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Geometrical Series with Multiple Reasons

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Abstract : *the present article deals with geometric series, a numerical sequence of exponential behavior used in financial markets, study of bacterial growth, cell reproduction, population growth, atomic decay, etc.However, what is presented in the literature on this topic, talks about a single factor(reason), interfering in its growth, which makes predictive work limited to this characteristic.The purpose of this study is to discuss the possibilities of working with several factors (reasons) of interference in the growth or decrease of the series, making them attractive to those who need a more robust and real simulation.In this paper we present an advance related to the already published article on the Arithmetic Series by the same author, in which a general term equation for any value of the sequence is defined, as well as an equation for the calculation of the sum of these terms.*

Keywords –*Modular Arithmetic, Simulations, Reasons, Geometry, series.*

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I. Introduction

Geometric Series that use whole domain are defined as Geometric Progressions [1], and in this work will be called G.P.They make possible the most varied studies, being widely explored in the financial market, in the population growth, within the chemistry of atomic regression and growth of a bacterium or cell [2].

Within the common literature for this type of application, conventional formulas are presented that allow their use in the simulation or prediction of a certain result [3] and [4].These models use in their algorithms only one reason as a growth or decrease factor, thus requiring the user, several simulations under different conditions, effective statistical analysis and a manipulation of these results to arrive at the best predictions for the object being studied [5].

This work offers an option for people searching within these areas, or others who need this calculation model, to make use of a varied number of reasons or factors for growth / decay.Thus, in a single calculation line, one can simulate a sequence with indeterminate values, limited only by the researcher's need [1].

In the following sections, two equations can be used to find any term within a sequence with this characteristic, as well as an equation that allows you to find the sum of all terms within the series up to the term displayed.It is intended with this work, to help the execution of activities of people who need to manipulate these concepts in their research.

II. InitialSettings

It is characterized as a geometric series with multiple reasons a series that presents the following behavior:
Being:

$$a_1 = firstvalue, a_2 = Secondvalue, a_n = overallvalue, \; q_1 = firstreason,$$
$$q_2 = secondreason, q_n = overallreason,$$

then building the series will follow the steps:

$$a_{\,1} = (first\ value), a_2 = a_1 * q_1, a_3 = a_2 * q_2 \ldots \; a_n = a_{n-1} * q_{n-1}$$

This type of series accepts as input values referring to the positioning of the terms, so it will consist of positive integers.Series with this behavior receive the name of Geometric Progression [2], soon to be denominated only G.P. When the value of the quantity of reasons that is finite ends, the next term returns to the first reason by repeating the cycle of substitutions [1]. Therefore, the construction of a progression in these conditions, behaves as a sequence in module, where:

$$a_n = Qn\,mod(k)$$

Other variables will also be adopted:$k = nº\ q,\ Q = \prod q,\ y = \frac{n}{k}, wherey \in \mathbb{N}$ e $x = n - yk.$

III. General term of a Geometric Progression

When you want to find any value within a geometric series, the following procedure is used:

Lemma 01: $\boldsymbol{x = 0 \rightarrow a_n = a_k Q^{y-1}}$

The first cycle will be considered initially in the module.

$$a_1 = a_1$$
$$a_2 = a_1 * q_1$$
$$a_3 = a_2 * q_2 \rightarrow a_1 * q_1 * q_2$$
$$a_4 = a_3 * q_3 \rightarrow a_1 * q_1 * q_2 * q_3$$
$$\vdots$$
$$a_k = a_{k-1} * q_{k-1} \rightarrow a_1 * q_1 * q_2 * q_3 \dots * q_{k-1}$$

Therefore, it can be summarized that:

$$a_k = a_1 * \prod_{i=1}^{k-1} q_i \quad (1)$$

At the moment, the term ends the due cycle$n = k$. However, the reason q_k has not yet been used, becoming the first in the new cycle.

$$a_{k+1} = a_k * q_k$$
$$a_{k+2} = a_k * q_k * q_1$$
$$a_{k+3} = a_k * q_k * q_1 * q_2$$
$$a_{k+4} = a_k * q_k * q_1 * q_2 * q_3$$
$$\vdots$$
$$a_{2k} = a_k * q_k * q_1 * q_2 * q_3 \dots * q_{k-1}$$

Soon:

$$a_{2k} = a_k * Q^1 \ (2)$$

In general, it can be inferred:

Table 01: Cycle and equations to $x = 0$

Cycle	How to determine
y=1	$a_n = a_k$
y=2	$a_n = a_k * Q^1$
y=3	$a_n = a_k * Q^2$
y=4	$a_n = a_k * Q^3$
⋮	⋮
y=n	$a_n = a_k * Q^{y-1}$ (3)

Therefore, as we wanted to demonstrate with **Lemma 1**, we have to:

$$se\ x = 0 \rightarrow a_n = a_k Q^{y-1}$$

Lemma 02: $\boldsymbol{x \neq 0 \rightarrow a_n = a_x Q^y}$

First it will be analyzed before the closing of the first cycle of the module:

In this case,$n = x$, and $1 \leq x \leq k - 1$, because, if:

$$x = k \rightarrow n = k, logo\ \frac{n}{k} = 1, \rightarrow x = 0, \rightarrow x \neq k\ (contradiction).$$

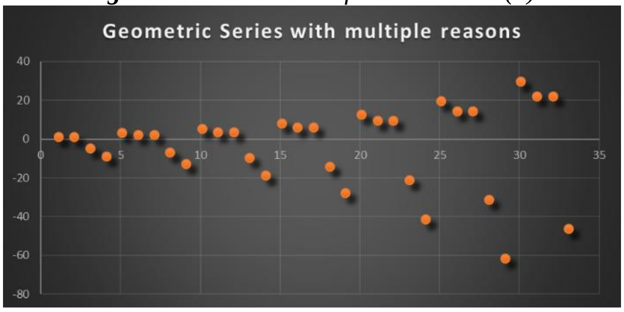
Like this,$\nexists\ x = k\ \forall\ n = x$

$$x = 1 \rightarrow a_x = a_1$$
$$x = 2 \rightarrow a_2 = a_1 * q_1$$
$$x = 3 \rightarrow a_3 = a_1 * q_1 * q_2$$
$$x = 4 \rightarrow a_4 = a_1 * q_1 * q_2 * q_3$$
$$\vdots$$
$$x = k - 1 \rightarrow a_{k-1} = a_1 * q_1 * q_2 * q_3 \dots * q_{k-1}$$
$$a_x = a_1 * \prod_{i=1}^{k-1} q_i \quad (4)$$

The next term will be a closed loop and is represented by **Lemma 1**.From this point, we will already have $y = 1$, and the values begin to repeat the terms of a_x, presenting a difference due to the beginning of the second cycle in the module.

It is observed that Fig. 1 below represents the simulation for a series with 5 real reasons:

Figure 1: Distribution of terms in mod (5)



It is noticed that the values are repeated in each cycle in the same positioning referring to the previous module but are displaced in a symmetrical unit with the passage of time.This unit is the same one presented and demonstrated in **Lemma 1**, that denominated of Q.

Therefore:

Table 02: Cycle and equations to $x \neq 0$	
Cycle	How to determine
y=0	$a_n = a_x$
y=1	$a_n = a_x * Q^1$
y=2	$a_n = a_x * Q^2$
y=3	$a_n = a_x * Q^3$
⋮	⋮
y=n	$a_n = a_x * Q^y$ (5)

Therefore, in general, one has the proof of **Lemma 02**:

$$se\ x \neq 0 \rightarrow a_n = a_x Q^y$$

IV. Sum of the Terms of G.P.

In order to find the sum of the desired terms in a G.P., two situations must be analyzed.One can be defined before the closure of a cycle and another after the closed cycle, so that the procedures already presented in [1], which provide the following situation, will be adopted:
For a non-closed cycle, the quantizer is adopted:

$$B = xa_1 + \sum_{i=1}^{x-1}(x-i)q_i \quad (6)$$

That represents the sum of all desired terms, prior to the close of the cycle.
Also [1] presents:

$$A = ka_1 + \sum_{i=1}^{k-1}(k-i)q_i \quad (7)$$

To represent the sum of all terms within a closed loop.

Thus, in considering the two cases, since one has to find the sum of terms that have several closed cycles and can be located at some point in a new cycle, it becomes necessary the following lemma:

Lemma 03: $\boldsymbol{S_n = \frac{A(1-Q^y)}{1-Q} + BQ^y}$

Before closing the first cycle, as stated, equation (6) is able to show the sum of the values for any$x$. At the beginning of the new cycle, the positioning of x begins to repeat that of the previous cycle, as it is formed by modular arithmetic, so it can be said that in the new cycle, the term will have the same positioning and characteristics of the previous one, with the change in its value due to its module.Therefore, it can be exemplified:

Table 03: Sum of terms before and after cycle 01	
Before	After
$\sum a_1 = a_1$	$\sum a_{k+1} = a_1 * Q$
$\sum a_2 = a_1 + a_2$	$\sum a_{k+2} = (a_1 + a_2) * Q$
⋮	⋮
$\sum a_x = a_1 + a_2 + \cdots + a_x$	$\sum a_{k+x} = (a_1 + a_2 + \cdots + a_x) * Q$

⋮	⋮
$\sum a_x = B$	$\sum a_x = BQ$

Following the same concept, one can then deduce a general equation for the terms of unclosed cycles in any part of the sequence:

Table 04: Sum of terms still all cycle

Cycle	Equation
0	$\sum a_x = B$
1	$\sum a_x = BQ$
2	$\sum a_x = BQ^2$
3	$\sum a_x = BQ^3$
⋮	⋮
y	$\sum a_x = BQ^y$ (8)

Now we will define the equation for sum of terms with complete cycles.

As for the complete cycles, there is no difference in the concept of a G.P. normal (single reason), because in this case, the multiple reasons cease to exist, because each closed loop has Q as the module's ratio.Each closed loop, represented by A, resembles the first term of a G.P. with a single reason.Thus, the definition could be the same already known in the literature, however, it will be deduced the sum equation for closed circles, for a better understanding of the article.

$$y = 1 \rightarrow \sum a_n = A$$

$$y = 2 \rightarrow \sum a_n = A + A\,Q$$

$$y = 3 \rightarrow \sum a_n = A + AQ + A\,Q^2$$

$$y = 4 \rightarrow \sum a_n = A + AQ + A\,Q^2 + AQ^3$$

$$\vdots$$

$$y = n \rightarrow \sum a_n = A + AQ + A\,Q^2 + AQ^3 \ldots AQ^{y-1} (9)$$

In this format equation (9) becomes difficult to operationalize, however, it is proposed the following change:

You must multiply both sides of the equation by Q.

$$Q(\sum a_n) = (A + AQ + A\,Q^2 + AQ^3 \ldots AQ^{y-1})Q$$

$$Q\sum a_n = AQ + A\,Q^2 + AQ^3 + AQ^4 \ldots AQ^y (10)$$

$$(09) - (10)$$

$$\sum a_n - Q\sum a_n = A + AQ + A\,Q^2 + AQ^3 \ldots AQ^{y-1} - AQ - AQ^2 - AQ^3 \ldots - AQ^y$$

$$\sum a_n - Q\sum a_n = A + \cancel{AQ} + \cancel{AQ^2} + \cancel{AQ^3} \ldots \cancel{AQ^{y-1}} - \cancel{AQ} - \cancel{AQ^2} - \cancel{AQ^3} \ldots - AQ^y$$

$$\sum a_n - Q\sum a_n = A - AQ^y$$

$$\sum a_n(1 - Q) = A(1 - Q^y)$$

$$\sum a_n = \frac{A(1 - Q^y)}{1 - Q} \quad (11)$$

As it is proposed to demonstrate, the sum equation of any number of terms in a geometric series will be the sum of equation (11), which gives the value for closed cycles, with equation (8), which also gives the values inside of a non-closed cycle.

$$S_n = (11) + (8)$$

$$S_n = \frac{A(1 - Q^y)}{1 - Q} + BQ^y \quad (12)$$

With this equation, we arrive at the sum value of any term.When the term is before the first cycle closes, the fraction is canceled and the result is provided by the operations of B.However, when the value is contained in a closed loop, the value of B cancels and the fraction of A determines the value of the sum. Finally, if the term is after the first cycle and within another non-closed cycle, the fraction of A determines the values of the closed cycles before the term, and the portion represented by the operation of B, gives the additional value within the new circle.

V. Conclusion

With equations (3), (5) and (12), it is believed to be able to assist the work of researchers who have the need to simulate series with several determining factors.This work advances concepts and allows one to improve existing studies in simulations, analyzes of events that are symmetrical to some factor, or factors.

It is understood that new questions arise with the nature of this approach, motivating the author in advancing his investigations within this analysis, in an attempt to reconcile a possible solution for a random distribution of the reasons within the series.This will be the scope that will guide future work.

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