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Application of Partial Differential Equations in Thermal Conduction Model

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Abstract: *Thermal protective clothing not only has the performance of ordinary protective clothing, but also has the function of protecting the human body under high temperature conditions[3]. It uses mathematical methods to study thermal protective clothing, aiming to reveal the heat transfer law inside the thermal protective clothing. Provide a theoretical basis for the development of thermal protective clothing[1][2][4][5][6]. At a certain ambient temperature, a partial differential equation model was established by using the heat conduction model to study the heat transfer model of the external temperature through heat protection clothing to the dummy skin under high temperature environment and solved by finite difference method.*

Keywords: *heat conduction, partial differential equation, finite difference method*

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I. Introduction

Thermal protective clothing refers to personal protective equipment worn in high temperature environment, which can promote the body's heat dissipation, prevent heatstroke, burns and burns. Its function is to protect the human body from various heats such as convection and conduction. Heat, radiant heat, molten metal sputtering, and damage from hot steam or hot gases. Among them, a professional garment consists of three layers, which are respectively recorded as layers I, II and III. It is known that the layer I is in direct contact with the outside world, and there is a gap between the layer III and the skin and the gap is recorded as an IV layer. In order to reduce the research and development costs and shorten the research and development cycle to design such special clothing [7] to control the surface temperature of the dummy, this paper establishes a corresponding model to determine the temperature change of the skin outside the family and solve it.

II. The Establishment Of Partial Differential Equations

The heat transfer mainly includes three forms of heat convection, heat conduction and heat radiation. However, the external temperature simulated in this study is much lower than that of thermal radiation (>600K), so the whole heat transfer process ignores the effect of heat radiation. Only heat convection and heat conduction are considered [1]. When the external environment is in contact with protective clothing, this study considers thermal convection; in the three-layer structure of protective clothing, only heat conduction is considered in this paper; in the IV layer, only heat conduction is considered. Because of the complexity of heat transfer in the air layer, according to research [1][3], the heat transfer between the skin and the air layer of the fabric is dominated by the exchange of radiant heat. However, when the thickness of the air layer between the skin and the fabric is less than 8 mm, since the air layer gap is too small, convective motion cannot be formed, and at this time, the heat transfer of the air layer is mainly heat conduction.

2.1.The ambient temperature and the heat transfer of the first layer

According to Newton's law of cooling, when there is a temperature difference between the surface of the object and the surrounding, the heat lost per unit area per unit time is proportional to the temperature difference, and the relationship is $q = h(T_{air} - T_{fab_1})$. Among them, h convective heat transfer coefficient, T_{air} is the ambient temperature, and T_{fab_1} is the zone boundary temperature. For the change of boundary temperature with heat transfer time during heat conduction, it is assumed that the thermal energy density of the I zone boundary is q in unit time and unit area, and the energy value of the boundary is k in unit time and unit length. The relationship between the thermal energy density q , the temperature T , and the time period dt of the boundary under heat conduction conditions is $qdt = c_1 p_1 \Delta T Dx + k_1 dt$. Where c_1 is the specific heat capacity of zone I, p_1 is the density of zone I, dt recorded as a small time period, and □T is the temperature rise of zone I. Get forward difference and get $q\Delta t = c_1 p_1 (T_2 - T_1)\ \Delta x + k_1 \Delta t$,equivalent to $T_2 = \dfrac{q\Delta t}{c_1 p_1 \Delta x + k_1} + T_1$, Among them, T_1 is the surface temperature of the first I zones of time Δt , T_2 is the surface temperature of I zones after Δt hours, and Δx is the microscopic distance (step size).

2.2 Heat transfer between fabrics of layers I, II and III

$\dfrac{\partial T}{\partial t} = a_k \dfrac{\partial T^2}{\partial x^2}, x > 0, t > 0, k = 1,2,3,4$, Among them, $a_k = k_k / (c_k p_k), u = \Delta t / \Delta x^2$,and k_k are the thermal conductivity of each layer, c_k is the specific heat of each layer, and p_k is the density of each layer.The boundary value condition is $T|_{x=0} = T(0,t)\ , t > 0$. The initial value condition is $T|_{t=0} = C\ , 0 \le x \le l$.

So the heat transfer model is $$\begin{cases} \dfrac{\partial T}{\partial t} = a_k \dfrac{\partial T^2}{\partial x^2} \\ T|_{x=0} = T(0,t) \\ T|_{t=0} = C \end{cases}$$.

III. The Solution Of Partial Differential Equations

In this paper, the model is solved by the finite difference method. The basic idea of the finite difference method is to replace the continuous fixed solution region with a grid composed of a finite number of discrete points, where the discrete points are called the nodes of the mesh; the discrete variable functions defined on the mesh are approximated to replace the continuous The function of the continuous variable on the solution region; the derivative of the original equation and the fixed solution condition is approximated by the difference quotient. Then use the interpolation method to obtain the approximation of the solution in the whole region from the discrete solution.

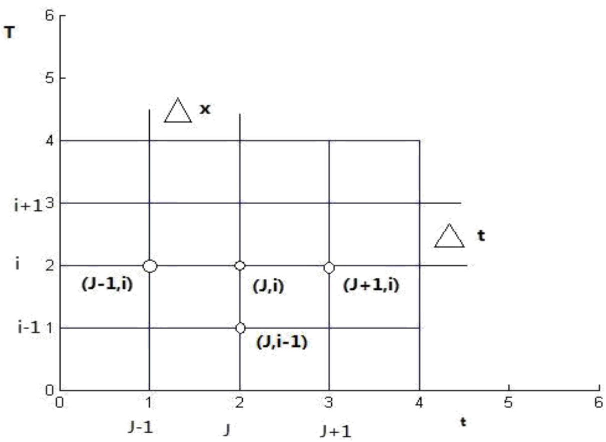

Figure 1 Finite difference method grid diagram

In this regard, we establish a difference equation to replace the original partial differential equation. $\dfrac{T_j^{i+1} - T_j^i}{\Delta t} = a_k \dfrac{T_{j+1}^i - 2T_j^i + T_{j-1}^i}{\Delta x^2}, k = 1,2,3,4$, Simplify the formula above and get

$T_j^{i+1} = T_j^i + a_k u(u_{j+1}^i - 2u_j^i + u_{j-1}^i), k = 1, 2, 3, 4$. The results obtained after iterating over the above equation are shown in Figure 2.

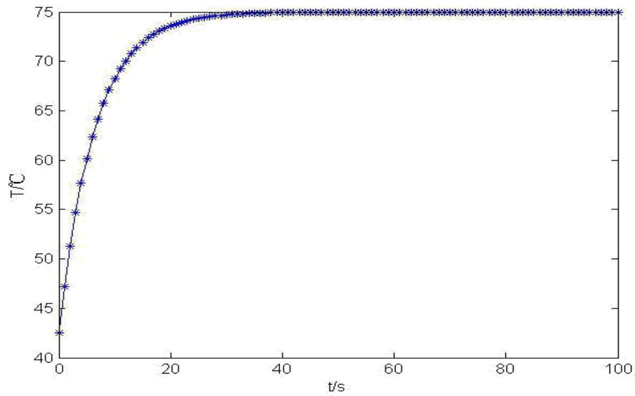

Figure 2 Temperature change with time at x=0

It can be seen from the figure that when x ≈ 37s, the temperature curve tends to be smooth, and the slope of the curve tends to 0 from large to small, which is consistent with the actual situation.
In the matlab environment, the temperature region distribution map can be obtained [8], as shown in Figure 3.

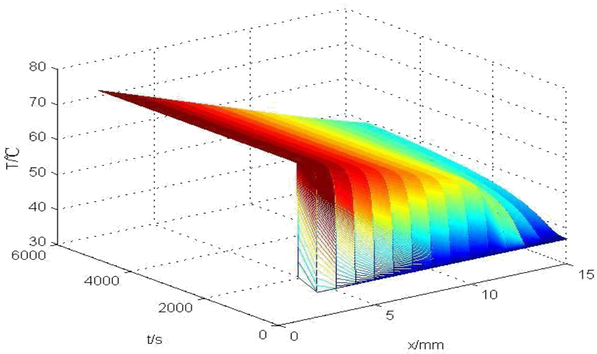

Figure 3 Temperature zone map

When viewed from the x-axis, the initial temperature T = 37 ° C, before 11 mm, solid heat transfer, resulting in a slight difference in temperature trends, after 11 mm, gas heat transfer, resulting in a significant increase in temperature changes.

The temperature change data outside the skin of the dummy is shown in the following table (partial display). For detailed data, see [7].

Temperature change data outside the skin of the dummy

Time	...	1055	1056	1057	1058	1059	1061	...
Temperature	...	41.9918	41.9968	42.0017	42.0067	42.0117	42.0166	...

The temperature change data outside the human skin obtained by solving the partial differential equation is compared with the original data given in [7], as shown in Fig. 4.

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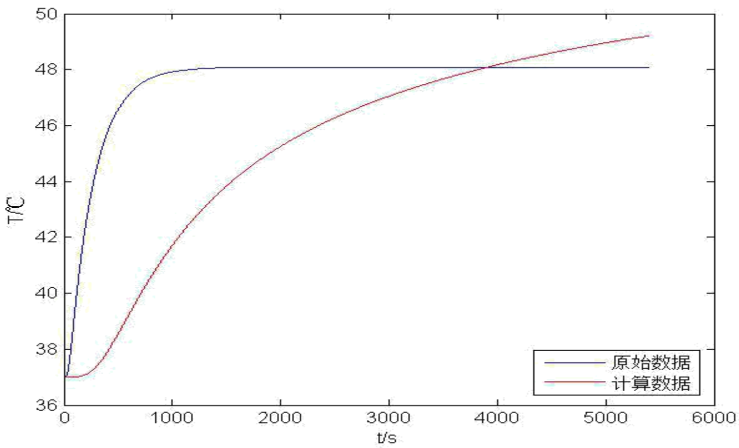

Figure 4 Comparison of temperature changes on the outside of human skin

It can be seen from Fig. 4 that the original data differs greatly from the data calculated by the model because it is found that during the programming process, the air layer cannot converge during the heat conduction process, of which $a_4u = 23.6 > 1/2$, so we have a_4u in the previous formula. Multiplying the convergence factor f=0.0003, f has an accidental factor, so the result is quite different from the original data.

IV. Conclusion

Thermal protective clothing not only has good thermal protection properties, but also requires good performance. However, in the current research and development of thermal protective clothing, various difficulties have been encountered. This article uses a partial differential equation model to determine the temperature outside the skin of the dummy, which is conducive to the accurate design of thermal protective clothing. I hope this paper can provide some help for the thermal protective clothing to control the temperature outside the skin.

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