That is $\rho_1 \subset f(G)$ and $\rho_2 \subset f(H)$, $f(G)$ and $f(H)$ are disjoint $Nrg^{**}b$-open sets in V. Since f is strongly $Nrg^{**}b$-open function. Therefore V is $Nrg^{**}b$-normal space.

References

[1]. Dhanis Arul Mary. A. and Arockiarani. I., Note on Nano b normal spaces, Bulletin of Mathematics and Statistics Research, 3(3) (2015), 124-136.

[2]. Maheswari. S.N. and Prasad. R., On s-regular spaces, Glasnik Mat.Ser.III, 10(1975), 347-350.

[3]. Maheswari.S.N. and Prasad. R., On s- normal spaces, Bull. Math. Soc. Sci. Math. R.S. Roumanie, 22(1978), 27-29.

[4]. Munshi. B.M., Separation axioms, Acta Ciencia Indica, 12(1986), 140-144.

[5]. Lellis Thivagar. M. and Carmel Richard, On Nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, 1(1) (2013), 31 -37.

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