IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 14, Issue 4 Ver. III (Jul - Aug 2018), PP 52-55
www.iosrjournals.org

Two - Sided Partial Differential Equations with Non-Local Boundary Conditions

Imanisho
Materials Engineering Department, University of Technology, Iraq
Corresponding Author: Imanisho

Abstract: *This paper presents an initial value problem with non-local boundary conditions for the partial differential equation. Modification of adomian decomposition method is introduced for solving problem. Simulation results for example illustrate the comparison of the analytical and numerical solution.*
Key words: *Partial differential equation, two-sided, modified decomposition method, non-local boundary condition problem.*

Date of Submission: 27-07-2018 Date of acceptance: 13-08-2018

I. Introduction

Partial differential equations with non-local boundary conditions and partial differential equations arise in many fields of science and engineering such as chemical diffusion, heat conduction processes, population dynamics, medical science, electrochemistry and control theory [1-13].

There are many literatures developed concerning Adomian decomposition method [15, 21, 22] and the related modification to investigate various sciences model [16, 17, 18, 19]. The theoretical treatment of the convergence of Adomian decomposition method has been considered in [20].

In this paper, we present computationally efficient numerical method for solving the partial differential equation with boundary integral conditions:

$$D_t\Omega(x,t) - D_{+xx}\Omega(x,t) - D_{-xx}\Omega(x,t) + \Omega(x,t) = f(x,t) \tag{1}$$

with the initial condition

$$\Omega(x,0) = q(x), 0 \le x \le T$$

and the non-local boundary conditions

$$\int_0^1 \Omega(x,t)dx = g_1(t), 0 < t \le T$$

$$\int_0^1 \mathrm{p(x)}\Psi(x,t)dx = g_2(t)\,, 0 < t \le T$$

Where q, g_1, g_2, Ω and f are known functions, T is given constant. In the present work, we apply the modified Adomian's decomposition method for solving eq.(1) and compare the results with exact solution. The paper is organized as follows: In section 2 the partial differential equations with non-local boundary conditions and its solution by modified decomposition method is presented. In section 3 an example is solved numerically using the modified decomposition method. Finally, we present conclusion about solution of two-sided partial differential equation.

II. Proposed Method:

The aim of this section is to discuss the use of modified decomposition method for solving of two-sided partial differential equations with non-local boundary given in eq.(1). In this method we assume that:

$$\Omega(x,t) = \sum_{n=0}^{\infty} \Omega_n(x,t)$$

Can be rewritten eq.(1):

$$L_t\Omega(x,t) = L_{+xx}\Omega(x,t) + L_{-xx}\Omega(x,t) - \Omega(x,t) + f(x,t) \qquad (2)$$

Where

$$L_t(\cdot) = \frac{\partial}{\partial t}(\cdot)$$

and $$L_{xx} = \frac{\partial^2}{\partial x^2}$$

The inverse L^{-1} is assumed an integral operator given by

$$L^{-1} = \int_0^t (\cdot)dt \qquad (3)$$

Take the operator L^{-1} of eq (2) we have

$$L^{-1}(L_t\Omega((x,t))) = L^{-1}(L_{+xx}(\Omega(x,t)) + L_{-xx}(\Omega(x,t))) - L^{-1}(\Omega(x,t)) + L^{-1}(f(x,t))$$

Therefore, we can write,

$$\Omega(x,t) = \Omega(x,0) + L_t^{-1}\left(L_{+xx}\left(\sum_{n=0}^{\infty}\Omega_n\right) + L_{-xx}\left(\sum_{n=0}^{\infty}\Omega_n\right)\right) - L_t^{-1}(\Omega(x,t)) + L_t^{-1}\left(f(x,t)\right) \qquad (4)$$

The modified decomposition method was introduced by Wazwaz [4]. This method is based on the assumption that the function $K(x)$ can be divided into two parts, namely $K_1(x)$ and $K_2(x)$. Under this assumption we set

$$K(x) = K_1(x) + K_2(x)$$

Then the modification

$$\Omega_0 = K_1$$

$$\Omega_1 = K_2 + L_t^{-1}\left(L_{+xx}\Omega_0\right) + L_t^{-1}\left(L_{-xx}\Omega_0\right) - L_t^{-1}\left(\Omega_0\right)$$

$$\Omega_{n+1} = L_t^{-1}\left(L_{+xx}\left(\sum_{n=0}^{\infty}\Omega_n\right)\right) + L_t^{-1}\left(L_{-xx}\left(\sum_{n=0}^{\infty}\Omega_n\right)\right) - L_t^{-1}\left(\sum_{n=0}^{\infty}\Omega_n\right), \ n>1$$

Numerical Example:

Consider partial differential equation with boundary integral condition for the equation (1), as taken in [23]:

$$D_t\Omega - D_{+xx}\Omega - D_{-xx}\Omega + \Omega = 2t + t^2 + x^2$$

$$\Omega(x,0) = x, \qquad x \in (0,1), \quad 0 \le t \le T$$

$$\int_0^1 \Psi(x,t)dx = t^2 + \frac{1}{3} \qquad 0 \le t \le T$$

$$\int_0^1 \mathrm{x}^2\Psi(x,t)dx = \frac{t^2}{3} + \frac{1}{5} \qquad 0 \le t \le T$$

We apply the above proposed method; we obtain:

$\Omega_0(x,t) = x^2 + t^2$

$\Omega_1(x,t) = 0$

$\Omega_2(x,t) = 0$

$\Omega_3(x,t) = 0$

Then the series form is given by:

$$\Omega(x,t) = \Omega_0(x,t) + \Omega_1(x,t) + \Omega_2(x,t) + \Omega_3(x,t)$$
$$= x^2 + t^2$$

This is the exact solution $u(x,t) = x^2 + t^2$.

Table 1 shows the analytical solutions for partial differential equation with boundary integral condition obtained for different values and comparison between exact solution and analytical solution.

Table1. Comparison between exact solution and analytical solution

x	**t**	**Exact Solution**	**Modified Adomian Decomposition Method**	$\lvert u_{ex}-u_{MADM}\rvert$
0	3	9.00	9.00	0.0000
0.1	3	9.01	9.01	0.0000
0.2	3	9.04	9.04	0.0000
0.3	3	9.09	9.09	0.0000
0.4	3	9.16	9.16	0.0000
0.5	3	9.25	9.25	0.0000
0.6	3	9.36	9.36	0.0000
0.7	3	9.49	9.49	0.0000
0.8	3	9.64	9.64	0.0000
0.9	3	9.81	9.81	0.0000
1	3	10.0	10.0	0.0000

III. Conclusion

In this paper, we have presented the modified decomposition method for the solution of two-sided partial differential equation with non-local boundary condition. This algorithm is easy to implement. Furthermore, numerical example is presented to show that good agreement between the numerical solution and exact solution has been noted.

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