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Estimation of Availability Measures and Confidence Interval for two unit system with Common Cause Shock failures and Human Errors

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Abstract: *This paper discusses maximum likelihood estimation (M L) approach to derive availability measures of a two unit system with common cause shocks (CCS) as well as Human errors. Availability estimates for time-dependent and steady-state availability were developed in the case of series and parallel systems. The purpose of this investigation is to incorporate the possibility of Human errors in reliability analysis. We developed empirical evidence to establish the validity of the proposed estimates by Monte Carlo Simulation.*
Keywords*: Availability, Common cause shock failures, Human errors, M L Estimation, Simulation*

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I. Introduction

There has been a considerable interest in human initiated equipment failures and their effect on system reliability [5]. In real life most of the systems require some human participation irrespective of the degree of automation. According to Meister [8] about 30 percept of failures are directly or indirectly due to Human errors. In shock models, the Common Cause events from the outside environment such as lightning, flood, earth quake, hurricane, fire, thunderstorm etc, occur at random times, causing simultaneous failure of several components of the system [3, 4]. The reliability and availability of the system with Human Errors (HE) as well as Common Cause shock failures (CCSF) may be affected considerably due to outage of several units [1,2]. Researchers have considered them in the assessment of reliability and availability measures and performance of the system very much. If the data (samples) is available one can try to find the estimates of availability measures of the system performance [7].

In this paper, an attempt is made to find an approach of estimation method which could establish a formal estimation procedure to estimate the availability measures such as Availability ($A_s(t)$; $A_s(\infty)$) for series and parallel systems under the influence of CCS failures as well as Human errors. The estimation of availability of system is considered in the CCS and Human errors of chance that is the components in the system will fail by external cause like CCS and Human errors as well as individual failures with chance c_1, c_2 and c_3 $\exists$ $c_1+c_2+c_3 = 1$ respectively [3, 6]. In fact, the assumptions lead to compound type of Poisson process application. Intuitively, maximum likelihood approach is considered to develop the availability measures like $A_s(t)$ and $A_s(\infty)$. Also the approach used is empirical one with Monte Carlo Simulation, because no exact and closed form mathematical probability density function of the estimates is found.

II. Assumptions and Notations

We consider a two-unit system with the following assumptions:

1. The units fail individually and also simultaneously due to Human errors as well as Common Cause Shock in Poisson fashion.
2. Individual, CCS failures and Human errors are independent each other.
3. The components in the system will fail singly at the constant rate λ_i and failure probability is c_1
4. The components may fail due to common causes at the constant rate λ_c and with failure probability is c_2
5. The components may fail due to Human errors at the constant rate λ_h and with failure probability is c_3 s.t $c_1 + c_2 + c_3 = 1$
6. Time occurrences of CCS failures, Human errors and individual failures follow exponential law.
7. The failed components are repaired singly and repair time follows exponential distribution with rate of service μ

We use the Notations:

$\hat{A}^*_{sh}(t)$	:	M L Estimate of time-dependent availability of series system
$\hat{A}^*_{ph}(t)$	:	M L Estimate of time-dependent availability of parallel system
$\hat{A}^*_{sh}(\infty)$	:	M L Estimate of steady-state availability of series system
$\hat{A}^*_{ph}(\infty)$	:	M L Estimate of steady-state availability of parallel system
$\bar{x}, \bar{y}$ & $\bar{w}$	:	Sample means of the occurrence of individual, CCS failures and human errors respectively
$\bar{z}$	:	Sample mean of repair time of the components
$\hat{\bar{x}}, \hat{\bar{y}}$ & $\hat{\bar{w}}$	:	Sample estimates of individual failure rate, CCS failure rate and human errors respectively
$\hat{\bar{z}}$	:	Sample estimate of repair time of the components
n	:	Sample size
N	:	Number of simulated samples
θ	:	$(\lambda_i, \lambda_c, \lambda_h, \mu)$
M S E	:	Mean square error

III. The Model

Under the stated assumptions, Markovian model can be formulated to derive the time-dependent and steady-state availability functions $A_s(t)$ and $A_s(\infty)$ under the influence of individual, Common cause shocks as well as human errors and the Markovian diagram is shown in Fig.1. The quantities $\lambda_0 = 2\lambda_i c_1$, $\lambda_1 = \lambda_i c_1$, $\lambda_2=\lambda_c c_2$, $\lambda_3=\lambda_h c_3$, $\mu_1=\mu$ & $\mu_2=2\mu$. The probability equations governing the model are $P_0(t)$, $P_1(t)$ and $P_2(t)$ are derived by Sagar G Y [5].

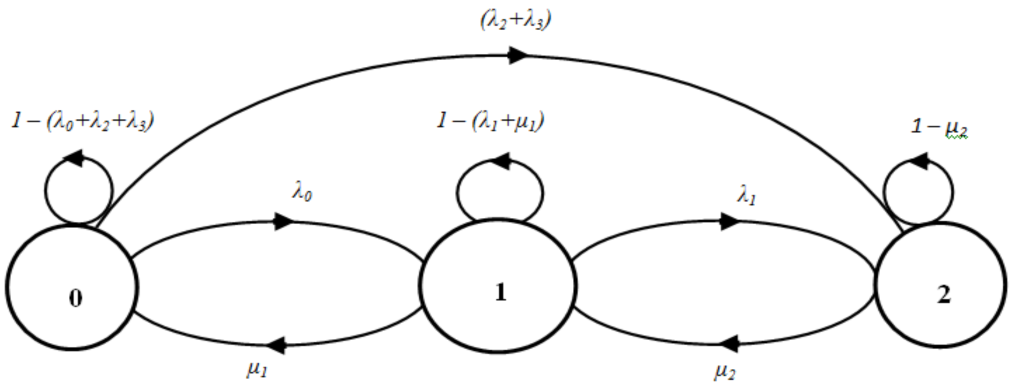

FIG.1. STATE TRANSITION DIAGRAM FOR HUMAN ERROR MODEL

IV. Maximum Likelihood Estimation (MLE) – Availability Function

The maximum likelihood estimation approach for estimating time-dependent Availability function of two component series and parallel systems, which is under the influence of human errors and common cause shock failures in addition to individual failures.

Let $x_1, x_2, \ldots\ldots x_n$ be a sample of 'n' number of times between individual failures which will obey exponential law.

Let $y_1, y_2, \ldots\ldots y_n$ be a sample of 'n' number of times between Common cause shock failures which follow exponential as well.

Let $w_1, w_2, \ldots\ldots w_n$ be a sample of 'n' number of times between Human errors which follow exponential as well.

Let $z_1, z_2, \ldots\ldots z_n$ be a sample of 'n' number of times repair of the components with exponential population law.

$\hat{\bar{x}}, \hat{\bar{y}}, \hat{\bar{w}}$ & $\hat{\bar{z}}$ are the M L estimates of individual failure rate (λ_i), CCS failure rate (λ_c), human errors rate (λ_h) and repair rate 'μ' of the system respectively.

Where,

$$\hat{\bar{x}} = \frac{1}{\bar{x}}; \hat{\bar{y}} = \frac{1}{\bar{y}}; \hat{\bar{w}} = \frac{1}{\bar{w}}; \hat{\bar{z}} = \frac{1}{\bar{z}} \text{ and } \bar{x} = \frac{\sum x_i}{n}; \bar{y} = \frac{\sum y_i}{n}; \bar{w} = \frac{\sum w_i}{n}; \bar{z} = \frac{\sum z_i}{n}$$

are the sample estimates of the rate of individual failure times, rate of CCS failure times, rate of human error times and rate of repair times of the components respectively.

4.1 Estimation of Time – Dependent Availability

The M L estimates of time-dependent availability for both series and parallel systems in the case of individual failures and CCS failures as well as human errors are derived.

Series System

Thus, the expression of time-dependent availability function for series system is given by

$A^*_{sh}(t) = [2\mu^2 / G] + [\ l_1 \exp(\gamma_1 t) - l_2 \exp(\gamma_2 t)\ ] / (\gamma_1 - \gamma_2)$ ----------- (4.1)

Where

$G = (2\mu^2 + 4\lambda_i c_1\mu + 3\lambda_c c_2\mu + 3\lambda_h c_3\mu)$

$l_1 = (\gamma_1^2 + \gamma_1 3\mu + 2\mu^2) / \gamma_1$

$l_2 = (\gamma_2^2 + \gamma_2 3\mu + 2\mu^2) / \gamma_2$

$\gamma_1, \gamma_2 = ½[-(3\mu + 2\lambda_i c_1 + \lambda_c c_2 + \lambda_h c_3) \pm \text{SQRT}((\mu - 2\lambda_i c_1 - \lambda_c c_2 - \lambda_h c_3)^2 - 4\lambda_c c_2\mu - 4\lambda_h c_3\mu)\ ]$

Therefore, the expression of maximum likelihood estimate of time-dependent availability function for series system is derived as

$$\hat{A}^*_{sh}(t) = \left(\frac{2\hat{z}.\hat{z}}{H}\right) + \left(\frac{D_1 \exp(r_1.t) - D_2 \exp(r_2.t)}{(r_1 - r_2)}\right) - - - - - (4.2)$$

Where

$$H = 2\hat{z}.\hat{z} + 4\hat{x}.\hat{z}.c_1 + 3\hat{y}.\hat{z}.c_2 + 3\hat{w}.\hat{z}.c_3$$

$$D_1 = \frac{\left(r_1^2 + 3\hat{z}.r_1 + 2\hat{z}.\hat{z}\right)}{r_1}$$

$$D_2 = \frac{\left(r_2^2 + 3\hat{z}.r_2 + 2\hat{z}.\hat{z}\right)}{r_2}$$

$$r_1, r_2 = \frac{1}{2}\left[-\left(3\hat{z} + 2\hat{x}.c_1 + \hat{y}.c_2 + \hat{w}.c_3\right) \pm sqrt\left(\left(\hat{z} - 2\hat{x}.c_1 - \hat{y}.c_2 - \hat{w}.c_3\right)^2 - 4\hat{y}.\hat{z}.c_2 - 4\hat{w}.\hat{z}.c_3\right)\right]$$

Where, $\hat{x}$, $\hat{y}$, $\hat{w}$ & $\hat{z}$ are the maximum likelihood estimates of individual failure rate (λ_i), Common cause failures rate (λ_c), human error (λ_h) and repair rate (μ) of system respectively.

Parallel System

The expression of time-dependent availability function for parallel system is given by

$A^*_{ph}(t) = (B_1 + L_1)\exp(\gamma_1 t) - (B_2 + L_2)\exp(\gamma_2 t) + (B_3 + L_3)$ ------------- (4.3)

Where $B_1 = [\ \gamma_1^2 + \gamma_1(3\mu + \lambda_i c_1) + 2\mu^2\ ] / \gamma_1(\gamma_1 - \gamma_2)$

$B_2 = [\ \gamma_2^2 + \gamma_2(3\mu + \lambda_i c_1) + 2\mu^2\ ] / \gamma_2(\gamma_1 - \gamma_2)$

$B_3 = 2\mu^2 / \gamma_1\gamma_2$

$L_1 = [\gamma_1(2\lambda_i c_1) + 2\mu(2\lambda_i c_1 + \lambda_c c_2 + \lambda_h c_3)] / \gamma_1(\gamma_1 - \gamma_2)$

$L_2 = [\gamma_2(2\lambda_i c_1) + 2\mu(2\lambda_i c_1 + \lambda_c c_2 + \lambda_h c_3)] / \gamma_2(\gamma_1 - \gamma_2)$

$L_3 = [2\mu(2\lambda_i c_1 + \lambda_c c_2 + \lambda_h c_3)] / \gamma_1\gamma_2$

$\gamma_1, \gamma_2 = ½[-(3\mu + 3\lambda_i c_1 + \lambda_c c_2 + \lambda_h c_3) \pm \text{SQRT}((\mu + \lambda_i c_1 + \lambda_c c_2 + \lambda_h c_3)^2 - 8\mu(\lambda_c c_2 + \lambda_h c_3))]$

Therefore, the expression of M L estimate of time-dependent availability function for parallel system is derived as

$$\hat{A}^*_{ph}(t) = (J_1 + K_1)\exp(r_1.t) - (J_2 + K_2)\exp(r_2.t) + (J_3 + K_3) - - - - - (4.4)$$

Where

$$J_1 = \frac{\left(r_1^2 + r_1(3\hat{z} + \hat{x}.c_1) + 2\hat{z}.\hat{z}\right)}{r_1(r_1 - r_2)}; J_2 = \frac{\left(r_2^2 + r_2(3\hat{z} + \hat{x}.c_1) + 2\hat{z}.\hat{z}\right)}{r_2(r_1 - r_2)}; J_3 = \frac{\left(2\hat{z}.\hat{z}\right)}{r_1.r_2}$$

$$K_1 = \frac{\left[r_1(2\hat{x}.c_1) + 2\hat{z}.\hat{z}(2\hat{x}.c_1 + \hat{y}.c_2 + \hat{w}.c_3)\right]}{r_1(r_1 - r_2)}; K_2 = \frac{\left[r_2(2\hat{x}.c_1) + 2\hat{z}.\hat{z}(2\hat{x}.c_1 + \hat{y}.c_2 + \hat{w}.c_3)\right]}{r_2(r_1 - r_2)}$$

$$K_3 = \frac{\left[2\hat{\hat{z}}(2\hat{\hat{x}}.c_1 + \hat{\hat{y}}.c_2 + \hat{\hat{w}}.c_3)\right]}{r_1.r_2}$$

$$r_1, r_2 = \frac{1}{2}\left[-\left(3\hat{\hat{z}} + 3\hat{\hat{x}}.c_1 + \hat{\hat{y}}.c_2 + \hat{\hat{w}}.c_3\right) \pm sqrt\left(\left(\hat{\hat{z}} + \hat{\hat{x}}.c_1 + \hat{\hat{y}}.c_2 + \hat{\hat{w}}.c_3\right)^2 - 8.\hat{\hat{z}}(\hat{\hat{y}}.c_2 + \hat{\hat{w}}.c_3)\right)\right]$$

Where $\hat{\hat{x}}, \hat{\hat{y}}, \hat{\hat{w}}, \hat{\hat{z}}$ are the sample estimates.

4.2 Estimation of Steady – State Availability

The M L Estimates of steady-state availability for both series and parallel systems in the case of individual, CCS failures as well as Human errors are derived.

Series System

The expression for steady-state availability of a series system is obtained as

$A^*_{sh}(\infty) = (2\mu^2) / (2\mu^2 + 4\lambda_i c_1\mu + 3\lambda_c c_2\mu + 3\lambda_h c_3\mu)$ --------------- (4.5)

Therefore, the expression of M L estimate of steady-state availability function for series system is derived as

$$\hat{A}^*_{sh}(\infty) = \frac{2\hat{\hat{z}}.\hat{\hat{z}}}{(2\hat{\hat{z}}.\hat{\hat{z}} + 4\hat{\hat{x}}.\hat{\hat{z}}.c_1 + 3\hat{\hat{y}}.\hat{\hat{z}}.c_2 + 3\hat{\hat{w}}.\hat{\hat{z}}.c_3)} - - - - - (4.6)$$

Parallel System

In the long run usage of the system the steady-state availability of the parallel system can be obtained as

$A^*_{ph}(\infty) = (2\mu\,(2\lambda_i c_1 + \lambda_c c_2 + \lambda_h c_3 + \mu)) / (2\mu^2 + 2\,(\lambda_i c_1)^2 + 4\mu\,\lambda_i c_1 + 3\lambda_c c_2\,\mu +$
$3\lambda_h c_3\mu + \lambda_i c_1 \lambda_c c_2 + \lambda_i c_1 \lambda_h c_3)$ ------------ (4.7)

Therefore, the expression of M L estimate of steady-state availability function for parallel system is derived as

$$\hat{A}^*_{ph}(\infty) = \left[\frac{2\hat{\hat{z}}(2\hat{\hat{x}}.c_1 + \hat{\hat{y}}.c_2 + \hat{\hat{w}}.c_3 + \hat{\hat{z}})}{(2\hat{\hat{z}}.\hat{\hat{z}} + 2\hat{\hat{x}}.c_1.\hat{\hat{x}}.c_1 + 4\hat{\hat{z}}.\hat{\hat{x}}.c_1 + 3\hat{\hat{y}}.\hat{\hat{z}}.c_2 + 3\hat{\hat{w}}.\hat{\hat{z}}.c_3 + \hat{\hat{x}}.c_1.\hat{\hat{y}}.c_2 + \hat{\hat{x}}.c_1.\hat{\hat{w}}.c_3)}\right] - - - - - (4.8)$$

V. Confidence Interval

The estimates presented in equations (4.2, 4.4, 4.6 & 4.8) are functions of $\bar{x}, \bar{y}, \bar{w}$ & $\bar{z}$ which are differentiable. Now from (RAO 1974) multivariate central limit theorem:

$\sqrt{n}[(\bar{x}, \bar{y}, \bar{w}\ \&\ \bar{z}) - (\lambda_i, \lambda_c, \lambda_h, \mu)] \sim N_4(0, \Sigma)$ for $n \to \infty$

Where $\Sigma = (\sigma_{ij})_{4\times4}$ co-variance matrix, $\Sigma = \mathrm{dig}\,(\lambda_i^2, \lambda_c^2, \lambda_h^2\,\mu^2)$

Also we have $\sqrt{n}\,[A_s^*(\infty) - \hat{A}_s^*(\infty)] \sim N(0, \sigma_\theta^2)$ as $n \to \infty$ and θ is the vector. By the properties of M L method of estimation $\hat{A}_s^*(\infty)$ is CAN estimate of $A_s(\infty)$ respectively. Also $\sigma^2_{(\hat{\theta})}$ be the estimator of $\sigma^2_{(\theta)}$

Where $(\hat{\theta}) = (\hat{\hat{x}}, \hat{\hat{y}}, \hat{\hat{w}}\ \&\ \hat{\hat{z}})$ and

Let us consider $\psi = \sqrt{n}\,[\hat{A}_s(\infty) - A_s(\infty)] / \sigma^2_\theta \sim N(0,1)$

from Slutsky theorem, we have $P[-Z_{\alpha/2} \le \psi \le Z_{\alpha/2}] = 1-\alpha$

Where $Z_{\alpha/2}$ are the α/2 percentiles points of normal distribution and are available from normal tables. Hence (1–α)% confidence interval for availability function are given by are given by

$$A^*_{sh}(\infty) \pm Z_{\alpha/2}\ \sigma^2_{A_{sh}(\infty)} / \sqrt{n}$$
$$A^*_{ph}(\infty) \pm Z_{\alpha/2}\ \sigma^2_{A_{sh}(\infty)} / \sqrt{n}$$

Respectively. $\hat{A}^*_{sh}(t)$, $\hat{A}^*_{ph}(t)$, $\hat{A}^*_{sh}(\infty)$, $\hat{A}^*_{ph}(\infty)$ are the M L estimates of $A^*_{sh}(t)$, $A^*_{ph}(t)$ $A^*_{sh}(\infty)$, $A^*_{ph}(\infty)$ and $\hat{\sigma}_{(\hat{A}^*_{sh}(t), \hat{A}^*_{ph}(t), \hat{A}^*_{sh}(\infty), \hat{A}^*_{ph}(\infty))}$ is M L estimate of $\sigma_{(A^*_{sh}(t), A^*_{ph}(t), A^*_{sh}(\infty), A^*_{ph}(\infty))}$. We know that $\hat{A}^*_{sh}(t)$, $\hat{A}^*_{ph}(t)$, $\hat{A}^*_{sh}(\infty)$, $\hat{A}^*_{ph}(\infty)$ are M L E's and CAN (Consistently Asymptotic Normal) estimates of $A^*_{sh}(t)$, $A^*_{ph}(t)$ $A^*_{sh}(\infty)$, $A^*_{ph}(\infty)$.

VI. Simulation And Validity

In this paper, we developed empirical evidence of M L estimation by Monte Carlo simulation procedure for validity of results. For a range of specified values of the rates of individual (λ_i), common cause failures (λ_c), human errors (λ_h) and repair rates (μ) and for the samples of sizes n = 5 (5) 30 are using computer package developed in this research work and M L Estimates are computed for N = 10,000 (20,000) 90,000 and mean square error (MSE) and confidence interval of the estimates for$A^*_{sh}(t)$,$A^*_{ph}(t)$,$A^*_{sh}(\infty)$, $A^*_{ph}(\infty)$ were obtained and given in numerical illustration. For large samples Maximum Likelihood estimators are undisputedly better since they are CAN estimators. However it is interesting to note that for a sample size as low as five i.e (n=5) M L estimate is still seem to be reasonably good giving near accurate estimate in this case.

6.1 Numerical Illustration

Table.1 Simulation results for Time-dependent Availability function for series system with λ_i,= 0.1; λ_c = 0.2; λ_h = 0.05; μ = 5 c_1 = 0.5; c_2 = 0.25; c_3 = 0.25; t =1

Sample size (n = 5)			
N	$A^*_{sh}(t)$	$\hat{A}^*_{sh}(t)$	M S E
10000	0.952642	0.952929	0.000253
30000	0.952642	0.952867	0.000243
50000	0.952642	0.952852	0.000249
70000	0.952642	0.952846	0.000251
90000	0.952642	0.952764	0.000254
Sample size (n =10)			
N	$A^*_{sh}(t)$	$\hat{A}^*_{sh}(t)$	M S E
10000	0.952642	0.954544	0.000100
30000	0.952642	0.954502	0.000104
50000	0.952642	0.954434	0.000102
70000	0.952642	0.954476	0.000102
90000	0.952642	0.954453	0.000103
Sample size (n =15)			
N	$A^*_{sh}(t)$	$\hat{A}^*_{sh}(t)$	M S E
10000	0.952642	0.954976	0.000080
30000	0.952642	0.955015	0.000080
50000	0.952642	0.954897	0.000080
70000	0.952642	0.954887	0.000080
90000	0.952642	0.954938	0.000079
Sample size (n =25)			
N	$A^*_{sh}(t)$	$\hat{A}^*_{sh}(t)$	M S E
10000	0.952642	0.955298	0.000048
30000	0.952642	0.955341	0.000048
50000	0.952642	0.955292	0.000048
70000	0.952642	0.955312	0.000048
90000	0.952642	0.955298	0.000047
Sample size (n =30)			
N	$A^*_{sh}(t)$	$\hat{A}^*_{sh}(t)$	M S E
10000	0.952642	0.955460	0.000042
30000	0.952642	0.955378	0.000041
50000	0.952642	0.955410	0.000041
70000	0.952642	0.955382	0.000041
90000	0.952642	0.955382	0.000041

Table.2 Simulation results for Time-dependent Availability function for parallel system with λ_i = 0.1; λ_c = 0.2; λ_h = 0.05; μ = 5; c_1 = 0.5; c_2 = 0.25; c_3 = 0.25; t =1

Sample size (n = 5)			
N	$A^*_{ph}(t)$	$\hat{A}^*_{ph}(t)$	MSE
10000	0.989410	0.989498	0.000000
30000	0.989410	0.989635	0.000000
50000	0.989410	0.989739	0.000000
70000	0.989410	0.989788	0.000000
90000	0.989410	0.989827	0.000000

Sample size (n = 10)			
N	$A^*_{ph}(t)$	$\hat{A}^*_{ph}(t)$	MSE
10000	0.989410	0.989575	0.000000
30000	0.989410	0.989757	0.000000
50000	0.989410	0.989843	0.000000
70000	0.989410	0.989887	0.000000
90000	0.989410	0.989913	0.000000

Sample size (n = 15)			
N	$A^*_{ph}(t)$	$\hat{A}^*_{ph}(t)$	M S E
10000	0.989410	0.989593	0.000000
30000	0.989410	0.989795	0.000000
50000	0.989410	0.989872	0.000000
70000	0.989410	0.989907	0.000000
90000	0.989410	0.989928	0.000000

Sample size (n = 20)			
N	$A^*_{ph}(t)$	$\hat{A}^*_{ph}(t)$	MSE
10000	0.989410	0.989596	0.000000
30000	0.989410	0.989807	0.000000
50000	0.989410	0.989885	0.000000
70000	0.989410	0.989917	0.000000
90000	0.989410	0.989935	0.000000

Sample size (n = 25)			
N	$A^*_{ph}(t)$	$\hat{A}^*_{ph}(t)$	MSE
10000	0.989410	0.989606	0.000000
30000	0.989410	0.989821	0.000000
50000	0.989410	0.989890	0.000000
70000	0.989410	0.989923	0.000000
90000	0.989410	0.989939	0.000000
Sample size (n = 30)			
N	$A^*_{ph}(t)$	$\hat{A}^*_{ph}(t)$	MSE
10000	0.989410	0.989613	0.000000
30000	0.989410	0.989824	0.000000
50000	0.989410	0.989895	0.000000
70000	0.989410	0.989925	0.000000
90000	0.989410	0.989942	0.000000

Table.3 Simulation results for steady-state Availability function for series system with λ_i,= 0.1; λ_c = 0.2; λ_h = 0.05; μ.= 5; c_1 = 0.5; c_2 = 0.25; c_3 = 0.25

Sample size (n =5)			
N	$A^*_{sh}(\infty)$	$\hat{A}^*_{sh}(\infty)$	M S E
10000	0.240000	0.248818	0.008504
30000	0.240000	0.249210	0.008499
50000	0.240000	0.249459	0.008494
70000	0.240000	0.249171	0.008636
90000	0.240000	0.249688	0.008662

Sample size (n =10)			
N	$A^*_{sh}(\infty)$	$\hat{A}^*_{sh}(\infty)$	M S E
10000	0.240000	0.244141	0.004415
30000	0.240000	0.244222	0.004574
50000	0.240000	0.244780	0.004491
70000	0.240000	0.244474	0.004488
90000	0.240000	0.244707	0.004465

Sample size (n =15)			
N	$A^*_{sh}(\infty)$	$\hat{A}^*_{sh}(\infty)$	M S E
10000	0.240000	0.242741	0.002917
30000	0.240000	0.242440	0.002925
50000	0.240000	0.243248	0.002946

70000	0.240000	0.243370	0.002956
90000	0.240000	0.243042	0.002927

Sample size (n =20)			
N	$A^*_{sh}(\infty)$	$\hat{A}^*_{sh}(\infty)$	M S E
10000	0.240000	0.243529	0.002209
30000	0.240000	0.242200	0.002190
50000	0.240000	0.242209	0.002153
70000	0.240000	0.242091	0.002174
90000	0.240000	0.242148	0.002162

Sample size (n =25)			
N	$A^*_{sh}(\infty)$	$\hat{A}^*_{sh}(\infty)$	M S E
10000	0.240000	0.241726	0.001739
30000	0.240000	0.241428	0.001729
50000	0.240000	0.241780	0.001719
70000	0.240000	0.241642	0.001725
90000	0.240000	0.241662	0.001741

Sample size (n =30)			
N	$A^*_{sh}(\infty)$	$\hat{A}^*_{sh}(\infty)$	M S E
10000	0.240000	0.240836	0.001451
30000	0.240000	0.241479	0.001462
50000	0.240000	0.241241	0.001432
70000	0.240000	0.241421	0.001439
90000	0.240000	0.241388	0.001437

Table.4 Simulation results for steady-state Availability function for Parallel system with λ_i= 0.1; λ_c = 0.2; λ_h = 0.05; μ = 5; c_1 = 0.5; c_2 = 0.25; c_3 = 0.25

Sample size (n =5)				
N	$A^*_{ph}(\infty)$	$\hat{A}^*_{ph}(\infty)$	M S E	Confidence - Intervals (95%)
10000	0.999616	0.999602	0.000000	(0.996311 , 1.000000)
30000	0.999616	0.999731	0.000000	(0.996311 , 1.000000)
50000	0.999616	0.999809	0.000000	(0.996311 , 1.000000)
70000	0.999616	0.999846	0.000000	(0.996311 , 1.000000)
90000	0.999616	0.999872	0.000000	(0.996311 , 1.000000)

Sample size (n =10)				
N	$A^*_{ph}(\infty)$	$\hat{A}^*_{ph}(\infty)$	M S E	Confidence - Intervals (95%)
10000	0.999616	0.999679	0.000000	(0.997279 , 1.000000)
30000	0.999616	0.999835	0.000000	(0.997279 , 1.000000)
50000	0.999616	0.999897	0.000000	(0.997279 , 1.000000)
70000	0.999616	0.999927	0.000000	(0.997279 , 1.000000)
90000	0.999616	0.999943	0.000000	(0.997279 , 1.000000)

Sample size (n =15)				
N	$A^*_{ph}(\infty)$	$\hat{A}^*_{ph}(\infty)$	M S E	Confidence - Intervals (95%)
10000	0.999616	0.999693	0.000000	(0.997708 , 1.000000)
30000	0.999616	0.999865	0.000000	(0.997708 , 1.000000)
50000	0.999616	0.999918	0.000000	(0.997708 , 1.000000)
70000	0.999616	0.999942	0.000000	(0.997708 , 1.000000)
90000	0.999616	0.999954	0.000000	(0.997708 , 1.000000)

Sample size (n =20)				
N	$A^*_{ph}(\infty)$	$\hat{A}^*_{ph}(\infty)$	M S E	Confidence - Intervals (95%)
10000	0.999616	0.999697	0.000000	(0.997963 , 1.000000)
30000	0.999616	0.999878	0.000000	(0.997963 , 1.000000)
50000	0.999616	0.999927	0.000000	(0.997963 , 1.000000)
70000	0.999616	0.999947	0.000000	(0.997963 , 1.000000)
90000	0.999616	0.999959	0.000000	(0.997963 , 1.000000)

Sample size (n =25)				
N	$A^{*}_{ph}(\infty)$	$\hat{A}^{*}_{ph}(\infty)$	M S E	Confidence - Intervals (95%)
10000	0.999616	0.999704	0.000000	(0.998138 , 1.000000)
30000	0.999616	0.999888	0.000000	(0.998138 , 1.000000)
50000	0.999616	0.999930	0.000000	(0.998138 , 1.000000)
70000	0.999616	0.999952	0.000000	(0.998138 , 1.000000)
90000	0.999616	0.999961	0.000000	(0.998138 , 1.000000)

Sample size (n =30)				
N	$A^{*}_{ph}(\infty)$	$\hat{A}^{*}_{ph}(\infty)$	M S E	Confidence - Intervals (95%)
10000	0.999616	0.999709	0.000000	(0.998266 , 1.000000)
30000	0.999616	0.999890	0.000000	(0.998266 , 1.000000)
50000	0.999616	0.999934	0.000000	(0.998266 , 1.000000)
70000	0.999616	0.999952	0.000000	(0.998266 , 1.000000)
90000	0.999616	0.999964	0.000000	(0.998266 , 1.000000)

VII. Conclusion

We have derived maximum likelihood estimates for availability indices such as time-dependent availability $A_s(t)$ and Steady-state availability $A_s(\infty)$ for two unit system with Human errors and Common cause shock failures in the case of series and parallel systems. The model considered is motivated by potential applications in engineering systems and simplicity of the estimation approach used as is clear from simulation process. The possibility of human errors as well as CCS failures is included. Therefore, the empirical evidence was developed which indicate that MSE is found very small and satisfactory for the estimation process.

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