IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 14, Issue 4 Ver. II (Jul - Aug 2018), PP 10-17
www.iosrjournals.org

Slip Hydromagnetic Radiating Flow In Wavy Isothermal Channel With Porous Medium Considering Joule And Frictional Heating

P.F. Fasogbon[1], R.A. Adetona[2] D.A. Adeyemi[3]
[1,2,3]*Department Of Mathematics, Obafemi Awolowo University, Ile-Ife, Nigeria*
Corresponding Author: P.F. Fasogbon[1]

Abstract: *The problem of hydromagnetic mixed convection heat transfer across a porous medium due to buoyancy force, difference in temperature , the presence of pressure gradient and radiation is considered between two finitely long vertical isothermal walls, one of which is wavy. The presence of the heat source and combined effects of viscous dissipation and Joule heating in slip flow regime is taken into account. The resulting nonlinear dimensionless governing equations are linearized by perturbation technique and then solved numerically using the software MAPLE. As a result, the dimensionless velocity, temperature, skin friction coefficient and the Nusselt number are illustrated through the graphs and discussed for different physical parameters.*
Keywords*- Frictional heating, hydromagnetic slip flow, isothermal wavy channel, Joule heating, porous medium.*

Date of Submission: 15-07-2018 Date of acceptance: 31-07-2018

I. Introduction

Investigation of flow of viscous fluid over or through wavy wall(s), in both horizontal and vertical orientations, has been of much interest to scholars because of its relevance to physical problems (see for example, Vajravelu and Sastri [1] and the literature cited therein), for different wall temperature conditions, where the flow and transfer processes are limited to buoyancy effect of thermal diffusion. Since then, many scholars have extended the work on the same geometry; Das and Ahmed [2] and Fasogbon [3] account for the presence of an external transverse magnetic field which is applied uniformly with constant heat source or sink, Taneja and Jain [4] examined the slip effects on hydromagnetic flow in presence of temperature-dependent heat source, Fasogbon and Omolehin [5] examined the radiation effect, Fasogbon [6] analysed the combined effects of thermal and mass diffusion in the presence of uniform internal heat source or sink, Devika et al. [7] discussed steady chemical reaction effects on MHD free convection flow through porous medium, Ahmed et al. [8] carried out the Soret and Dufour effects on an MHD free convective flow, Soret and Dufour effects on steady free convective MHD viscoelastic fluid flow was studied by Das [9], Amini et al. [10] considered effects of wall waviness and temperature variations on the fluid flow and natural convection, and most recently, Gbadeyan et al. [11] studied the problem of coupled heat and mass transfer by free convection of a chemically-reacting viscous incompressible and electrically conducting fluid in the presence of diffusion-thermo, thermal-diffusion and constant internal heat source or sink. This present work is primarily motivated by the flow of a viscous fluid over and through a porous medium. This is due to its natural occurrence in the movement of water and oil inside the earth and the flow of rivers through porous banks. Hydromagnetic laminar flow through a porous medium is also relevant for its wide range of application in many branches of science and technology, chemical engineering and petroleum technology (cf. Ravikumar et al. [12]).

Most of the previous studies of the flows of heat or heat and mass convection have focused mainly on no-slip boundary condition where the fluid velocity takes the velocity of the solid boundary. However, the non-adherence of the fluid to a solid boundary known as velocity slip, has been observed under certain practical problems, such as the walls of air-craft and rockets moving at very high altitude. When the particles adjacent to the wall no longer take the velocity of the wall but possess a finite tangential velocity which slips along the wall. The flow regime thus induced is referred to as slip flow regime, and its effect is relevant in this time of modern science, technology and fast growing industrialization. Other vital practical applications are where a thin film of light oils is attached to the moving walls or when the wall is coated with special coating such as thick monolayer of hydrophobic mechanical device where a thin of lubricant is attached to the wall slipping over one another or when the walls are coated with special coating to minimize the friction between them, and others abound. Another subject of interest is the combined action of viscous dissipation and Joule heating on the hydromagnetic flow and heat transfer for its application on heat exchanger designs, wire and glass fiber drawing and in nuclear engineering in connection with the cooling of reactors. Another vital application is in various devices which are subjected to large variations of gravitational force. In view of these applications, Chauhan and Agrawal [13] presented hydrodynamic convection effects with viscous and Ohmic dissipation in a vertical channel partially filled by a porous medium. Jaber [14] investigated Joule heating and viscous dissipation effects on hydromagnetic flow over stretching porous sheet subjected to power law heat flux in presence of heat source. Ohmic heating and viscous dissipation effects over a vertical plate in the presence of porous medium was performed by Loganathan and Sivapoornapriya [15]. To the best of our knowledge, combined effects of viscous dissipation and Joule heating on free and force convection through a porous medium which is bounded by finitely long vertical isothermal walls (see [1-11]) and the present work demonstrate the issue.

II. Formulation Of The Problem

A steady, laminar, two-dimensional hydromagnetic mixed convection flow of a viscous incompressible electrically conducting dissipative fluid which is an optically thin non gray gas through a porous medium between two finitely long vertical walls of distance d apart is considered. A Cartesian co-ordinates system where X-axis is in the upward direction; direction of flow, and the axis of Y normal to it is chosen. The hot wavy wall at Y = 0 is sinusoidal $\left(Y=\varepsilon^{*}\cos(k_w X)\right)$ and has a uniform temperature T_w while the wall at Y=d is cold smooth flat subjected to slip velocity $u_s^{'}$ and a uniform temperature T_1, where $u_s^{'}$ is proportional to the local wall shear stress and is given by $u_s{'}=L'\dfrac{\partial u'}{\partial y'}$ and $L'=\left(\dfrac{2-m_1}{m_1}\right)L$. Orthogonal to the channel walls is a uniform magnetic field of strength B_0. Properties of the fluid are assumed constant except that density change with temperature that brings about the buoyancy forces in a manner corresponding to the equation of state $\rho_e=\rho\left(1+\beta\left(T-T_e\right)\right)$. There is no applied magnetic field in the Y-direction and magnetic Reynold's number is much less than unit, so that Hall, ion slip current and induced magnetic field are ignored. $\nabla\cdot B=0$ (Gauss's law of magnetism) gives B_y =constant= B_0 in the flow, thus $B=\left(0,B_0,0\right)$ and Lorentz force $\vec{F}=\vec{J}\times\vec{B}=\sigma_e\left(\vec{u}\times\vec{B}\right)\times\vec{B}=-\sigma_e B_0^2 u$, $\nabla\times\vec{B}=\vec{J}$, $\nabla\cdot\vec{E}=0$ and $\vec{J}=\sigma_e\vec{u}\times\vec{B}$ (Ohm's law).

Omitted again are Maxwell currents displacement and free charges. Now, we assume the fluid is in the optically thin limit; the fluid does not absorb its own radiation but it only absorbs radiations emitted by the boundaries. Accordingly, Cogley et al. [16] showed that the relation $\dfrac{\partial q_r}{\partial r}=4I\left(T-T_e\right)$

where $I=\int_0^\infty \alpha_{\lambda_e}\left(\dfrac{\partial B_{\lambda_p}}{\partial T}\right)_e d\lambda$, evaluated at the temperature T_e holds.

The equations which govern the problem when the velocity and the temperature are functions of X and Y are in the dimensionless form as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial(P_0-P_e)}{\partial x}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+G_r\theta-\frac{1}{A}u-Mu \qquad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial P}{\partial y}+\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}-\frac{1}{A}v \qquad (3)$$

$$p_r\left(u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}\right)=\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}-R\theta+S\theta+je(ec)u^2+p_r(ec)\left(\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}-4\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}\right)$$

(4)

$$A=\frac{k^{'}}{d^2},M=\frac{\sigma_e B_0^2 d^2}{\rho\nu},G_r=\frac{d^3 g\beta(T_w-T_e)}{\nu^2},\ je=\frac{\sigma_e B_0^2 d^2 c_p}{k},ec=\frac{\nu^2}{c_p(T_w-T_e)d^2}$$

The above equations were non-dimensionalized upon using dimensionless variables

$$\left(x,y\right)=\frac{1}{d}\left(X,Y\right),\left(u,v\right)=\frac{d}{\nu}\left(u',v'\right),\theta=\frac{T-T_e}{T_w-T_e},T_w>T_e,P=\frac{P'd^2}{\rho\nu^2},P_e=\frac{P_e'd^2}{\rho\nu^2},\lambda=k_w d,$$

$$\varepsilon=\frac{\varepsilon^*}{d},p_r=\frac{\mu c_p}{k},S=\frac{Qd^2}{k},n=\frac{T_1-T_e}{T_w-T_e},R=\frac{4Id^2}{k}$$

The boundary conditions for the present problem are:

$$u=0,v=0,\theta=1\,at\;y=\varepsilon\cos(\lambda x)$$

$$u=h\frac{\partial u}{\partial y},v=0,\theta=n\;at\;y=1 \tag{5}$$

III. Solution Of The Problem

To solve the equations (1) to (4), we assume

$$u\left(x,y\right)=u_0(y)+\varepsilon u_1\left(x,y\right),v=\varepsilon v_1\left(x,y\right)$$

$$p\left(x,y\right)=p_0(y)+\varepsilon p_1\left(x,y\right) \tag{6}$$

$$\theta\left(x,y\right)=\theta_0(y)+\varepsilon\theta_1\left(x,y\right)$$

where the zeroth- and perturbed quantities denoted by the subscripts 0 and 1 respectively.

Substituting the equation (6) in equations (1) to (4) and equating ε^0, ε^1 and neglecting $O\left(\varepsilon^2\right)$, we obtain

$$u_0''(y)-\left(M+\frac{1}{A}\right)u_0(y)=k_p-G_r\theta_0(y) \tag{7}$$

$$\theta_0''(y)-\left(R-S\right)\theta_0(y)+je(ec)\left(u_0(y)\right)^2+p_r(ec)\left(u_0'(y)\right)^2=k_p-G_r\theta_0(y) \tag{8}$$

Satisfying

$$u_0=0,\,\theta_0=1,\,on\;y=0\qquad u_0=h\,u_0'(y),\,\theta_0=n,\,on\;y=1 \tag{9}$$

$$u_{1,x}+v_{1,y}=0 \tag{10}$$

$$u_0u_{1,x}+v_1u_0'=-P_{1,x}+u_{1,xx}+u_{1,yy}-\frac{1}{A}u_1+G_r\theta_1-Mu_1 \tag{11}$$

$$u_0v_{1,x}=-P_{1,y}+v_{1,xx}+v_{1,yy}-\frac{1}{A}v_1 \tag{12}$$

$$p_r\left(u_0\theta_{1,x}+v_1\theta_0'\right)=\theta_{1,xx}+\theta_{1,yy}-\frac{1}{A}u_1-R\theta_1-S\theta_1+2je(ec)u_0u_1+2p_r(ec)u_0'\left(u_{1,y}+v_{1,x}\right) \tag{13}$$

subject to the boundary conditions

$$u_0=-u_0,v_1=0,\,\theta_1=-\theta_0'(y),\,on\;y=0$$

$$u_1=h\,u_1'(y),\,v_1=0,\,\theta_1=0,\,on\;y=1 \tag{14}$$

To solve (10) - (14), we introduce a stream function

Ψ_1

$$u_1=-\Psi_{1,y},\,v_1=\Psi_{1,x} \tag{15}$$

so that (10) can be satisfied identically and P_1 eliminated. We then assume wave-like solutions of the form

$$\Psi_1(x,y)=\varepsilon e^{i\lambda x}\psi(x,y)$$

(16)

$$\theta_1(x,y)=\varepsilon e^{i\lambda x}\phi(x,y)$$

So that (15) becomes

$$u_1(x,y)=-\varepsilon e^{i\lambda x}\psi'(x,y) \qquad v_1(x,y)=\varepsilon i e^{i\lambda x}\psi(x,y)$$

and equations in terms of stream function reduce to

$$\psi^{iv}+\lambda^2\left(\frac{1}{A}\psi-2\psi''\right)-\frac{1}{A}\psi''-M\psi''-G_r\phi'=i\lambda\left(u_0\psi''-u_0''\psi\right)$$

(17)

$$\phi''-R\phi+S\phi+2je(ec)u_0\psi'+2p_r(ec)u_0'\left(-\psi''-\lambda^2\psi\right)-\lambda^2\phi=i\lambda p_r\left(u_0\phi+\theta_0'\psi\right)$$

(18)

where *i* is the complex unit. If we further assume that λ is much less than unity or $\left(k_w<<1\right)$, we write

$$\psi\left(\lambda,y\right)=\sum_{j=0}^{\infty}\lambda^j\psi_j$$

(19)

$$\psi\left(\lambda,y\right)=\sum_{j=0}^{\infty}\lambda^j\psi_j$$

j=(0,1,2, …)

Putting (19) into (17) and (18) in terms of ψ_j and comparing the like-power terms of λ to the order of λ^2, we have overall set of ordinary differential equations and the boundary conditions for this work:

$$\frac{d^4}{dy^4}\Psi_0(y)-\left(M+\frac{1}{A}\right)\left(\frac{d^2}{dy^2}\Psi_0(y)\right)=G_r\left(\frac{d}{dy}\phi_0(y)\right)$$

(20)

$$\frac{d^2}{dy^2}\phi_0(y)-(R-S)\phi_0(y)+2je\,ec\,u_0(y)\left(\frac{d}{dy}\Psi_0(y)\right)-2P_r ec\left(\frac{d}{dy}u_0(y)\right)\left(\frac{d^2}{dy^2}\Psi_0(y)\right)=0$$

(21)

$$\frac{d^4}{dy^4}\Psi_1(y)-\left(M+\frac{1}{A}\right)\left(\frac{d^2}{dy^2}\Psi_1(y)\right)=G_r\left(\frac{d}{dy}\phi_1(y)\right)+u_0(y)\left(\frac{d^2}{dy^2}\Psi_0(y)\right)-\left(\frac{d^2}{dy^2}u_0(y)\right)\Psi_0(y)$$

(22)

$$\frac{d^2}{dy^2}\phi_1(y)-(R-S)\phi_1(y)+2je\,ec\,u_0(y)\left(\frac{d}{dy}\Psi_1(y)\right)-2P_r ec\left(\frac{d}{dy}u_0(y)\right)\left(\frac{d^2}{dy^2}\Psi_1(y)\right)=u_0(y)\phi_0(y)+\Psi_0(y).\left(\frac{d}{dy}\theta_0(y)\right)$$

(23)

$$\frac{d^4}{dy^4}\Psi_2(y)-\left(M+\frac{1}{A}\right)\left(\frac{d^2}{dy^2}\Psi_2(y)\right)+\frac{\Psi_0(y)}{A}=2\left(\frac{d^2}{dy^2}\Psi_0(y)\right)+G_r\left(\frac{d}{dy}\phi_2(y)\right)-u_0(y)\left(\frac{d^2}{dy^2}\Psi_1(y)\right)+\left(\frac{d^2}{dy^2}u_0(y)\right)\Psi_1(y)$$

(24)

$$\frac{d^2}{dy^2}\phi_2(y) - (R-S)\phi_2(y) + 2je\,ec\,u_0(y)\left(\frac{d}{dy}\psi_2(y)\right) - 2P_r ec\left(\frac{d}{dy}u_0(y)\right)\left(\frac{d^2}{dy^2}\psi_2(y)\right) - 2P_r ec\left(\frac{d}{dy}u_0(y)\right)\psi_0(y) = \phi_0(y) - u_0(y)\phi_1(y) + \psi_1(y)\left(\frac{d}{dy}\theta_0(y)\right) \tag{25}$$

The solutions for u and θ have been obtained, and the physical quantities of interest at any point in the field ; skin friction C_f and Nusselt number N_u emerged but are not presented here for the sake of brevity. From equations (6), (15), (16) and (19), after obvious simplification, on the assumption that $\lambda x = \frac{\pi}{2}$

$$u(y) = u_0(y) + \varepsilon\lambda_1\psi_1'(y)$$
$$\theta(y) = \theta_0(y) - \varepsilon\lambda\phi_1'(y)$$
$$c_f(y) = u_0'(y) + \varepsilon\lambda_1\psi_1''(y)$$
$$N_u(y) = \theta_0'(y) - \varepsilon\lambda\phi_1'(y)$$

IV. Results And Discussion

To study the combined influence of friction and ohmic heating on the hydromagnetic free and force convection on heat transfer through a porous medium in finitely long vertical isothermal wavy channel , the dimensionless velocity u, the temperature θ, skin friction coefficient C_f and Nusselt number N_u are illustrated graphically against y for selected values of physical parameters.

Here we took the geometric parameters $\lambda x = \frac{\pi}{2}$, λ =0.01, and $\varepsilon = 0.025$, $p_r = 0.71$, $h = 0/0.2$, for no slip/slip condition, $k_p = 0/2$ means absence/presence of pressure gradient. The default parameters used throughout the numerical calculations, unless otherwise stated are: $G_r = 5, A = 0.3, p_r = 0.71, h = 0.2\ n = 0.4, ec = 0.5, je = 0.5\ M = 3, k_p = 2, R = 3, S = 2$

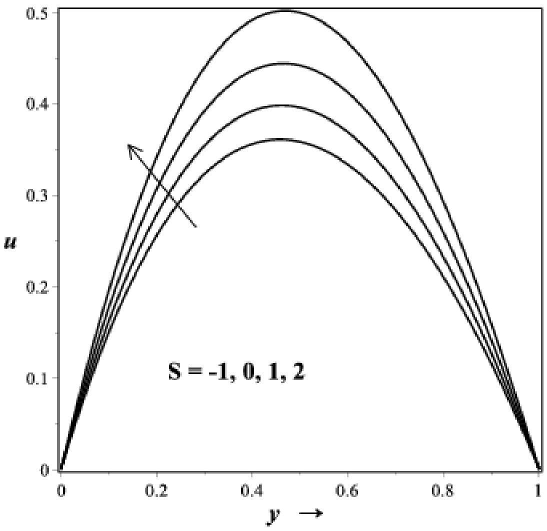


Fig 1. Variation of S on u against y when kp=0

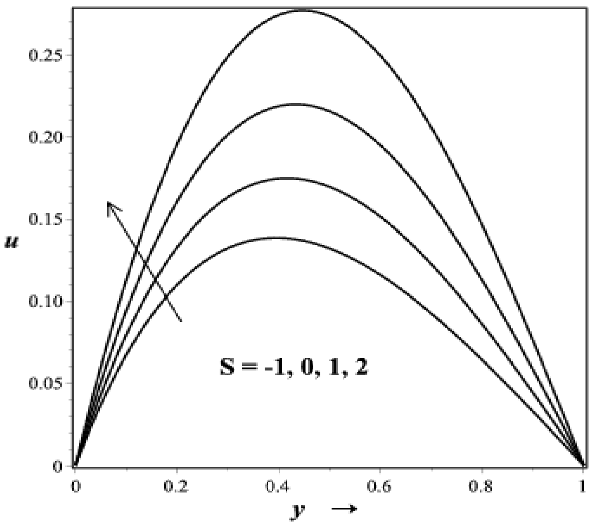
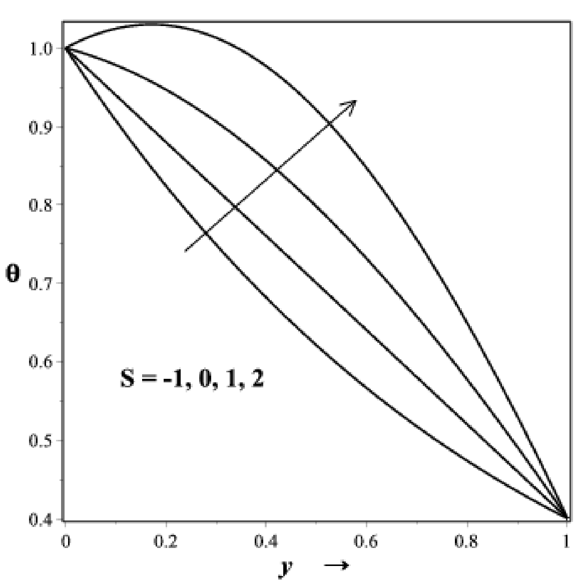


Fig 2. Variation of S on u against y when kp=2



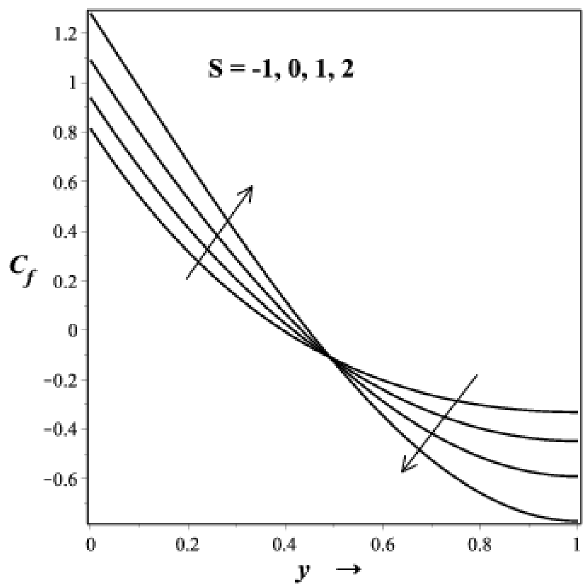


Fig 3. Variation of S on theta against y when kp=0, 2 **Fig 4**. Variation of S on skin friction cf against y when kp=2

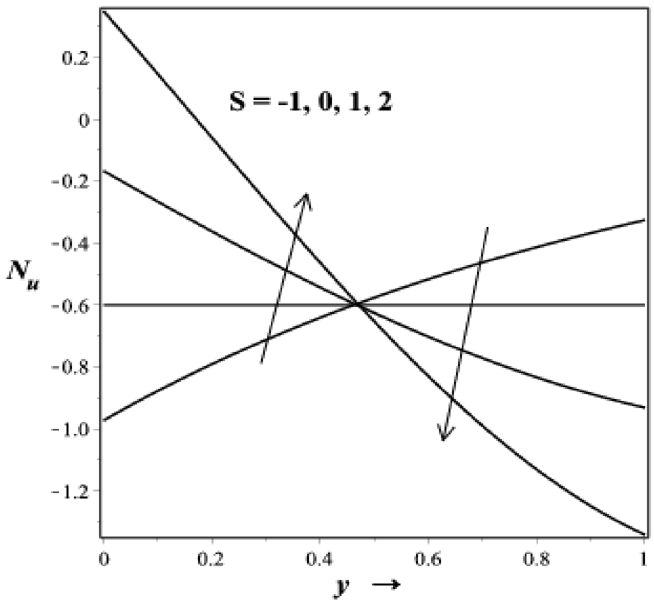


Fig 5. Variation of S on Nu against y when kp=2

We set the values je, ec, A, R, M, to zero and n=0.4, A=1 and kp=0, 2 introduced in the present study and obtained the results in Fig. 1-5 . The results compared with the famous work of Vajravelu ans Sastri [1] and found to be in good agreement.

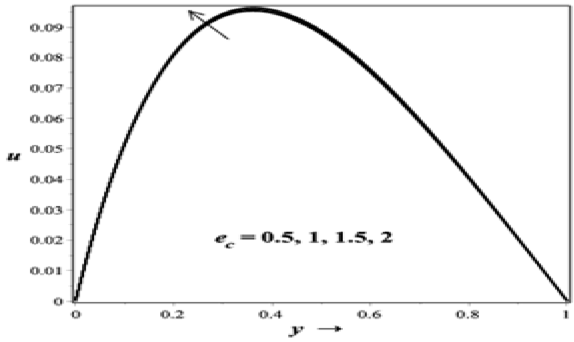


Fig 6. Variation of ec on u against y when kp=2

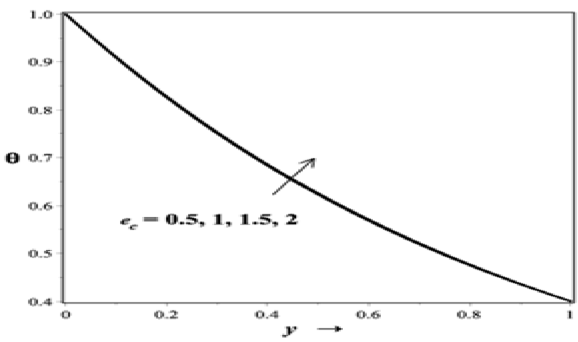


Fig 7. Variation of ec on theta against y when kp=2

Fig. 6 and 7 showed that viscous and Joule heating has very slight influence on velocity profile both in no-slip and slip condition.

V. Conclusion

The study reveals that the rate of heat transfer is most effective in a sinusoidal wavy isothermal wall than in an isothermal smooth flat wall. In mixed convection, the influence of buoyancy forces on a forced flow or the influence of forced flow on a buoyant flow is significant, hence, immensely affected by increasing pressure gradient.

References

[1]. Vajravelu K and Sastri K.S, (1978), "Free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall", Journal of Fluid Mechanics, Vol. 86, No. 2, pp. 365-383.

[2]. Das U.N and Ahmed N, (1992), "Free convective MHD flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall", Indian Journal of Pure and Applied. Mathematics, Vol. 23, No. 4, pp. 299-304.

[3]. Fasogbon P. F, (2007), "MHD flows in corrugated channel", International Journal of Applied Mathematics, Vol. 20, No. 2, pp. 137-147.

[4]. Taneja R and Jain N.C, (2004), "MHD flow with slip effects and temperature-dependent heat source in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall", Defence Science Journal, Vol. 54, No.1, pp. 21-29.

[5]. Fasogbon P.F and Omolehin J.O, (2008), "Radiation effect on natural convection in spirally enhanced channel", International Electronic Journal of Engineering Mathematics.: Theory and Application (IeJEMTA), Vol. 3, No. 1, pp.1-28.

[6]. Fasogbon P.F, (2010), "Analytical studies of heat and mass transfer by free convection in a two-dimensional irregular channel. International Journal of Applied Mathematics and Mechanics, Vol. 6, No. 4, pp. 17-37.

[7]. Devika B, Narayana P.V.S and Venkataramana S, (2013), "Chemical reaction effects on MHD Free convection flow in an irregular channel with porous medium", International Journal of Mathematical. Archive, Vol. 4, No. 4, pp. 282-295.

[8]. Ahmed N, Sarma K and Deka H, (2014), "Soret and Dufour effects on an MHD free convective flow through a channel bounded by a long wavy wall and a parallel flat wall", Turkish Journal of Physics, Vol. 38, pp. 50-63.

[9]. Das U.J, (2014), "Soret and Dufour effects on steady free convective MHD viscoelastic fluid flow confined between a long vertical wavy wall and parallel flat wall", Thammasat International Journal of Science and Technology, Vol. 19, No.2, pp. 383-393.

[10]. Amini M, Najafi M, Dorosti Q, Biglarian H and Ghasemi E, (2015), "Effects of wall waviness and temperature variations on the fluid flow and natural convection in an inclined corrugated channel", Int J Appl Mathem Resh., Vol. 4, No. 1, pp. 163-176.

[11]. Gbadeyan J. A, Oyekunle T. L,Fasogbon, P. F and Abubakar J. U, (2018), "Soret and Dufour effects on heat and mass transfer in chemically reacting MHD flow through a wavy channel", Journal of Taibah university for Science, DOI:10.1080/16583655.2018.1492221

[12]. Ravikumar V, Raju M.C and Raju G.S.S, (2012), "MHD three dimensional Couette flow past a porous plate with heat transfer", IOSR Journal of Mathematics (IOSR-JM), Vol. 1, Issue 3, pp. 03-09.

[13]. Chauhan D.S and Agrawal R, (2012), "Magnetohydrodynamic convection effects with viscous and Ohmic dissipation in a vertical channel partially filled by a porous medium", Journal of Applied Science and Engineering, Vol. 15, No. 1, pp. 1-10.

[14]. Jaber K.K, (2016), "Joule heating and viscous dissipation effects on MHD flow over stretching porous sheet subjected to power law heat flux in presence of heat source", Open Journal of Fluid Dynamics, Vol. 6, pp. 156-165.

[15]. Loganathan P and Sivapoornapriya C, (2016), "Ohmic heating and viscous dissipation effects over a vertical plate in the presence of porous medium", Journal of Applied Fluid Mechanics, Vol. 9, No. 1, pp.225-232.

[16]. Cogley A. C., Vinceti and Giles S. E., (1968), "Differential approximation for radiative transfer in a non-gray gas near equilibrium", AIAA J., 6, No.3, 551 – 556.

VI. Nomenclature

A	Permeability parameter	p_r	Prandtl number
B_0	Intensity of the applied magnetic field	q_r	Radiation heat flux
c_p	Specific heat at constant pressure	R	Radiation parameter
d	Distance between the walls	S	Heat source/sink parameter
ec	Eckert number		
g	Acceleration due to gravity		Greek symbols
G_r	Grashof number	β	Thermal expansion coefficient

h Velocity slip parameter $\left(h = \frac{L'}{d} \right)$

je Joule heating

k Thermal conductivity

k' Permeability of the porous medium

k_p Pressure gradient

M magnetic field parameter

N Temperature difference ratio

P pressure

T Fluid temperature

T_w Temperature of the wavy wall

T_1 Temperature of the smooth wall

T_e Temperature in the equilibrium state

(u, v) Dimensionless velocity components

(u', v') velocity components in (x', y') direction

(X, Y) Coordinate system

(x, y) dimensional coordinate

ρ_e density of the fluid in the equilibrium state

ρ Fluid density

μ Dynamic viscosity

ν Kinematic viscosity

m_1 Maxwell's reflection coefficient

L mean free path

k_w wave number

λ Wavelength

α_{λ_e} mean absorption coefficient

P.F. Fasogbon "Slip Hydromagnetic Radiating Flow In Wavy Isothermal Channel With Porous Medium Considering Joule And Frictional Heating" IOSR Journal of Mathematics (IOSR-JM) 14.4 (2018) PP: 10-17.